Learning Targets:
• Identify and represent patterns using models, tables, and expressions.
• Write and evaluate algebraic expressions that represent patterns with constant differences.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Use Manipulatives, Think-Pair-Share

People have been investigating number patterns for thousands of years. Legend has it that Pythagoras and his students arranged pebbles in the sand to represent number patterns. One pattern they studied is shown below.

1. Draw the fourth, fifth and sixth figures.

2. Organize the number of pebbles in each figure into a table.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Pebbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

3. Extend the pattern to determine how many pebbles are in the 10th figure.
20 pebbles

4. Describe the patterns you observe in the pebble drawings and the table in words.
   Answers will vary. Each figure has 2 more pebbles than the previous one.

5. How many pebbles are in the 53rd figure? Explain your reasoning.
   106 pebbles. Sample answer: I multiplied the figure number by 2.

6. Write a numeric expression using the number 3 for the number of pebbles in the third figure.
   2(3)

A legend is a story handed down by tradition that is popularly regarded as historical but unverified.

Activity Standards Focus
Unit 2 focuses on linear equations. Activity 9 introduces some of the ideas that will be needed when analyzing equations by first looking at those ideas in the context of patterns. Before they see algebraic equations, students will be introduced to algebraic expressions as they use them to identify and represent patterns. They will write and evaluate algebraic expressions that represent patterns—some with constant differences and some without. This idea of constant difference will serve as an introduction to the concepts they will come to understand in later activities as rate of change and slope.

Lesson 9-1

PLAN

Pacing: 2 class periods

Chunking the Lesson
#1—5  #6—8
Check Your Understanding
#14  #15—16  #17
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Give students a few minutes to write down examples of patterns in everyday life. Then have volunteers share their ideas with the class. Ask them what makes a sequence a pattern. Explain that this activity will deal with patterns in numbers.

1–5 Look for a Pattern, Use Manipulatives, Create Representations, Think-Pair-Share
Give students a few minutes to use the manipulatives if needed to form figures 4, 5 and 6. Then have them complete the table and extend the pattern by either drawing more figures or adding rows to the table before pairing with a partner to discuss Items 4 and 5. Partners should then share with other pairs to confirm responses and to add to individual patterns observed. Groups should be given an opportunity to share patterns with the entire class. Most students will notice that the number of pebbles increases by 2 each time. Some might say that the numbers of pebbles are the even numbers, 2, 4, 6, and so on. Students who notice that the number of pebbles is twice the figure number are ready to generalize a rule for this pattern.
7. Write a similar numeric expression using the number 7 for the number of pebbles in the seventh figure. 
   2(7)

8. Let \( n \) represent the figure number.
   a. Use \( n \) to write an expression that could be used to determine the number of pebbles in figure \( n \).
   \( 2n \)
   b. What value would you substitute for \( n \) to determine the number of pebbles in the third figure?
   3
   c. Check to see that your expression from part a is correct by evaluating it for the value you chose in part b.
   \( 2(3) \); 6 pebbles
   d. Use your expression to determine the number of pebbles in the 100th figure.
   \( 2(100) \); 200 pebbles

Check Your Understanding
A pattern of small squares is shown. Use the pattern to answer Items 9–13 that follow.

9. How many small squares are in each figure?

10. Draw the fourth, fifth, and sixth figures and determine the number of small squares in each figure.

11. Create a table to organize the number of squares in each figure into a table.

<table>
<thead>
<tr>
<th>Number of Squares</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
</tr>
</tbody>
</table>

12. Answers will vary. The number of squares increases by 2 as the figure number increases by 1.
13. 22 squares. Answers may vary. I multiplied 10 by 2 and then added 2.
Lesson 9–1
Representing Patterns

14. Another pebble arrangement is shown below.

![Pebble Arrangement Diagram](Image)

a. Draw the fourth, fifth, and sixth figures.

b. Organize the number of pebbles in each figure in the table below.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pebbles</th>
<th>Difference in Number of Pebbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>19</td>
<td>2</td>
</tr>
</tbody>
</table>

c. Describe any patterns you observe in the pebble drawings and in the table above.

Sample answer: The number of pebbles in each figure is 1 less than 2 times the figure number.

15. Subtract consecutive terms in the pebbles column and record this information in the last column in the table.

See the table in Item 14 part b.

16. Reason quantitatively. How does the constant difference in the new column relate to the patterns you observed?

Sample answer: Each consecutive figure increases by 2 pebbles. This consistency allows me to observe that the number of pebbles in each figure is one less than twice the figure number.

MINI-LESSON: Evaluating Expressions

For additional student support in evaluating algebraic expressions, a mini-lesson is available. This lesson will feature some simple practice problems in which students evaluate an expression for different values of the same variable, helping them to develop the concept of a function as a set of related input and output values.

See SpringBoard’s eBook Teacher Resources for a student page for this mini-lesson.
### 17. The number of pebbles in a specific figure can be written using repeated additions of the constant difference. For example, the third figure is $1 + 2 + 2$ or $1 + 2(2)$.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pebbles</th>
<th>Expression Using Repeated Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$1 + 2(0)$</td>
</tr>
<tr>
<td>2</td>
<td>1 + 2</td>
<td>$1 + 2(1)$</td>
</tr>
<tr>
<td>3</td>
<td>1 + 2 + 2</td>
<td>$1 + 2(2)$</td>
</tr>
<tr>
<td>4</td>
<td>1 + 2 + 2 + 2</td>
<td>$1 + 2(3)$</td>
</tr>
<tr>
<td>5</td>
<td>1 + 2 + 2 + 2 + 2</td>
<td>$1 + 2(4)$</td>
</tr>
</tbody>
</table>

a. Write the number of pebbles in the fourth and fifth figure using repeated addition of the constant difference.

fourth: $1 + 2 + 2 + 2$ or $1 + 2(3)$

fifth: $1 + 2 + 2 + 2 + 2$ or $1 + 2(4)$

b. Let $n$ represent the figure number. Use $n$ to write an expression that could be used to determine the number of pebbles in figure $n$.

$1 + 2(n - 1)$

c. What value would you substitute for $n$ to determine the number of pebbles in the third figure?

3

d. Check to see that your expression from part a is correct by evaluating it when $n = 5$.

$1 + 2(5 - 1)$; 9 pebbles

e. Use your expression to determine the number of pebbles in the 100th figure.

$1 + 2(100 - 1)$; 199 pebbles
Lesson 9-1
Representing Patterns

Check Your Understanding

18. A pattern of pebbles is shown. Use the pattern to answer parts a–c.

a. Draw the fourth, fifth, and sixth figures.
b. Create a table to organize the number of pebbles in each figure for the first six figures.
c. What is the constant difference?
d. Write the number of pebbles for each figure using repeated addition of the constant difference.
e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

LESSON 9-1 PRACTICE

19. Check to see that your expression from Item 18 part e is correct by evaluating when \( n = 6 \).

20. Use your expression from Item 18 part e to determine the number of pebbles in the 51st figure.

21. Critique the reasoning of others. The expression that represents the number of squares in the \( n \)th figure of a pattern is given by \( 2 + 3(n - 1) \). Mia claims the constant difference is 2. Do you agree? Explain.

Check Your Understanding

Use the Check Your Understanding section to debrief Items 14–17. At this point, students should be comfortable recognizing and identifying expressions for patterns characterized by repeated addition of a constant difference.

Answers

18. a.

b. See table below.
c. 2
d. See table in part b.
e. \( 3 + 2(n - 1) \)

ASSESS

Students’ answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Answers

19. \( 3 + 2(6 - 1) = 13 \)
20. \( 3 + 2(51 - 1) = 103 \)
21. No. Answers may vary. The first 3 terms are 2, 5, and 8, so the constant difference is 3. The first term is 2.

Answers

18. b.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pebbles</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Expression (part d)</td>
<td>( 3 + 2(0) )</td>
<td>( 3 + 2(1) )</td>
<td>( 3 + 2(2) )</td>
<td>( 3 + 2(3) )</td>
<td>( 3 + 2(4) )</td>
<td>( 3 + 2(5) )</td>
</tr>
</tbody>
</table>
22. A pattern of small squares is shown.

b. Draw the fourth, fifth, and sixth figures.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Squares</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>

c. Answers will vary. Each figure has 2 more squares than the previous one.

d. 2

e. 5

f. $3 + 2(n - 1)$

23. Answers may vary. They both have a constant difference of 2. The number of pebbles in the first figure is 1. The number of squares in the first figure is 5.

ADAPT

This lesson has dealt with recognizing patterns characterized by repeated addition of a constant difference, and deriving expressions for those patterns. Use Activity Practice Items 1–3 to evaluate students’ grasp of the concepts involved here. Extra support to help students master these concepts may be provided along with Items 1–2 in the next lesson. These items will review and extend the topic before the lesson moves on to different kinds of patterns.
Lesson 9-2  
Using Patterns to Write and Evaluate Expressions

**Learning Targets:**
- Identify patterns that do not have a constant difference.
- Write and evaluate algebraic expressions that represent patterns that do not have a constant difference.

**SUGGESTED LEARNING STRATEGIES:** Look for a Pattern, Create Representations, Discussion Group, Group Presentation

1. Four different pebble patterns are shown. Your teacher will assign one to your group. Use your selected pattern to answer parts a–e that follow and then prepare a group presentation of your results.

   **Pattern A**
   - Figure 1
   - Figure 2
   - Figure 3

   **Pattern B**
   - Figure 1
   - Figure 2
   - Figure 3

   **Pattern C**
   - Figure 1
   - Figure 2
   - Figure 3

   **Pattern D**
   - Figure 1
   - Figure 2
   - Figure 3

   a. Draw a few additional figures and then organize the information in a table. Identify the constant difference.

   The next figure in each pattern is shown below.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pattern A</th>
<th>Pattern B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pattern C</th>
<th>Pattern D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td>22</td>
</tr>
</tbody>
</table>

   Constant difference: Pattern A: 4, Pattern B: 3, Pattern C: 4, Pattern D: 4

**ACTIVITY 9  Continued**
2. Based on the class’s work for Item 1, how does the constant difference in a pebble pattern relate to the algebraic expression that can be written to describe the pattern? The constant difference is the coefficient of \( n \) in the expression.
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

Check Your Understanding

Tables representing two pebble patterns are shown below.

<table>
<thead>
<tr>
<th>Pebble Pattern A</th>
<th>Pebble Pattern B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Figure</strong></td>
<td><strong>Pebbles</strong></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

3. What is the constant difference for each pebble pattern shown in the tables?
4. For each pebble pattern, use the variable \( n \) to write an expression for the \( n \)th figure.
5. How many pebbles are in the 50th figure for each pebble pattern?
6. Both pebble patterns have 7 pebbles in Figure 2. If the patterns continue, will they ever have the same number of pebbles as another figure? Explain your reasoning.

7. The Pythagoreans also studied the following pebble pattern.

```
  .
  .  .
  .  .  .
  .  .  .  .
  .  .  .  .  .
```

   Figure 1   Figure 2   Figure 3

a. How many pebbles are there in the fourth, fifth, and sixth figures?
   fourth: 16; fifth: 25; sixth: 36

b. Does this pattern have a constant difference? Explain your response.
   No. The difference between the number of pebbles in each figure is not constant.

c. Describe the pattern in words.
   Sample answer: The number of pebbles in each figure is the figure number times itself, or the figure number squared.

d. How many pebbles are there in the 10th figure? How did you determine your answer?
   100 pebbles. Sample answer: I multiplied 10 by itself.

ACTIVITY 9 Continued

Check Your Understanding
Debrief this section of the activity by asking students to explain in their own words how to write an algebraic expression from a visual pattern, a table or a written description of a pattern.

Answers
3. The constant difference for A is 4. The constant difference for B is 5.
4. A: \( 4n - 1 \); B: \( 5n - 3 \)
5. A: \( 4(50) - 1 \); 199 pebbles
   B: \( 5(50) - 3 \); 247 pebbles
6. No. Answers will vary. Pattern B increases by 5 pebbles in each additional figure while Pattern A only increases by 4 pebbles in each additional figure. Therefore, Pattern B will always have more pebbles.

Teacher to Teacher
In this portion of the activity, students explore non-linear patterns represented by figurate numbers. Students will observe that while the differences have a pattern, it is not constant. The quadratic patterns come from relating the figure number to the dimensions of the pebble shape and considering the total number of pebbles as an array. For example, in Item 7, the pebbles are arranged in an \( n \times n \) square.

7 Look for a Pattern, Think-Pair-Share, Create Representations, Sharing and Responding
Students should quickly notice that the difference is not constant but that there is a pattern. Rather than having them extend the pattern of adding the next odd number through the 10th figure, groups should be looking for a way to relate the figure number to the total number of pebbles. If a group is stuck, you might ask them what shape is formed by each pebble arrangement. Alternatively, ask students how many pebbles are in each row and column. As groups share responses, be sure students notice that they should be multiplying two variables together rather than multiplying the variable by a constant.
Lesson 9-2

Using Patterns to Write and Evaluate Expressions

e. How many pebbles are there in the 40th figure? In the nth figure?
   40th: 1,600 pebbles; nth: \(n^2\)

f. The Pythagoreans called the numbers in this pattern the square numbers. Why do you think this is so?
   Sample answer: The number of pebbles in each figure is the square of the figure number. Also, the pebbles in each figure form a square.

8. The numbers in the pebble pattern shown below are called the rectangular numbers.

\[
\text{Figure 1} \quad \text{Figure 2} \quad \text{Figure 3}
\]

a. How many pebbles are in the fourth, fifth, and sixth figures?
   fourth: 20; fifth: 30; sixth: 42

b. Describe the pattern in words. Is there a constant difference? Explain your response.
   The base and height of each consecutive rectangle both increase by 1 pebble. There is no constant difference. The difference between the number of pebbles in each figure is different.

c. Describe how to find the number of pebbles in the 10th figure.
   The base and height of each consecutive rectangle both increase by 1 pebble. So, the base of the 10th rectangle will have 10 pebbles. The rectangle will be 11 pebbles high. The number of pebbles will be 10(11), or 110 pebbles.

d. How many pebbles are there in the 30th figure? In the nth figure?
   30th: 930 pebbles; nth: \(n(n + 1)\)

9. The Pythagoreans called the numbers represented by the pebbles in this pebble pattern the triangular numbers.

\[
\text{Figure 1} \quad \text{Figure 2} \quad \text{Figure 3}
\]

a. Why do you think the Pythagoreans called these numbers triangular?
   Sample answer: The pebbles are arranged in a triangular shape.
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

b. How is the triangular number pebble pattern related to the pebble pattern of the rectangular numbers?
   If each rectangle in the rectangular number pattern is cut in half along the diagonal, it results in two of the same triangular numbers.

c. Use your response to part b to write an algebraic expression for the number of pebbles in the \( n \)th triangular number.
   \[
   \frac{n(n + 1)}{2}
   \]

d. Verify your expression by substituting \( n = 4 \). Is the result the number of pebbles in the fourth triangular number?
   Yes, \( \frac{4(4 + 1)}{2} = 10 \) pebbles

e. Use your expression to predict the number of pebbles in the 30th triangular number.
   \[
   \frac{30(30 + 1)}{2} = 465 \text{ pebbles}
   \]

Check Your Understanding

10. Is the number 72 a square number, rectangular number, or triangular number? Explain your reasoning.
   Use the figures below to answer Items 11–15.

   ![Figure 1](image1)
   ![Figure 2](image2)
   ![Figure 3](image3)

11. Assume each side of each pentagon is 1 cm. What is the perimeter of each figure shown?

12. Draw the next three figures and determine the perimeter of each.

13. Organize the results of Items 11 and 12 in a table. What would be the perimeter of the 10th figure? Explain your reasoning.

14. Use \( n \) to represent the figure number. Write an expression that could be used to determine the perimeter of the \( n \)th figure.

15. Use your expression to determine the perimeter of the 50th figure.

ACTIVITY 9
Check Your Understanding

Debrief this section of the activity by having students share within their groups their individual opinions. Groups should then share the consensus with the class. Focus should be on students’ explanations.

Answers

10. Rectangular. It would be the 8th rectangular figure. This would give it a base of 8 and a height of 9. \( 8(9) = 72 \) pebbles.

11. Figure 1: 5 cm; Figure 2: 8 cm; Figure 3: 11 cm

12. Figure 4 Figure 5 Figure 6
   Perimeter of figure 4: 14 cm; Perimeter of figure 5: 17 cm; Perimeter of figure 6: 20 cm

13. 32 cm; Begin with a perimeter of 5 cm in Figure 1 and add 3 nine times.

14. \( 5 + 3(n - 1) \) OR \( 3n + 2 \)

15. \( 3(50) + 2 = 152 \text{ cm} \)
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

ASSESS

Students’ answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning. See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Answers

16. Answers may vary. Nate does not realize the base and height of a rectangular number differ by 1. He is correct that 56 is a rectangular number. However the reason is the base is 7 and the height is 8.

17. 36; Answers may vary. A square number is the figure number squared. I squared 6.

18. \(45 = \frac{9(9 + 1)}{2}\)

19. a. Figure 1: 8 ft; Figure 2: 14 ft; Figure 3: 20 ft
   
   b.  
   Figure 4: 26 ft  
   Figure 5: 32 ft  
   Figure 6: 38 ft  

c. \(8 + 6(n - 1)\)

d. \(8 + 6(6 - 1) = 38\)

20. Model with mathematics. Octagonal blocks are being used to make a walkway along a garden. Use your expression from Item 19 part c to find the perimeter of the walkway if 30 octagonal blocks, each side 1 foot long, are used for the walkway.
Lesson 9-1

1. Use the figures below to answer parts a–c.

   Figure 1  Figure 2  Figure 3

   a. What is the perimeter of each figure shown?
      Assume each side is 1 unit.
   b. Draw the next three figures.
   c. What would be the perimeter of the 10th figure? Justify your response.

2. Write an expression, using the variable n, that could be used to determine the perimeter of the nth figure in Item 1. Use the expression to determine the perimeter of the 50th figure.

3. A pattern of pebbles is shown.

   Figure 1  Figure 2  Figure 3

   a. Draw the fourth, fifth, and sixth figures.
   b. Create a table showing the number of the figure and the number of pebbles in each figure.
   c. Describe the patterns you observe in the pebble drawings and the table in words.
   d. What is the constant difference?
   e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in the nth figure.
   f. Use the expression in part e to determine the number of pebbles in the 51st figure.
4. a. Figure 1: 4 square units; Figure 2: 12 square units; Figure 3: 24 square units
   b. What is the area of each figure if each small square has an area of 1 unit?
   c. 220 square units. Figure 10 would be 11 squares by 20 squares
5. \((n + 1)(2n)\); 2520 square units
6. a. Figure 1: 8, Figure 2: 14; Figure 3: 20; Figure 4: 26; Figure 5: 32; Figure 6: 38
   b. Yes; Answer may vary. The perimeters differ by 6, so the constant difference is 6.
   c. \(2 + 6n\)
   d. \(2 + 6(35) = 212\)
7. a. 
   b. \(\frac{10(10 + 1)}{2}\); 55 dots

**ADDITIONAL PRACTICE**
If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.
Writing Expressions
Pebbles in the Sand
Lesson 9-1 Representing Patterns

Learning Targets:
- Identify and represent patterns using models, tables, and expressions.
- Write and evaluate algebraic expressions that represent patterns with constant differences.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Use Manipulatives, Think-Pair-Share

People have been investigating number patterns for thousands of years. Legend has it that Pythagoras and his students arranged pebbles in the sand to represent number patterns. One pattern they studied is shown below.

1. Draw the fourth, fifth and sixth figures.

2. Organize the number of pebbles in each figure into a table.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Pebbles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Extend the pattern to determine how many pebbles are in the 10th figure.

4. Describe the patterns you observe in the pebble drawings and the table in words.

5. How many pebbles are in the 53rd figure? Explain your reasoning.

6. Write a numeric expression using the number 3 for the number of pebbles in the third figure.

Activity 9 • Writing Expressions
Lesson 9-1
Representing Patterns

7. Write a similar numeric expression using the number 7 for the number of pebbles in the seventh figure.

8. Let $n$ represent the figure number.
   a. Use $n$ to write an expression that could be used to determine the number of pebbles in figure $n$.
   b. What value would you substitute for $n$ to determine the number of pebbles in the third figure?
   c. Check to see that your expression from part a is correct by evaluating it for the value you chose in part b.
   d. Use your expression to determine the number of pebbles in the 100th figure.

MATH TERMS
An expression is a mathematical phrase using numbers or variables or both. $1 + 1$ and $3x - 5$ are examples of expressions.

You do not solve an expression; you evaluate it for a specific value. To do this, substitute a value for the variable and simplify.

Check Your Understanding
A pattern of small squares is shown. Use the pattern to answer Items 9–13 that follow.

9. How many small squares are in each figure?
10. Draw the fourth, fifth, and sixth figures and determine the number of small squares in each figure.
11. Create a table to organize the number of squares in each figure into a table.
12. Describe in words the patterns you see in the square pattern and in the table.
13. How many squares would be in the 10th figure? Explain your reasoning.
Lesson 9-1
Representing Patterns

14. Another pebble arrangement is shown below.

Figure 1
Figure 2
Figure 3

a. Draw the fourth, fifth, and sixth figures.

b. Organize the number of pebbles in each figure in the table below.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pebbles</th>
<th>Difference in Number of Pebbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Describe any patterns you observe in the pebble drawings and in the table above.

15. Subtract *consecutive terms* in the pebbles column and record this information in the last column in the table.

<table>
<thead>
<tr>
<th>n</th>
<th>term</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

16. Reason quantitatively. How does the *constant difference* in the new column relate to the patterns you observed?

MATH TERMS

*Consecutive terms* follow each other directly in a sequence. In the table below, 5 and 8 are *consecutive terms*. There is a *constant difference* between the terms in this table that is equal to 3.
Lesson 9-1
Representing Patterns

17. The number of pebbles in a specific figure can be written using repeated additions of the constant difference. For example, the third figure is $1 + 2 + 2$ or $1 + 2(2)$.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pebbles</th>
<th>Expression Using Repeated Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$1 + 2(0)$</td>
</tr>
<tr>
<td>2</td>
<td>$1 + 2$</td>
<td>$1 + 2(1)$</td>
</tr>
<tr>
<td>3</td>
<td>$1 + 2 + 2$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Write the number of pebbles in the fourth and fifth figure using repeated addition of the constant difference.

b. Let $n$ represent the figure number. Use $n$ to write an expression that could be used to determine the number of pebbles in figure $n$.

c. What value would you substitute for $n$ to determine the number of pebbles in the third figure?

d. Check to see that your expression from part a is correct by evaluating it when $n = 5$.

e. Use your expression to determine the number of pebbles in the 100th figure.
Lesson 9-1
Representing Patterns

Check Your Understanding

18. A pattern of pebbles is shown. Use the pattern to answer parts a–c.

![Figure 1](image1)
![Figure 2](image2)
![Figure 3](image3)

a. Draw the fourth, fifth, and sixth figures.
b. Create a table to organize the number of pebbles in each figure for the first six figures.
c. What is the constant difference?
d. Write the number of pebbles for each figure using repeated addition of the constant difference.
e. Let \( n \) represent the figure number. Use \( n \) to write an expression that could be used to determine the number of pebbles in figure \( n \).

LESSON 9-1 PRACTICE

19. Check to see that your expression from Item 18 part e is correct by evaluating when \( n = 6 \).

20. Use your expression from Item 18 part e to determine the number of pebbles in the 51st figure.

21. Critique the reasoning of others. The expression that represents the number of squares in the \( n \)th figure of a pattern is given by \( 2 + 3(n - 1) \). Mia claims the constant difference is 2. Do you agree? Explain.
Lesson 9-1
Representing Patterns

22. A pattern of small squares is shown.

![Figure 1](image1.png) ![Figure 2](image2.png) ![Figure 3](image3.png)

**a.** Draw the fourth, fifth, and sixth figures.
**b.** Create a table showing the number of the figure and the number of squares in each figure.
**c.** Describe the patterns you observe in the square drawings and in your table.
**d.** What is the constant difference?
**e.** How many small squares are in the first figure?
**f.** Let \( n \) represent the figure number. Use \( n \) to write an expression that could be used to determine the number of squares in the \( n \)th figure.

23. Compare the expression you found in Item 22 part f with the expression you found in Item 17 part b. How are they the same? How are they different?
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

Learning Targets:
• Identify patterns that do not have a constant difference.
• Write and evaluate algebraic expressions that represent patterns that do not have a constant difference.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Group, Group Presentation

1. Four different pebble patterns are shown. Your teacher will assign one to your group. Use your selected pattern to answer parts a–e that follow and then prepare a group presentation of your results.

   **Pattern A**
   - Figure 1
   - Figure 2
   - Figure 3

   **Pattern B**
   - Figure 1
   - Figure 2
   - Figure 3

   **Pattern C**
   - Figure 1
   - Figure 2
   - Figure 3

   **Pattern D**
   - Figure 1
   - Figure 2
   - Figure 3

   a. Draw a few additional figures and then organize the information in a table. Identify the constant difference.
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

b. Describe the pattern in words.

c. **Reason abstractly.** Write an expression using the variable \( n \) that could be used to determine the number of pebbles in figure \( n \).

d. Use your expression to determine the number of pebbles in the 10th, 53rd, and 200th figures.

<table>
<thead>
<tr>
<th>Figure</th>
<th>10th</th>
<th>53rd</th>
<th>200th</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e. For the pattern you selected, is it possible to have a figure with 100 pebbles? Explain your reasoning.

2. Based on the class’s work for Item 1, how does the constant difference in a pebble pattern relate to the algebraic expression that can be written to describe the pattern?
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

Check Your Understanding

Tables representing two pebble patterns are shown below.

<table>
<thead>
<tr>
<th>Pebble Pattern A</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Pebbles</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pebble Pattern B</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Pebbles</td>
<td>2</td>
<td>7</td>
<td>12</td>
<td>17</td>
<td>22</td>
</tr>
</tbody>
</table>

3. What is the constant difference for each pebble pattern shown in the tables?
4. For each pebble pattern, use the variable \( n \) to write an expression for the \( n \)th figure.
5. How many pebbles are in the 50th figure for each pebble pattern?
6. Both pebble patterns have 7 pebbles in Figure 2. If the patterns continue, will they ever have the same number of pebbles as another figure? Explain your reasoning.

7. The Pythagoreans also studied the following pebble pattern.

```
  .  
 Figure 1

  ..  
 Figure 2

  ..  
 Figure 3
```

a. How many pebbles are there in the fourth, fifth, and sixth figures?

b. Does this pattern have a constant difference? Explain your response.

c. Describe the pattern in words.

d. How many pebbles are there in the 10th figure? How did you determine your answer?
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

e. How many pebbles are there in the 40th figure? In the \(n\)th figure?

f. The Pythagoreans called the numbers in this pattern the square numbers. Why do you think this is so?

8. The numbers in the pebble pattern shown below are called the rectangular numbers.

- [Figure 1]
- [Figure 2]
- [Figure 3]

a. How many pebbles are in the fourth, fifth, and sixth figures?

b. Describe the pattern in words. Is there a constant difference? Explain your response.

c. Describe how to find the number of pebbles in the 10th figure.

d. How many pebbles are there in the 30th figure? In the \(n\)th figure?

9. The Pythagoreans called the numbers represented by the pebbles in this pebble pattern the triangular numbers.

- [Figure 1]
- [Figure 2]
- [Figure 3]

a. Why do you think the Pythagoreans called these numbers triangular?
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

b. How is the triangular number pebble pattern related to the pebble pattern of the rectangular numbers?

c. Use your response to part b to write an algebraic expression for the number of pebbles in the \( n \)th triangular number.

d. Verify your expression by substituting \( n = 4 \). Is the result the number of pebbles in the fourth triangular number?

e. Use your expression to predict the number of pebbles in the 30th triangular number.

Check Your Understanding

10. Is the number 72 a square number, rectangular number, or triangular number? Explain your reasoning. Use the figures below to answer Items 11–15.

11. Assume each side of each pentagon is 1 cm. What is the perimeter of each figure shown?

12. Draw the next three figures and determine the perimeter of each.

13. Organize the results of Items 11 and 12 in a table. What would be the perimeter of the 10th figure? Explain your reasoning.

14. Use \( n \) to represent the figure number. Write an expression that could be used to determine the perimeter of the \( n \)th figure.

15. Use your expression to determine the perimeter of the 50th figure.
Lesson 9-2
Using Patterns to Write and Evaluate Expressions

LESSON 9-2 PRACTICE

16. Critique the reasoning of others. Nate claims 56 is a rectangular number because a rectangle with base 4 and height 14 can be formed. What is his error?

17. What is the sixth square number? How did you get your answer?

18. Use the expression from Item 9 part c to show that 45 is a triangular number.

19. Use the figures below to answer parts a–d. Assume each figure is a regular octagon with sides of 1 foot.

   a. What is the perimeter of each figure?
   b. Draw the next three figures and determine the perimeter of each.
   c. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.
   d. Verify the expression in part c by substituting n = 6.

20. Model with mathematics. Octagonal blocks are being used to make a walkway along a garden. Use your expression from Item 19 part c to find the perimeter of the walkway if 30 octagonal blocks, each side 1 foot long, are used for the walkway.
**ACTIVITY 9 PRACTICE**

**Writing Expressions**

**Pebbles in the Sand**

**Lesson 9-1**

1. Use the figures below to answer parts a–c.

![Figure 1](image1.png)  ![Figure 2](image2.png)  ![Figure 3](image3.png)

   **a.** What is the perimeter of each figure shown? Assume each side is 1 unit.

   **b.** Draw the next three figures.

   **c.** What would be the perimeter of the 10th figure? Justify your response.

2. Write an expression, using the variable \( n \), that could be used to determine the perimeter of the \( n \)th figure in Item 1. Use the expression to determine the perimeter of the 50th figure.

3. A pattern of pebbles is shown.

![Figure 4](image4.png)  ![Figure 5](image5.png)  ![Figure 6](image6.png)

   **a.** Draw the fourth, fifth, and sixth figures.

   **b.** Create a table showing the number of the figure and the number of pebbles in each figure.

   **c.** Describe the patterns you observe in the pebble drawings and the table in words.

   **d.** What is the constant difference?

   **e.** Let \( n \) represent the figure number. Use \( n \) to write an expression that could be used to determine the number of pebbles in the \( n \)th figure.

   **f.** Use the expression in part e to determine the number of pebbles in the 51st figure.
Lesson 9-2

4. Use the pattern of unit squares shown to answer parts a–c.

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Figure 2</th>
<th>Figure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Figure 1" /></td>
<td><img src="image2" alt="Figure 2" /></td>
<td><img src="image3" alt="Figure 3" /></td>
</tr>
</tbody>
</table>

a. What is the area of each figure if each small square has an area of 1 unit?
b. Draw the next three figures in the pattern and determine the area of each.
c. What would be the area of the 10th figure? Justify your response.

5. Write an expression that could be used to determine the area of the $n$th figure in Item 4. Use the expression to determine the area of the 35th figure.

6. Use the figures from Item 4 to answer parts a–c.
   a. What is the perimeter of each of the six figures?
   b. Is there a constant difference? Explain.
   c. Write an expression that could be used to find the perimeter of the $n$th figure.
   d. Use the expression in part c to find the perimeter of the 35th figure.

MATHEMATICAL PRACTICES

Model with Mathematics

7. Use the expression $\frac{n(n + 1)}{2}$.
   a. Create a pattern using circles or dots and show the first three figures.
   b. Determine the number of circles or dots in the 10th figure. Explain how you determined the number of dots in this figure.