Greg Carpenter works for the Greene Construction Company. The company is building a new recreation hall, and the roof of the hall will be supported by triangular trusses, like the ones shown below.

Each of the trusses contains pairs of congruent triangles. Greg’s boss tells him that his first job will be to determine the side lengths and angle measures in the triangles that make up one of the trusses.

Activity Standards Focus
In Activity 11, students explore congruence using a transformational approach. They investigate triangle congruence criteria, including SSS, SAS, SSA, AAS, and HL. After writing proofs for each of the triangle congruence criteria, students apply them to solve real-world problems.

Lesson 11-1

PLAN

Materials
- Ruler
- Protractor
- Tracing paper, patty paper, or acetate; or coffee stirrers, stapler, and scissors

Pacing: 1 class period

 Chunking the Lesson
#1–3 Example A
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Determine whether or not each transformation is a rigid motion.
1. a reflection across a line [rigid motion]
2. a rotation about a point [rigid motion]
3. a dilation by a scale factor of 3 [not a rigid motion]

ELL Support
To begin the unit, all students should have an understanding of the properties of a truss. A truss is used to support the roof of a structure. It is in the shape of a triangle and is usually built from wood or steel.

Introduction Close Reading, Activating Prior Knowledge
Have students read the introductory paragraphs in their groups. Elicit from students that a truss is made up of smaller triangles. Point out in the Connect to Careers signal box that triangles are strong and rigid because their angles and sides do not change under pressure.

Common Core State Standards for Activity 11

HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding sides and corresponding angles are congruent.

HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Greg wonders, "If I know that two triangles are congruent, and I know the side lengths and angle measures in one triangle, do I have to measure all the sides and angles in the other triangle?"

Greg begins by examining two triangles from a truss. According to the manufacturer, the two triangles are congruent.

1. Because the two triangles are congruent, can one triangle be mapped onto the other? If yes, what are the criteria for the mapping?

Each triangle can be mapped onto the other triangle by a sequence of rigid motions.

2. Suppose you use a sequence of rigid motions to map $\triangle ABC$ to $\triangle DEF$. Find the image of each of the following under this sequence of transformations.

$\triangle ABC \rightarrow \triangle DEF$

$A \rightarrow D$

$B \rightarrow E$

$C \rightarrow F$

3. Make use of structure. What is the relationship between $AB$ and $DE$? What is the relationship between $\angle B$ and $\angle E$? How do you know?

$AB \cong DE$ and $\angle B \cong \angle E$ because there is a sequence of rigid motions that maps $AB$ to $DE$ and a sequence of rigid motions that maps $\angle B$ to $\angle E$.

The triangles from the truss that Greg examined illustrate an important point about congruent triangles. In congruent triangles, corresponding pairs of sides are congruent and corresponding pairs of angles are congruent. These are corresponding parts.

When you write a congruence statement like $\triangle ABC \cong \triangle DEF$, you write the vertices so that corresponding parts are in the same order. So, you can conclude from this statement that $AB \cong DE$, $BC \cong EF$, $AC \cong DF$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. 
Lesson 11-1
Congruent Triangles

Example A
For the truss shown below, Greg knows that \( \triangle JKL \cong \triangle MNP \).

\[
\begin{align*}
J & \quad L \\
K & \quad P \\
2.1 \text{ m} & \\
M & \quad N
\end{align*}
\]

Greg wants to know if there are any additional lengths or angle measures that he can determine.

Since \( \triangle JKL \cong \triangle MNP \), \( KL \cong NP \). This means \( KL = NP \), so \( NP = 2.1 \text{ m} \).

Also, since \( \triangle JKL \cong \triangle MNP \), \( \angle K \cong \angle N \). This means \( m\angle K = m\angle N \), so \( m\angle K = 50^\circ \).

Try These A
In the figure, \( \triangle RST \cong \triangle XYZ \). Find each of the following, if possible.

\[
\begin{align*}
a. & \quad m\angle X \\
& \quad \text{not possible} \\
b. & \quad YZ \\
& \quad 15 \text{ cm} \\
c. & \quad m\angle T \\
& \quad 47^\circ \\
d. & \quad XZ \\
& \quad \text{not possible} \\
e. & \quad \text{Both } \triangle JKL \text{ and } \triangle MNP \text{ are equilateral triangles in which the measure of each angle is } 60^\circ \text{. Can you tell whether or not } \triangle JKL \cong \triangle MNP? \\
& \quad \text{No. All equilateral triangles have the same angle measures, but can have different side measures. The corresponding sides will always be proportional, but may not be equal. Therefore, the triangles may not be congruent.}
\end{align*}
\]

MATH TIP
Two line segments are congruent if and only if they have the same length. Two angles are congruent if and only if they have the same measure.

Differentiating Instruction
It may be beneficial for some students to explore congruence in the truss structure using hands-on strategies. For example, students could use patty paper to trace the truss diagram. Then, by turning and folding the paper, they could identify rotations and reflections that map various triangles onto each other. Encourage students to label the vertices so that they more easily identify corresponding parts.
ACTIVITY 11 Continued

Check Your Understanding

Debrief students’ answers to these items to ensure that they understand concepts related to corresponding parts of congruent triangles. When students answer Item 7, you may also want to have them explain how they arrived at their answers.

Answers
4. Yes. Congruent triangles have side lengths with the same measures, and therefore the sum of their side lengths will be the same.
5. No. Congruent triangles have the same angle measures. Since all angles of an acute triangle are acute angles, and since one angle of a right triangle is a right angle, the angles of the two triangles do not have the same measures, and therefore are not congruent.
6. a. CD
   b. DA
   c. ∠DCA
   d. ∠CAD
7. a. TF
   b. TR
   c. ∠I
   d. ∠P

ASSESS

Students’ answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 11-1 PRACTICE

8. 18 in.
9. 22 in.
10. 73 in.
11. Yes. Since ∆XYZ is congruent to ∆TUV, corresponding side lengths have the same measure. In a congruence statement corresponding parts are in the same position, so XY corresponds to TU. Since XY is the longest side in ∆XYZ, TU is the longest side in ∆TUV.

LEsson 11-1 Practice

In the figure, ∆ABC ≅ ∆DEF.

8. Find the length of AB.
9. Find the measure of all angles in ∆DEF that it is possible to find.
10. What is the perimeter of ∆DEF? Explain how you know.
11. Construct viable arguments. Suppose ∆XYZ ≅ ∆TUV and that XY is the longest side of ∆XYZ. Is it possible to determine which side of ∆TUV is the longest? Explain.

ADAPT

Check students’ answers to the Lesson Practice to ensure that they understand basic concepts related to finding measures of corresponding parts of congruent figures. Remind students that the triangle congruence statement can help them identify the corresponding vertices.

SpringBoard® Mathematics Geometry, Unit 2 • Transformations, Triangles, and Quadrilaterals
Learning Targets:
• Develop criteria for proving triangle congruence.
• Determine which congruence criteria can be used to show that two triangles are congruent.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Use Manipulatives, Think-Pair-Share

As you have seen, congruent triangles have six pairs of congruent corresponding parts. The converse of this statement is also true. That is, if two triangles have three pairs of congruent corresponding sides and three pairs of congruent corresponding angles, the triangles are congruent.

Greg’s boss asks him to check that two triangles in a truss are congruent. Greg wonders, “Must I measure and compare all six parts of both triangles?” He decides that a shortcut will allow him to conclude that two triangles are congruent without checking all six pairs of corresponding parts.

1. Greg begins by checking just one pair of corresponding parts of the two triangles.
   a. In your group, each student should draw a triangle that has a side that is 2 inches long. The other two sides can be any measure. Draw the triangles on acetate or tracing paper.
      Check students’ work

   b. To check whether two triangles are congruent, place the sheets of acetate on the desk. If the triangles are congruent, you can use a sequence of translations, reflections, and rotations to map one triangle onto the other.
      Are all of the triangles congruent to each other? Why or why not?
      The triangles are not congruent. No sequence of rigid motions will map one triangle to another.

   c. Cite your results from part b to prove or disprove this statement: “If one part of a triangle is congruent to a corresponding part of another triangle, then the triangles must be congruent.”
      Students’ triangles should serve as counterexamples to disprove the statement.

MATH TIP
A counterexample is a single example that shows that a statement is false.
Now Greg wonders if checking two pairs of corresponding parts suffices to show that two triangles are congruent.

2. Greg starts by considering $\triangle ABC$ below.

   a. Draw triangles that each have one side congruent to $AB$ and another side congruent to $AC$. Use transformations to check whether every triangle is congruent to $\triangle ABC$. Explain your findings.

   Every such triangle is not congruent to $\triangle ABC$. The angle that corresponds to $\angle A$ can vary between 0 and 180 degrees.

   b. Draw triangles that each have an angle congruent to $\angle A$ and an adjacent side congruent to $AB$. Is every such triangle congruent to $\triangle ABC$? Explain.

   No. The side that corresponds to $AC$ can have any length.

   c. Draw triangles that each have an angle congruent to $\angle A$ and an opposite side congruent to $CB$. Is every such triangle congruent to $\triangle ABC$? Explain.

   No; see the drawing below.

   d. Draw triangles that each have an angle congruent to $\angle A$ and an angle congruent to $\angle B$. Is every such triangle congruent to $\triangle ABC$? Explain.

   No. These triangles all have the same shape but may have different sizes.

   e. Consider the statement: “If two parts of one triangle are congruent to the corresponding parts in a second triangle, then the triangles must be congruent.” Prove or disprove this statement. Cite the triangles you constructed.

   Disprove. Students’ triangles serve as counterexamples.
Lesson 11-2
Congruence Criteria

Greg decides that he must have at least three pairs of congruent parts in order to conclude that two triangles are congruent. In order to work more efficiently, he decides to make a list of all possible combinations of three congruent parts.

3. Greg uses A as an abbreviation to represent angles and S to represent sides. For example, if Greg writes SAS, it represents two sides and the included angle, as shown in the first triangle below. Here are the combinations in Greg’s list: SAS, SSA, ASA, AAS, SSS, and AAA.

a. Mark each triangle below to illustrate the combinations in Greg’s list. Answers may vary. Sample answers.

   ![SAS, SSA, ASA, AAS, SSS, AAA triangles]

b. Are there any other combinations of three parts of a triangle? If so, is it necessary for Greg to add these to his list? Explain. SAA and ASS. It is not necessary to add these to the list since SAA is the same as AAS and ASS is the same as SSA.

© 2015 College Board. All rights reserved.

Activity 11 • Congruence Transformations and Triangle Congruence
ACTIVITY 11 Continued

4 Debriefing, Use Manipulatives

Provide students with materials for copying angles and segments. There are many different materials that can be used: tracing or patty paper, acetate, and coffee stirrers are a few suggestions. To use acetate or tracing paper, each student will need six separate pieces. Students should carefully copy the given figures, one per piece, onto the paper or acetate. They should then use the figure numbers to label each figure. To use coffee stirrers, students cut a coffee stirrer to the same length as a segment. To copy an angle, students put two coffee stirrers together to form an angle the same size as those in the figures shown, and then staple the stirrers together at the vertex. Reinforce the understanding that when copying an angle, the lengths of the sides of the angle may vary, but the degree measure of the angle must remain constant.

Lesson 11-2
Congruence Criteria

Now Greg wants to find out which pairs of congruent corresponding parts guarantee congruent triangles.

4. Three segments congruent to the sides of ΔABC and three angles congruent to the angles of ΔABC are given in Figures 1–6, shown below.

a. If needed, use manipulatives supplied by your teacher to recreate the six figures given below. Check students’ figures

b. Identify which of the figures in part a is congruent to each of the parts of ΔABC.

△A: Figure 1
△B: Figure 3
△C: Figure 2
AB: Figure 4
AC: Figure 5
BC: Figure 6
5. For each combination in Greg’s list in Item 3, choose three appropriate triangle parts from Item 4. Each student should create a triangle using these parts. Then use transformations to check whether every such triangle is congruent to $\triangle ABC$. Use the table to organize your results.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Name the Three Figures Used by Listing the Figure Numbers</th>
<th>Is Every Such Triangle Congruent to $\triangle ABC$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSS</td>
<td>4, 5, 6</td>
<td>Yes</td>
</tr>
<tr>
<td>SAS</td>
<td>4, 1, 5, 5, 2, 6, 6, 3, 4</td>
<td>Yes</td>
</tr>
<tr>
<td>ASA</td>
<td>1, 5, 2, 2, 6, 3, 3, 4</td>
<td>Yes</td>
</tr>
<tr>
<td>AAS</td>
<td>1, 2, 6, 1, 3, 6, 2, 1, 4, 3, 2, 4, 2, 3, 4, 3, 2, 5</td>
<td>Yes</td>
</tr>
<tr>
<td>AAA</td>
<td>1, 2, 3</td>
<td>No</td>
</tr>
<tr>
<td>SSA</td>
<td>1, 5, 6, 2, 6, 4, 4, 5, 2, 3, 6, 4, 2, 6, 4, 1</td>
<td>No</td>
</tr>
</tbody>
</table>

6. Express regularity in repeated reasoning. Compare your results from Item 5 with those of students in other groups. Then list the different combinations that seem to guarantee a triangle congruent to $\triangle ABC$. These combinations are called **triangle congruence criteria**. SSS, SAS, ASA, AAS

7. Do you think there is an AAA triangle congruence criterion? Why or why not? No. We have shown that it is possible for two triangles to have three pairs of congruent corresponding angles but not be congruent.

**ACADEMIC VOCABULARY**

A **criterion** (plural: criteria) is a standard or rule on which a judgment can be based. Criteria exist in every subject area. For example, a scientist might use a set of criteria to determine whether a sample of water is safe for human consumption.
Greg realizes that it is not necessary to check all six pairs of corresponding parts to determine if two triangles are congruent. The triangle congruence criteria can be used as “shortcuts” to show that two triangles are congruent.

Example A
For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a. Since three pairs of corresponding sides are congruent, the triangles are congruent by SSS.

b. Although they are not marked as such, the vertical angles in the figure are congruent. The triangles are congruent by ASA.

Try These A
For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a. None

b. SAS

You can use theorems about vertical angles, midpoints, bisectors, and parallel lines that are cut by a transversal to identify additional congruent parts that may not be marked in the figure.

ELL Support
Understanding and using the terms criterion and its plural criteria may be challenging for most students, in part because the plural is not formed in the usual way (by adding an s). Reinforce for ELL students that criterion and criteria are the singular and plural forms of the same word.

MATH TIP
You can use theorems about vertical angles, midpoints, bisectors, and parallel lines that are cut by a transversal to identify additional congruent parts that may not be marked in the figure.
Lesson 11-2
Congruence Criteria

Check Your Understanding

8. Two triangles each have two sides that are 8 feet long and an included angle of 50°. Must the two triangles be congruent? Why or why not?
9. Two equilateral triangles each have a side that is 5 cm long. Is it possible to conclude whether or not the triangles are congruent? Explain.
10. The figure shows a circle and two triangles. For both triangles, two vertices are points on the circle and the other vertex is the center of the circle. What information would you need in order to prove that the triangles are congruent?

LESSON 11-2 PRACTICE
For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

11. none
12. SSS
13. ASA
14. ASA
15. Make sense of problems. What one additional piece of information do you need, if any, in order to conclude that \( \triangle ABD \cong \triangle CBD \)? Is there more than one set of possible information? Explain.

16. \( \triangle ABD \cong \triangle CBD \)

ACTIVITY 11

Check Your Understanding

Debrief students’ answers to these items to ensure that they understand concepts related to triangle congruence criteria. When students answer Item 10, you may also want to have them explain how they arrived at their answer.

Answers

8. Yes. Two corresponding sides and the corresponding included angle are congruent, so the triangles are congruent by SAS congruence.
9. Yes. Sample answer. An equilateral triangle has three sides with the same measure. If one side of an equilateral triangle measures 5 cm, then all sides measure 5 cm. Because the two triangles both have sides measuring 5 cm, the triangles are congruent by SSS congruence.
10. If the distance between the two vertices on the circle are the same for both triangles, then the triangles are congruent by SSS congruence. Also, if the vertex at the center of the circle has the same measure for both triangles, then the triangles are congruent by SAS congruence.

ASSESS

Students’ answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning. See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

ADAPT

Check students’ answers to the Lesson Practice to ensure that they understand basic concepts related to triangle congruence criteria. Remind students that there is no SSA triangle congruence criterion.
Lesson 11-3

ACTIVITY 11 Continued

Learning Targets:
- Prove that congruence criteria follow from the definition of congruence.
- Use the congruence criteria in simple proofs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think Aloud, Discussion Groups, Visualization

You can use the SSS, SAS, or ASA congruence criteria as shortcuts to show that two triangles are congruent. In order to prove why these criteria work, you must show that they follow from the definition of congruence in terms of rigid motions.

1. To justify the SSS congruence criterion, consider the two triangles below.

   ![Triangles](image)

   a. What given information is marked in the figure? \( \overline{AB} \cong \overline{DE} \), \( \overline{AC} \cong \overline{DF} \), and \( \overline{BC} \cong \overline{EF} \)

   b. Based on the definition of congruence, what do you need to do in order to show that \( \triangle ABC \cong \triangle DEF \)? Show that there is a sequence of rigid motions that maps \( \triangle ABC \) to \( \triangle DEF \).

   c. It is given that \( \overline{AB} \cong \overline{DE} \). What does this tell you? There is a sequence of rigid motions that maps \( \overline{AB} \) to \( \overline{DE} \).

   d. Draw a figure to show the result of the sequence of rigid motions that maps \( \overline{AB} \) to \( \overline{DE} \). Assume that this sequence of rigid motions does not map point \( C \) to point \( F \).

   e. Which points coincide in your drawing? Which segments coincide? Points \( A \) and \( D \) coincide; points \( B \) and \( E \) coincide; \( \overline{AB} \) and \( \overline{DE} \) coincide.

   f. Mark the line segments that you know are congruent in the figure below.

   ![Marked Segments](image)
Lesson 11-3
Proving and Applying the Congruence Criteria

1. In the proof, is it important to know exactly which rigid motions map \( AB \) to \( DE \)? Explain.

   Although it is not necessary to specify the exact sequence of rigid motions that maps \( AB \) to \( DE \), doing so is a useful strategy. It shows that such a rigid motion does exist.

2. Attend to precision. How is the definition of a reflection used in the proof?

   The definition of a reflection across a line \( \ell \) states that the reflection maps it to \( \ell \), and that any point \( P \) that is not on \( \ell \) is mapped to \( P' \) such that \( \ell \) is the perpendicular bisector of \( PP' \). In the proof, line \( \ell \) is \( ED \) and we show that it is the perpendicular bisector of \( FC \).

3. Consider the reflection of \( \triangle ABC \) across \( ED \). What is the image of \( \triangle ABC \)? How do you know?

   The image of \( \triangle ABC \) is \( \triangle DEF \). Points \( A \) and \( B \) are fixed under this reflection since they lie on the line of reflection. Point \( C \) maps to point \( F \) since the line of reflection is the perpendicular bisector of \( FC \).

   The argument in Item 1 shows that a sequence of rigid motions maps \( \triangle ABC \) to \( \triangle DEF \). Specifically, the sequence of rigid motions is a rotation that maps \( AB \) to \( DE \), followed by a reflection across \( ED \). By the definition of congruence, \( \triangle ABC \cong \triangle DEF \).

MATH TIP

The Perpendicular Bisector Theorem states that a point lies on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the segment.

Activity 11 continued

2–3 Summarizing, Levels of Questions, Activating Prior Knowledge, Debriefing

In Items 2 and 3, students reflect on the proof they just completed. They come to understand that on a universal level, it is only important that there exists a sequence of rigid motions that maps one triangle to the other, but it is not necessary to specify each one. On a more specific level, however, students can greatly benefit by identifying and analyzing each transformation in the sequence to understand exactly how it is used in the proof. This benefit is not only useful for understanding the proof at hand but also for developing a collection of strategies to be used in future proofs. For example, in Item 3, students revisit the basic definition of a reflection and translate it to apply to this particular proof.

Teacher to Teacher

As students become more proficient with writing proofs, they gain experience with various techniques and strategies that may not be available to the novice proof writer. To beginners, drawing the diagram that maps \( AB \) to \( DE \) might not seem like the next logical step in the argument. Because diagrams in which triangles share a side are often used in proofs, it is important to ensure that students have a complete understanding of why this step is taken.
Lesson 11-3
Proving and Applying the Congruence Criteria

4. To justify the SAS congruence criterion, consider the two triangles below.

a. What given information is marked in the figure?
\( AB \cong DE, AC \cong DF, \text{ and } \angle A \cong \angle D \)

b. The proof begins in the same way as in Item 1. Since \( AB \cong DE \), there is a sequence of rigid motions that maps \( AB \) to \( DE \). Draw a figure at the right to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does not map point \( C \) to point \( F \).

c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect \( \triangle ABC \) across \( ED \). What is the image of \( AC \)? Why?

The image of \( AC \) is \( DF \) because \( ED \) is the angle bisector of \( \angle CDF \).

e. When you reflect \( \triangle ABC \) across \( ED \), can you conclude that the image of point \( C \) is point \( F \)? Explain.

Yes. Since the image of \( AC \) is \( DF \), the image of point \( C \) must lie on \( DF \). However, \( AC \cong DF \), so the image of point \( C \) must be the same distance from point \( D \) as point \( F \). That means the image of point \( C \) is point \( F \).

The argument in Item 4 shows that there is a sequence of rigid motions that maps \( \triangle ABC \) to \( \triangle DEF \). Specifically, the sequence of rigid motions that maps \( AB \) to \( DE \), followed by the reflection across \( ED \), maps \( \triangle ABC \) to \( \triangle DEF \). By the definition of congruence, \( \triangle ABC \cong \triangle DEF \).
Lesson 11-3  
Proving and Applying the Congruence Criteria

5. To justify the ASA congruence criterion, consider the two triangles below.

\[ \triangle ABC \quad \text{and} \quad \triangle DEF \]

a. What given information is marked in the figure?
\[ AB \cong DE, \quad \angle B \cong \angle D, \quad \text{and} \quad \angle B \cong \angle E \]

b. The proof begins in the same way as in Item 1. Since \( AB \cong DE \), there is a sequence of rigid motions that maps \( AB \) to \( DE \). Draw a figure at the left to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does not map point \( C \) to point \( F \).

\[ \text{Diagram showing sequence of rigid motions} \]

\[ \triangle ABC \quad \text{and} \quad \triangle DEF \]

\[ C \quad B \quad A \quad F \quad E \quad D \]

C

b.

c. Mark the line segments and angles that you know are congruent in your figure.

\[ \overline{AC} \cong \overline{DF}, \quad \overline{BC} \cong \overline{EF}, \quad \angle C \cong \angle F \]

\[ \text{Diagram showing congruent segments and angles} \]

\[ \triangle ABC \quad \text{and} \quad \triangle DEF \]

\[ C \quad B \quad A \quad F \quad E \quad D \]

d. Suppose you reflect \( \triangle ABC \) across \( \overline{ED} \). What is the image of \( \overline{AC} \)?
What is the image of \( \overline{BC} \)? Why?
\[ \text{The image of} \ \overline{AC} \ \text{is} \ \overline{DF} \ \text{because} \ \overline{ED} \ \text{is the angle bisector of} \ \angle CDF. \]
\[ \text{The image of} \ \overline{BC} \ \text{is} \ \overline{EF} \ \text{because} \ \overline{ED} \ \text{is the angle bisector of} \ \angle CEF. \]

\[ \text{Diagram showing reflection of} \ \triangle ABC \ \text{across} \ \overline{ED} \]

e. When you reflect \( \triangle ABC \) across \( \overline{ED} \), can you conclude that the image of point \( C \) is point \( F \)? Explain.
\[ \text{Yes. Since the image of}\ \overline{AC} \ \text{is}\ \overline{DF}, \ \text{the image of point} \ C \ \text{must lie on}\ \overline{DF}. \ \text{Since the image of}\ \overline{BC} \ \text{is}\ \overline{EF}, \ \text{the image of point} \ C \ \text{must lie on}\ \overline{EF}. \ \text{The only point that lies on both}\ \overline{DF} \ \text{and} \ \overline{EF} \ \text{is point} \ F. \ \text{So the image of point} \ C \ \text{must be point} \ F. \]  

\[ \text{Diagram showing reflection and conclusion} \]
ACTIVITY 11 Continued

Example A Visualization, Close Reading, Summarizing, Debriefing

Students now use the triangle congruence postulates to solve a problem that requires proof. Review with students the real-world details of the situation and have them study the diagram. Ask students how Greg was able to determine the information that is given [by measuring]. Also point out the use of a definition of a midpoint as reason #2. Explain that using a geometric definition as a reason in a proof is a common strategy. Conclude the example by having students summarize what they have learned so far about triangle congruence criteria and how they can be used to solve problems.

Universal Access

A common error students make when proving triangles congruent is not stating that a side is congruent to itself by the Reflexive Property of Congruence. Although such a statement may seem redundant, it is mandatory when proving triangles congruent by any criteria in which the side is included.

Try These A

a. Yes.

b. No. Only two parts of the triangles can be proven congruent; an angle and a side. Since \( \overline{JK} \) bisects \( \angle JLK \), \( \angle XLJ \) and \( \angle XJK \) are congruent by the definition of angle bisection, and \( \angle LXJ \) \( \cong \) \( \angle LXK \) because congruence is reflexive. We would need to be able to prove that sides \( JL \) and \( JK \) are congruent in order to use the SAS Congruence Postulate, or prove that \( \angle LXJ \) and \( \angle LXK \) are congruent in order to use the ASA Congruence Postulate, or prove that \( \angle XLJ \) and \( \angle XJK \) are congruent in order to use the AAS Congruence Postulate.

Check Your Understanding

Debrief students’ answers to these items to ensure that they understand concepts proving that triangles are congruent.

Answers

6. Yes. The two triangles have been proven congruent in the example, and since \( \angle J \) and \( \angle K \) are corresponding angles in the congruent triangles, the conclusion can be made that the two angles are congruent.

7. Check students’ drawings and statements. Students must be able to use the drawing and statements to prove that two angles and the included side of one triangle are congruent to two angles and the included side of the other triangle.
LESSON 11-3 PRACTICE

For Items 8–10, write each proof as a two-column proof.

8. Given: \( \overline{AD} \cong \overline{CD} \);
   \( \angle ADB \cong \angle CDB \)
   Prove: \( \triangle ADB \cong \triangle CDB \)

9. Given: \( \overline{PQ} \cong \overline{QR} \equiv \overline{RS} \equiv \overline{SP} \)
   Prove: \( \triangle PQR \cong \triangle RSP \)

10. Given: \( \angle K \cong \angle L; \)
    \( \overline{KH} \cong \overline{TH} \)
   Prove: \( \triangle GKH \cong \triangle JLH \)

11. Critique the reasoning of others. A student wrote the proof shown below. Critique the student’s work and correct any errors he or she may have made.
Given: \( \overline{WX} \cong \overline{YZ}; \)
   \( \overline{ZW} \cong \overline{XY} \)
   Prove: \( \triangle ZWX \cong \triangle XYZ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{WX} \cong \overline{YZ} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{ZW} \cong \overline{XY} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle W ) and ( \angle Y ) are right angles.</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \angle W \cong \angle Y )</td>
<td>4. All right angles are congruent.</td>
</tr>
<tr>
<td>5. ( \triangle ZWX \cong \triangle XYZ )</td>
<td>5. SAS</td>
</tr>
</tbody>
</table>

12. Model with mathematics. A graphic designer made a logo for an airline, as shown below. The designer made sure that \( \overline{AB} \) bisects \( \angle CAD \) and that \( \overline{AC} \cong \overline{AD} \). Can the designer prove that \( \triangle ABC \cong \triangle ABD \)? Why or why not?

The correct proof is shown:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{WX} \cong \overline{YZ} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{ZW} \cong \overline{XY} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \overline{XZ} \cong \overline{XZ} )</td>
<td>3. Congruence is reflexive.</td>
</tr>
<tr>
<td>4. ( \triangle ZWX \cong \triangle XYZ )</td>
<td>4. SSS Congruence Postulate</td>
</tr>
</tbody>
</table>

11. The student incorrectly assumed that \( \angle W \) and \( \angle Y \) are right angles and wrote that statement as a given statement. Then the student concluded that \( \angle W \cong \angle Y \) and found the triangles congruent by SAS. The student should have proven the triangles congruent by SSS after proving side \( \overline{XZ} \cong \overline{XZ} \).

ACTIVITY 11 Continued

ASSESS

Students’ answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning. See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 11-3 PRACTICE

9.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{PQ} \cong \overline{QR} \equiv \overline{RS} \equiv \overline{SP} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{PQ} \cong \overline{RS} )</td>
<td>2. Transitive Property</td>
</tr>
<tr>
<td>3. ( \overline{QR} \cong \overline{SP} )</td>
<td>3. Transitive Property</td>
</tr>
<tr>
<td>4. ( \overline{PR} \cong \overline{PR} )</td>
<td>4. Congruence is reflexive.</td>
</tr>
<tr>
<td>5. ( \triangle PQR \cong \triangle RSP )</td>
<td>5. SSS Congruence Postulate</td>
</tr>
</tbody>
</table>

10. Given: \( \angle K \cong \angle L; \)
    \( \overline{KH} \cong \overline{TH} \)
   Prove: \( \triangle GKH \cong \triangle JLH \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{KH} \cong \overline{TH} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle KHG ) and ( \angle JHL ) are right angles.</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle LHJ \cong \angle GHJ )</td>
<td>3. Vertical angles are congruent.</td>
</tr>
<tr>
<td>4. ( \triangle GKH \cong \triangle JLH )</td>
<td>4. ASA Congruence Postulate</td>
</tr>
</tbody>
</table>

12. Yes, \( \overline{AD} \cong \overline{CD} \) is given. Since \( \overline{AB} \) bisects \( \angle CAD \), \( \angle CAB \) and \( \angle DAB \) are congruent, and \( \overline{AB} \cong \overline{AD} \) because congruence is reflexive. Therefore, the triangles can be proven congruent by the SAS Congruence Postulate.

ADAPT

Check students’ answers to the Lesson Practice to ensure that they understand basic concepts related to using triangle congruence postulates to prove that two triangles are congruent. Encourage students to mark their drawings to show given or otherwise known congruent corresponding sides and angles.
Learning Targets:

• Apply congruence criteria to figures on the coordinate plane.
• Prove the AAS criterion and develop the HL criterion.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing

You can use the triangle congruence criteria on the coordinate plane.

1. Reason quantitatively.

Greg’s boss hands him a piece of graph paper that shows the plans for a truss. Greg’s boss asks him if he can prove that \( \triangle DBE \) is congruent to \( \triangle FCE \).

a. Use the distance formula to find each length.

\[
BD = 3\sqrt{2} \quad CF = 3\sqrt{2}
\]

\[
DE = 5 \quad FE = 5
\]

\[
BE = \sqrt{13} \quad CE = \sqrt{13}
\]

b. Can Greg use this information to prove that \( \triangle DBE \cong \triangle FCE \)? Explain.

Yes; \( \triangle DBE \cong \triangle FCE \) by the SSS congruence criterion.

2. In Item 5 of Lesson 11-2, you discovered that SSS, SAS, and ASA are not the only criteria for proving two triangles are congruent. You also discovered that there is an AAS congruence criterion. What does the AAS congruence criterion state? Mark the triangles below to illustrate the statement.

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and nonincluded side of another triangle, then the triangles are congruent.
### Lesson 11-4
Extending the Congruence Criteria

3. The proof of the AAS congruence criterion follows from the other congruence criteria. Complete the statements in the proof of the AAS congruence criterion below.

**Given:** $\triangle MNO$ and $\triangle PQR$ with $\angle N \cong \angle Q$, $\angle O \cong \angle R$, and $MO \cong PR$

**Prove:** $\triangle MNO \cong \triangle PQR$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\triangle MNO$ and $\triangle PQR$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle M + m\angle N + m\angle O = 180^\circ; m\angle P + m\angle Q + m\angle R = 180^\circ$</td>
<td>2. The sum of the measures of the angles of a triangle is $180^\circ$.</td>
</tr>
<tr>
<td>3. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle Q + m\angle R$</td>
<td>3. Transitive Property of Equality</td>
</tr>
<tr>
<td>4. $\angle N \cong \angle Q; \angle O \cong \angle R$</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. $m\angle N = m\angle Q; m\angle O = m\angle R$</td>
<td>5. Definition of congruent angles</td>
</tr>
<tr>
<td>6. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle Q + m\angle R$</td>
<td>6. Substitution Property of Equality</td>
</tr>
<tr>
<td>7. $m\angle M = m\angle P$</td>
<td>7. Subtraction Property of Equality</td>
</tr>
<tr>
<td>8. $\angle M \cong \angle P$</td>
<td>8. Definition of congruent angles</td>
</tr>
<tr>
<td>9. $MO \cong PR$</td>
<td>9. Given</td>
</tr>
<tr>
<td>10. $\triangle MNO \cong \triangle PQR$</td>
<td>10. ASA</td>
</tr>
</tbody>
</table>

### Activity 11
Continued

3. **Think-Pair-Share, Close Reading, Marking the Text, Self Revision/ Peer Revision** In Item 3, students complete the reasons in a two-column proof of the AAS congruence criterion. Since the proof follows from the other congruence criteria, students may feel as if they have seen it before and therefore gloss over the details. Caution against this response as this proof is actually quite different from the others. Encourage students to closely read all statements in the proof and to mark the text and diagram freely, making copies if necessary. If students are having difficulties, suggest that they pair up, review what has been completed so far, and devise a strategy to complete the proof. Remind students that there may be more than one way to express some reasons. For example, one way to express reason #2 is as a sentence: “The sum of the measures of the angles of a triangle is $180^\circ$.” Another way is to simply write “Triangle Sum Theorem.”
ACTIVITY 11 Continued

4–5 Think-Pair-Share, Activating Prior Knowledge, Look for a Pattern
Each of the pairs of triangles should be marked SSA; however, only those shown in Items 4a and 4c are congruent triangles. Encourage students to look for patterns as they try to determine what the congruent triangles have in common. If students are having difficulty finding the common characteristic, ask them to think about ways that triangles can be classified.

Lesson 11-4
Extending the Congruence Criteria

4. Below are pairs of triangles in which congruent parts are marked. For each pair of triangles, name the angle and side combination that is marked and tell whether the triangles appear to be congruent.
   a. SSA; yes

   ![Diagram](image1)

   b. SSA; no

   ![Diagram](image2)

   c. SSA; yes

   ![Diagram](image3)

   d. SSA; no

   ![Diagram](image4)

5. We know that in general SSA does not always determine congruence of triangles. However, for two of the cases in Item 4 the triangles appear to be congruent. What do the congruent pairs of triangles have in common?
   They are right triangles.
Lesson 11-4
Extending the Congruence Criteria

6. In a right triangle, we refer to the correspondence SSA shown in Items 4a and 4c as hypotenuse-leg (HL). Write a convincing argument in the space below to prove that HL will ensure that right triangles are congruent.

Answers may vary. Using the Pythagorean Theorem, the third sides of the triangles can be shown to be congruent. If the third sides are congruent, then the triangles are congruent by SSS or SAS. The triangles from Item 4a can be labeled as shown below.

From the figure it follows that \( c^2 = a^2 + b^2 \) and \( f^2 = d^2 + e^2 \). Since \( c = f \), \( c^2 = f^2 \). Therefore, \( a^2 + b^2 = d^2 + e^2 \). Also, \( a = d \), so \( a^2 + b^2 = a^2 + e^2 \) and \( b^2 = e^2 \), which means \( b = e \) and the triangles can be proved congruent by SSS or SAS.

Check Your Understanding

7. Is it possible to prove that \( \triangle LKM \cong \triangle JKM \) using the HL congruence criterion? If not, what additional information do you need?

8. Do you think there is a leg-leg congruence criterion for right triangles? If so, what does the criterion say? If not, why not? Review your answers. Be sure to check that you have described the situation with specific details, included the correct mathematical terms to support your reasoning, and that your sentences are complete and grammatically correct.
**Lesson 11-4**

**Extending the Congruence Criteria**

**Lesson 11-4 Practice**

9. Construct viable arguments. On a coordinate plane, plot triangles $\triangle ABC$ and $\triangle DEF$ with vertices $A(-3, -1), B(-1, 2), C(1, 1), D(3, -4), E(1, -1),$ and $F(-1, -2)$. Then prove $\triangle ABC \cong \triangle DEF$.

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

10. $\triangle PQ$ isosceles with base $\overline{PQ}$.

11. $\triangle RST$ right with hypotenuse $\overline{RT}$.

12. $\triangle UVW$ congruent to $\triangle XYZ$.

13. $\overline{PQ}$ bisects $\angle SPT$.

**Assess**

Students’ answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning. See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

**Lesson 11-4 Practice**

9. Check students’ triangles. Using the Distance Formula, $AB = \sqrt{13}$, $BC = \sqrt{5}$, and $CA = 2\sqrt{5}$; $DE = \sqrt{13}$, $EF = \sqrt{5}$, and $FD = 2\sqrt{5}$. Since $AB = DE$, $BC = EF$, and $CA = FD$, by definition of congruent segments. Therefore, $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate.

10. HL

11. AAS

12. none

13. none

**Adapt**

Check students’ answers to the Lesson Practice to ensure that they understand basic concepts related to the HL congruence criterion and the application of congruence criteria to figures on the coordinate plane. Remind students that just because two triangles look congruent that does not mean that they are. To prove congruence one must show that all of the conditions of one of the triangle congruence criteria are met.
**ACTIVITY 11 PRACTICE**

Write your answers on notebook paper. Show your work.

1. \( \triangle \text{FIX} \cong \triangle \text{TOP} \). Complete the following.
   a. Name three pairs of corresponding vertices.
   b. Name three pairs of corresponding sides.
   c. Name three pairs of corresponding angles.
   d. Is it correct to say that \( \triangle \text{POT} \cong \triangle \text{XIP} \)? Why or why not?
   e. Is it correct to say that \( \triangle \text{IFX} \cong \triangle \text{PTO} \)? Why or why not?

In the figure, \( \triangle \text{MNP} \cong \triangle \text{UVW} \). Use the figure for items 2–4.

2. Write six congruence statements about line segments and angles in the figure.
3. Suppose \( MN = 14 \text{ cm} \) and \( \angle M = 24^\circ \). What other side lengths or angle measures can you determine? Why?
4. Suppose \( MN = 2PN \) and \( VW = 8 \text{ in} \). Find \( MN \).

5. If \( \triangle \text{MAP} \cong \triangle \text{TON} \), \( m\angle M = 80^\circ \), and \( m\angle N = 50^\circ \), name four congruent angles.
6. \( \triangle \text{ABC} \cong \triangle \text{DEF} \). \( AB = 15 \), \( BC = 20 \), \( AC = 25 \), and \( FE = 3x - 7 \). Find \( x \).
7. \( \triangle \text{MNO} \cong \triangle \text{PQR} \). \( m\angle N = 57^\circ \), \( m\angle P = 64^\circ \) and \( m\angle O = 5x + 4 \). Find \( x \) and \( m\angle R \).

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

8. \( \triangle \text{PQS} \cong \triangle \text{RSQ} \)
9. \( M \) bisects \( \overline{AB} \) and \( \overline{CT} \).
10. \( \triangle \text{FIX} \cong \triangle \text{PTO} \)
11. \( \angle \text{RSQ} \cong \angle \text{PQS} \)

**ACTIVITY 11 Continued**

**ACTIVITY PRACTICE**

1. \( \angle F \) and \( \angle T \), \( \angle A \) and \( \angle L \)
2. \( \overline{HT} \) and \( \overline{TO} \), \( \overline{TX} \) and \( \overline{OP} \), \( \overline{XF} \) and \( \overline{PT} \)
3. \( \triangle \text{FIX} \) and \( \triangle \text{TOP} \), \( \angle \text{IXF} \) and \( \angle \text{OPT} \), \( \angle \text{XFI} \) and \( \angle \text{PTO} \)
4. Yes. Since \( \triangle \text{FIX} \cong \triangle \text{TOP} \), \( \angle \text{I} \) corresponds with \( \angle \text{X} \), \( \angle \text{O} \) corresponds with \( \angle \text{L} \), and \( \angle \text{T} \) corresponds with \( \angle \text{F} \), so it is correct to say that \( \angle \text{POT} \cong \angle \text{XIF} \).
5. No. Since \( \triangle \text{FIX} \cong \triangle \text{TOP} \), \( \angle \text{I} \) and \( \angle \text{P} \) do not correspond, and \( \angle \text{X} \) and \( \angle \text{O} \) do not correspond, so it is not correct to say that \( \angle \text{IFX} \cong \angle \text{PTO} \).
6. \( \angle \text{M} \) and \( \angle \text{U} \), \( \angle \text{N} \) and \( \angle \text{V} \), \( \angle \text{P} \) and \( \angle \text{W} \), \( \overline{MN} \) and \( \overline{UV} \), \( \overline{NP} \) and \( \overline{VW} \), \( \overline{PM} \) and \( \overline{WU} \)
7. \( \triangle \text{M} \) congruent to \( \triangle \text{A} \), and \( \overline{UV} \) is congruent to \( \overline{MN} \).
8. 16 in.
9. \( \angle \text{A} \), \( \angle \text{P} \), \( \angle \text{O} \) and \( \angle \text{N} \)
10. \( x = 9 \)
11. \( x = 11 \), \( m\angle R = 59^\circ \)
12. None
13. None

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{EF} ) bisects ( \angle \text{GEH} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle \text{GEF} \cong \angle \text{HEF} )</td>
<td>2. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>3. ( \overline{EF} \cong \overline{EF} )</td>
<td>3. ( \cong ) is reflexive</td>
</tr>
<tr>
<td>4. ( \overline{EF} ) bisects ( \angle \text{GHF} )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( \angle \text{GFE} \cong \angle \text{HFE} )</td>
<td>5. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>6. ( \triangle \text{EGF} \cong \triangle \text{EHF} )</td>
<td>6. ASA</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{PQ} \cong \overline{SR} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{PQ} ) is ( \perp ) to ( \overline{SR} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle \text{PQS} \cong \angle \text{RSQ} )</td>
<td>3. Alt. int. ( \angle ) of ( \perp ) lines are ( \cong )</td>
</tr>
<tr>
<td>4. ( \overline{QS} \cong \overline{QS} )</td>
<td>4. ( \cong ) is reflexive</td>
</tr>
<tr>
<td>5. ( \triangle \text{PQS} \cong \triangle \text{RSQ} )</td>
<td>5. SAS</td>
</tr>
</tbody>
</table>
## ACTIVITY 11 Continued

### Statements and Reasons

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $M$ is mdpt. of $AB$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB \cong BC$</td>
<td>2. Def. of mdpt.</td>
</tr>
<tr>
<td>3. $\triangle AMD \cong \triangle BMC$</td>
<td>3. Vert. $\angle \cong$ are $\cong$</td>
</tr>
<tr>
<td>4. $M$ is mdpt. of $CD$</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. $MD \cong MC$</td>
<td>5. Def. of mdpt.</td>
</tr>
<tr>
<td>6. $\triangle AMD \cong \triangle BMC$</td>
<td>6. SAS</td>
</tr>
</tbody>
</table>

### ACTIVITY 11 Continued

In Items 12–15, write each proof as a two-column proof.

12. Given: $FE$ bisects $\angle GEH$; $FE$ bisects $\angle GFH$.
   Prove: $\triangle EGF \cong \triangle EHF$

13. Given: $PQ$ is parallel to $SR$; $PQ \cong SR$.
   Prove: $\triangle PQS \cong \triangle RSQ$

14. Given: $M$ is the midpoint of $\overline{CD}$.
   Prove: $\triangle AMD \cong \triangle BMC$

15. Given: $\overline{UV}$ is parallel to $\overline{WX}$; $\overline{UV} \cong \overline{WX}$.
   Prove: $\triangle UVY \cong \triangle XYW$

### MATHEMATICAL PRACTICES

**Look for and Make Use of Structure**

19. The opposite sides of a rectangle are congruent. Describe three different ways you could show that a diagonal divides a rectangle into two congruent triangles.

### ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

---

### Congruence Transformations and Triangle Congruence

**Truss Your Judgment**

16. Draw a figure that includes two triangles. Provide given information that would allow you to prove that the triangles are congruent by the AAS Congruence Postulate.

17. a. Explain why $\triangle ABD \cong \triangle CBD$.  
   b. Can you conclude that $\angle A \cong \angle C$? Why or why not?

18. a. Explain why $\overline{RS}$ and $\overline{PQ}$ are perpendicular to $\overline{QR}$.  
   b. Explain how to use the SAS Congruence Postulate to show that $\triangle PQT \cong \triangle RST$.

---

**Sample answer:** Use the Distance Formula to find that $\overline{PP}$ and $\overline{TT}$ are the same lengths and are therefore $\cong$. Use vertical angles to show that $\angle PTQ$ and $\angle RTS$ are $\cong$. Use the Distance Formula to find that $\overline{QT}$ and $\overline{ST}$ are $\cong$.

19. (1) Use the two opposite $\cong$ sides of the rectangle, along with the diagonal (reflective) to prove SSS congruence.  
   (2) Use the right $\angle$ opposite the diagonal, and the pair of $\cong$ sides of the rectangle to prove SAS congruence.  
   (3) Use the diagonal (reflexive) as the $\cong$ hypotenuse and use one of the $\cong$ sides to prove HL congruence.
Learning Targets:
- Use the fact that congruent triangles have congruent corresponding parts.
- Determine unknown angle measures or side lengths in congruent triangles.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing, Use Manipulatives, Think-Pair-Share

Greg Carpenter works for the Greene Construction Company. The company is building a new recreation hall, and the roof of the hall will be supported by triangular trusses, like the ones shown below.

Each of the trusses contains pairs of congruent triangles. Greg's boss tells him that his first job will be to determine the side lengths and angle measures in the triangles that make up one of the trusses.

CAREER CONNECT TO

Triangles are often used in construction for roof and floor trusses because of their strength and rigidity. Each angle of a triangle is held solidly in place by its opposite side. That means the angles will not change when pressure is applied—unlike other shapes.

MATH TIP

Congruent triangles are triangles that have the same size and shape. More precisely, you have seen that two triangles are congruent if and only if one can be obtained from the other by a sequence of rigid motions.
Lesson 11-1
Congruent Triangles

Greg wonders, “If I know that two triangles are congruent, and I know the side lengths and angle measures in one triangle, do I have to measure all the sides and angles in the other triangle?”

Greg begins by examining two triangles from a truss. According to the manufacturer, the two triangles are congruent.

1. Because the two triangles are congruent, can one triangle be mapped onto the other? If yes, what are the criteria for the mapping?

2. Suppose you use a sequence of rigid motions to map \( \triangle ABC \) to \( \triangle DEF \). Find the image of each of the following under this sequence of transformations.
   \[
   \begin{align*}
   AB \rightarrow & \quad BC \rightarrow \quad AC \rightarrow \\
   \angle A \rightarrow & \quad \angle B \rightarrow \quad \angle C \rightarrow 
   \end{align*}
   \]

3. Make use of structure. What is the relationship between \( AB \) and \( DE \)? What is the relationship between \( \angle B \) and \( \angle E \)? How do you know?

The triangles from the truss that Greg examined illustrate an important point about congruent triangles. In congruent triangles, corresponding pairs of sides are congruent and corresponding pairs of angles are congruent. These are **corresponding parts**.

When you write a congruence statement like \( \triangle ABC \cong \triangle DEF \), you write the vertices so that corresponding parts are in the same order. So, you can conclude from this statement that \( AB \cong DE, BC \cong EF, AC \cong DF, \angle A \cong \angle D, \angle B \cong \angle E, \) and \( \angle C \cong \angle F \).
Lesson 11-1
Congruent Triangles

Example A
For the truss shown below, Greg knows that \( \triangle JKL \cong \triangle MNP \).

\[
\begin{array}{c}
J \\
\downarrow \quad \downarrow 50^\circ \\
K \\
\downarrow 2.1 \text{ m} \\
L \\
\end{array}
\begin{array}{c}
P \\
\downarrow \quad \downarrow \\
M \\
\end{array}
\begin{array}{c}
N \\
\end{array}
\]

Greg wants to know if there are any additional lengths or angle measures that he can determine.

Since \( \triangle JKL \cong \triangle MNP \), \( KL \cong NP \). This means \( KL = NP \), so \( NP = 2.1 \text{ m} \).

Also, since \( \triangle JKL \cong \triangle MNP \), \( \angle K \cong \angle N \). This means \( m\angle K = m\angle N \), so \( m\angle K = 50^\circ \).

Try These A
In the figure, \( \triangle RST \cong \triangle XYZ \). Find each of the following, if possible.

\[
\begin{array}{c}
R \\
\downarrow \\
S \\
\downarrow 15 \text{ cm} \\
T \\
\end{array}
\begin{array}{c}
Y \\
\downarrow 47^\circ \\
Z \\
\end{array}
\begin{array}{c}
X \\
\end{array}
\]

a. \( m\angle X \)

b. \( YZ \)

c. \( m\angle T \)

d. \( XZ \)

e. Both \( \triangle JKL \) and \( \triangle MNP \) are equilateral triangles in which the measure of each angle is \( 60^\circ \). Can you tell whether or not \( \triangle JKL \cong \triangle MNP \)? Explain.
Lesson 11-1
Congruent Triangles

Check Your Understanding

4. If two triangles are congruent, can you conclude that they have the same perimeter? Why or why not?
5. Is it possible to draw two congruent triangles so that one triangle is an acute triangle and one triangle is a right triangle? Why or why not?
6. Rectangle $ABCD$ is divided into two congruent right triangles by diagonal $AC$.

Fill in the blanks to show the congruent sides and angles.

- $AB \cong ____$
- $BC \cong ____$
- $\angle BAC \cong ____$
- $\angle ACB \cong ____$

7. $\triangle PQR \cong \triangle GHJ$. Complete the following.

- $QR \cong ____$
- $GJ \cong ____$
- $\angle R \cong ____$
- $\angle G \cong ____$

**LESSON 11-1 PRACTICE**

In the figure, $\triangle ABC \cong \triangle DFE$.

8. Find the length of $\overline{AB}$.
9. Find the measure of all angles in $\triangle DEF$ that it is possible to find.
10. What is the perimeter of $\triangle DEF$? Explain how you know.
11. **Construct viable arguments.** Suppose $\triangle XYZ \cong \triangle TUV$ and that $\overline{XY}$ is the longest side of $\triangle XYZ$. Is it possible to determine which side of $\triangle TUV$ is the longest? Explain.
Learning Targets:

- Develop criteria for proving triangle congruence.
- Determine which congruence criteria can be used to show that two triangles are congruent.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Use Manipulatives, Think-Pair-Share

As you have seen, congruent triangles have six pairs of congruent corresponding parts. The converse of this statement is also true. That is, if two triangles have three pairs of congruent corresponding sides and three pairs of congruent corresponding angles, the triangles are congruent.

Greg’s boss asks him to check that two triangles in a truss are congruent. Greg wonders, “Must I measure and compare all six parts of both triangles?” He decides that a shortcut will allow him to conclude that two triangles are congruent without checking all six pairs of corresponding parts.

1. Greg begins by checking just one pair of corresponding parts of the two triangles.
   a. In your group, each student should draw a triangle that has a side that is 2 inches long. The other two sides can be any measure. Draw the triangles on acetate or tracing paper.

   b. To check whether two triangles are congruent, place the sheets of acetate on the desk. If the triangles are congruent, you can use a sequence of translations, reflections, and rotations to map one triangle onto the other.

   Are all of the triangles congruent to each other? Why or why not?

   c. Cite your results from part b to prove or disprove this statement: “If one part of a triangle is congruent to a corresponding part of another triangle, then the triangles must be congruent.”
Lesson 11-2
Congruence Criteria

Now Greg wonders if checking two pairs of corresponding parts suffices to show that two triangles are congruent.

2. Greg starts by considering \( \triangle ABC \) below.

```
A

C

B
```

a. Draw triangles that each have one side congruent to \( AB \) and another side congruent to \( AC \). Use transformations to check whether every triangle is congruent to \( \triangle ABC \). Explain your findings.

b. Draw triangles that each have an angle congruent to \( \angle A \) and an adjacent side congruent to \( AB \). Is every such triangle congruent to \( \triangle ABC \)? Explain.

c. Draw triangles that each have an angle congruent to \( \angle A \) and an opposite side congruent to \( CB \). Is every such triangle congruent to \( \triangle ABC \)? Explain.

d. Draw triangles that each have an angle congruent to \( \angle A \) and an angle congruent to \( \angle B \). Is every such triangle congruent to \( \triangle ABC \)? Explain.

e. Consider the statement: “If two parts of one triangle are congruent to the corresponding parts in a second triangle, then the triangles must be congruent.” Prove or disprove this statement. Cite the triangles you constructed.
Greg decides that he must have at least three pairs of congruent parts in order to conclude that two triangles are congruent. In order to work more efficiently, he decides to make a list of all possible combinations of three congruent parts.

3. Greg uses $A$ as an abbreviation to represent angles and $S$ to represent sides. For example, if Greg writes $SAS$, it represents two sides and the included angle, as shown in the first triangle below. Here are the combinations in Greg’s list: $SAS$, $SSA$, $ASA$, $AAS$, $SSS$, and $AAA$.

a. Mark each triangle below to illustrate the combinations in Greg’s list.

b. Are there any other combinations of three parts of a triangle? If so, is it necessary for Greg to add these to his list? Explain.
Lesson 11-2
Congruence Criteria

Now Greg wants to find out which pairs of congruent corresponding parts guarantee congruent triangles.

4. Three segments congruent to the sides of \( \triangle ABC \) and three angles congruent to the angles of \( \triangle ABC \) are given in Figures 1–6, shown below.

\[ \triangle ABC \]

a. If needed, use manipulatives supplied by your teacher to recreate the six figures given below.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

b. Identify which of the figures in part a is congruent to each of the parts of \( \triangle ABC \).

\[ \angle A: \quad \angle B: \quad \angle C: \quad \overline{AB}: \quad \overline{AC}: \quad \overline{CB}: \]
Lesson 11-2
Congruence Criteria

5. For each combination in Greg’s list in Item 3, choose three appropriate triangle parts from Item 4. Each student should create a triangle using these parts. Then use transformations to check whether every such triangle is congruent to \( \triangle ABC \). Use the table to organize your results.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Name the Three Figures Used by Listing the Figure Numbers</th>
<th>Is Every Such Triangle Congruent to ( \triangle ABC )?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Express regularity in repeated reasoning. Compare your results from Item 5 with those of students in other groups. Then list the different combinations that seem to guarantee a triangle congruent to \( \triangle ABC \). These combinations are called **triangle congruence criteria**.

7. Do you think there is an AAA triangle congruence **criterion**? Why or why not?

**ACADEMIC VOCABULARY**

A **criterion** (plural: **criteria**) is a standard or rule on which a judgment can be based. Criteria exist in every subject area. For example, a scientist might use a set of criteria to determine whether a sample of water is safe for human consumption.
Lesson 11-2
Congruence Criteria

Greg realizes that it is not necessary to check all six pairs of corresponding parts to determine if two triangles are congruent. The triangle congruence criteria can be used as “shortcuts” to show that two triangles are congruent.

Example A
For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a. Since three pairs of corresponding sides are congruent, the triangles are congruent by SSS.

b. Although they are not marked as such, the vertical angles in the figure are congruent. The triangles are congruent by ASA.

Try These A
For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a. 

b.
Lesson 11-2
Congruence Criteria

Check Your Understanding

8. Two triangles each have two sides that are 8 feet long and an included angle of 50°. Must the two triangles be congruent? Why or why not?

9. Two equilateral triangles each have a side that is 5 cm long. Is it possible to conclude whether or not the triangles are congruent? Explain.

10. The figure shows a circle and two triangles. For both triangles, two vertices are points on the circle and the other vertex is the center of the circle. What information would you need in order to prove that the triangles are congruent?

LESSON 11-2 PRACTICE

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

11. 

12. 

13. 

14. 

15. Make sense of problems. What one additional piece of information do you need, if any, in order to conclude that \( \triangle ABD \cong \triangle CBD \)? Is there more than one set of possible information? Explain.

CONNECT TO AP

The free-response items on the AP Calculus exam will often ask you to justify your answer. Such justifications should follow the same rules of deductive reasoning as the proofs in this geometry course.
Learning Targets:
- Prove that congruence criteria follow from the definition of congruence.
- Use the congruence criteria in simple proofs.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Marking the Text, Think Aloud, Discussion Groups, Visualization

You can use the SSS, SAS, or ASA congruence criteria as shortcuts to show that two triangles are congruent. In order to prove why these criteria work, you must show that they follow from the definition of congruence in terms of rigid motions.

1. To justify the SSS congruence criterion, consider the two triangles below.

   ![Triangles ABC and DEF](image)

   a. What given information is marked in the figure?

   b. Based on the definition of congruence, what do you need to do in order to show that $\triangle ABC \cong \triangle DEF$?

   c. It is given that $AB \cong DE$. What does this tell you?

   d. Draw a figure to show the result of the sequence of rigid motions that maps $AB$ to $DE$. Assume that this sequence of rigid motions does not map point $C$ to point $F$.

   e. Which points coincide in your drawing? Which segments coincide?

   f. Mark the line segments that you know are congruent in the figure below.

   ![Marked segments](image)
Lesson 11-3
Proving and Applying the Congruence Criteria

g. Based on the figure, how is \( ED \) related to \( FC \)? Why?

h. Consider the reflection of \( \triangle ABC \) across \( ED \). What is the image of \( \triangle ABC \)? How do you know?

The argument in Item 1 shows that a sequence of rigid motions maps \( \triangle ABC \) to \( \triangle DEF \). Specifically, the sequence of rigid motions is a rotation that maps \( AB \) to \( DE \), followed by a reflection across \( ED \). By the definition of congruence, \( \triangle ABC \cong \triangle DEF \).

2. In the proof, is it important to know exactly which rigid motions map \( AB \) to \( DE \)? Explain.

3. Attend to precision. How is the definition of a reflection used in the proof?
Lesson 11-3
Proving and Applying the Congruence Criteria

4. To justify the SAS congruence criterion, consider the two triangles below.

a. What given information is marked in the figure?

b. The proof begins in the same way as in Item 1. Since $AB \cong DE$, there is a sequence of rigid motions that maps $AB$ to $DE$. Draw a figure at the right to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does not map point $C$ to point $F$.

c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $ED$. What is the image of $AC$? Why?

e. When you reflect $\triangle ABC$ across $ED$, can you conclude that the image of point $C$ is point $F$? Explain.

The argument in Item 4 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $AB$ to $DE$, followed by the reflection across $ED$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$. 

MATH TIP
Your proof should use all three pieces of given information that you identified in part a. If you find that you are not using all of these facts, you may be missing an element of the proof.
Lesson 11-3
Proving and Applying the Congruence Criteria

5. To justify the ASA congruence criterion, consider the two triangles below.

a. What given information is marked in the figure?

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Draw a figure at the left to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does not map point $C$ to point $F$.

c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? What is the image of $\overline{BC}$? Why?

e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point $C$ is point $F$? Explain.
Lesson 11-3
Proving and Applying the Congruence Criteria

The argument in Item 5 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $AB$ to $DE$, followed by the reflection across $ED$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

Now you can use the SSS, SAS, and ASA congruence criteria to prove that triangles are congruent.

**Example A**

Greg knows that point $X$ is the midpoint of $JK$ in the truss shown below. He also makes measurements and finds that $JL \cong KL$. He must prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$.

Given: $X$ is the midpoint of $JK$; $JL \cong KL$.
Prove: $\triangle JXL \cong \triangle KXL$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $X$ is the midpoint of $JK$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $JX \cong KX$</td>
<td>2. Definition of midpoint</td>
</tr>
<tr>
<td>3. $JL \cong KL$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $LX \cong LX$</td>
<td>4. Congruence is reflexive.</td>
</tr>
<tr>
<td>5. $\triangle JXL \cong \triangle KXL$</td>
<td>5. SSS</td>
</tr>
</tbody>
</table>

**Try These A**

a. Suppose that Greg knew instead that $LX$ was perpendicular to $JK$ and suppose he made measurements to find that $JX \cong KX$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

b. Suppose that Greg knew instead that $LX$ bisects $\angle JLK$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

**Check Your Understanding**

6. In the Example, can Greg conclude that $\angle J \cong \angle K$? Why or why not?
7. Draw a figure that contains two triangles. Provide given information that would allow you to prove that the triangles are congruent by the ASA congruence criterion.
Lesson 11-3
Proving and Applying the Congruence Criteria

LESSON 11-3 PRACTICE
For Items 8–10, write each proof as a two-column proof.

8. Given: \( \overline{AD} \cong \overline{CD} \);
   \( \angle ADB \cong \angle CDB \)
   Prove: \( \triangle ADB \cong \triangle CDB \)

9. Given: \( \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP} \)
   Prove: \( \triangle PQR \cong \triangle RSP \)

10. Given: \( \angle K \cong \angle L \);
    \( \overline{KH} \cong \overline{LH} \)
    Prove: \( \triangle GKH \cong \triangle JLH \)

11. Critique the reasoning of others. A student wrote the proof shown below. Critique the student's work and correct any errors he or she may have made.

   Given: \( \overline{WX} \cong \overline{YZ} \);
   \( \overline{ZW} \cong \overline{XY} \)
   Prove: \( \triangle ZWX \cong \triangle XYZ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{WX} \cong \overline{YZ} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{ZW} \cong \overline{XY} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle W ) and ( \angle Y ) are right angles.</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \angle W \cong \angle Y )</td>
<td>4. All right angles are congruent.</td>
</tr>
<tr>
<td>5. ( \triangle ZWX \cong \triangle XYZ )</td>
<td>5. SAS</td>
</tr>
</tbody>
</table>

12. Model with mathematics. A graphic designer made a logo for an airline, as shown below. The designer made sure that \( \overline{AB} \) bisects \( \angle CAD \) and that \( \overline{AC} \cong \overline{AD} \). Can the designer prove that \( \triangle ABC \cong \triangle ABD \)? Why or why not?
Lesson 11-4
Extending the Congruence Criteria

Learning Targets:
- Apply congruence criteria to figures on the coordinate plane.
- Prove the AAS criterion and develop the HL criterion.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing

You can use the triangle congruence criteria on the coordinate plane.

1. **Reason quantitatively.** Greg’s boss hands him a piece of graph paper that shows the plans for a truss. Greg’s boss asks him if he can prove that \(\triangle DBE\) is congruent to \(\triangle FCE\).

   \[\begin{align*}
   BD &= \quad CF = \\
   DE &= \quad FE = \\
   BE &= \quad CE =
   \end{align*}\]

   **a.** Use the distance formula to find each length.

   **b.** Can Greg use this information to prove that \(\triangle DBE \cong \triangle FCE\)? Explain.

2. In Item 5 of Lesson 11-2, you discovered that SSS, SAS, and ASA are not the only criteria for proving two triangles are congruent. You also discovered that there is an AAS congruence criterion. What does the AAS congruence criterion state? Mark the triangles below to illustrate the statement.
3. The proof of the AAS congruence criterion follows from the other congruence criteria. Complete the statements in the proof of the AAS congruence criterion below.

Given: \( \triangle MNO \) and \( \triangle PQR \) with \( \angle N \cong \angle Q \), \( \angle O \cong \angle R \), and \( MO \cong PR \).  

Prove: \( \triangle MNO \cong \triangle PQR \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle MNO ) and ( \triangle PQR )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( m \angle M + m \angle N + m \angle O = 180^\circ ), ( m \angle P + m \angle Q + m \angle R = 180^\circ )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( m \angle M + m \angle N + m \angle O = m \angle P + m \angle Q + m \angle R )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( \angle N \cong \angle Q ); ( \angle O \cong \angle R )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( m \angle N = m \angle Q ); ( m \angle O = m \angle R )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( m \angle M + m \angle N + m \angle O = m \angle P + m \angle N + m \angle O )</td>
<td>6.</td>
</tr>
<tr>
<td>7. ( m \angle M = m \angle P )</td>
<td>7.</td>
</tr>
<tr>
<td>8. ( \angle M \cong \angle P )</td>
<td>8.</td>
</tr>
<tr>
<td>9. ( MO \cong PR )</td>
<td>9.</td>
</tr>
<tr>
<td>10. ( \triangle MNO \cong \triangle PQR )</td>
<td>10.</td>
</tr>
</tbody>
</table>
4. Below are pairs of triangles in which congruent parts are marked. For each pair of triangles, name the angle and side combination that is marked and tell whether the triangles appear to be congruent.

a.

b.

c.

d.

5. We know that in general SSA does not always determine congruence of triangles. However, for two of the cases in Item 4 the triangles appear to be congruent. What do the congruent pairs of triangles have in common?
Lesson 11-4
Extending the Congruence Criteria

6. In a right triangle, we refer to the correspondence SSA shown in Items 4a and 4c as hypotenuse-leg (HL). Write a convincing argument in the space below to prove that HL will ensure that right triangles are congruent.

Check Your Understanding

7. Is it possible to prove that $\triangle LKM \cong \triangle JKM$ using the HL congruence criterion? If not, what additional information do you need?

8. Do you think there is a leg-leg congruence criterion for right triangles? If so, what does the criterion say? If not, why not? Review your answers. Be sure to check that you have described the situation with specific details, included the correct mathematical terms to support your reasoning, and that your sentences are complete and grammatically correct.
Lesson 11-4
Extending the Congruence Criteria

**LESSON 11-4 PRACTICE**

9. **Construct viable arguments.** On a coordinate plane, plot triangles \( \triangle ABC \) and \( \triangle DEF \) with vertices \( A(-3, -1), B(-1, 2), C(1, 1), D(3, -4), E(1, -1), \) and \( F(-1, -2) \). Then prove \( \triangle ABC \cong \triangle DEF \).

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

10.

11.

12.

13. \( PQ \) bisects \( \angle SPT \).
ACTIVITY 11 PRACTICE
Write your answers on notebook paper.
Show your work.

1. \( \triangle \text{FIX} \cong \triangle \text{TOP} \). Complete the following.
   a. Name three pairs of corresponding vertices.
   b. Name three pairs of corresponding sides.
   c. Name three pairs of corresponding angles.
   d. Is it correct to say that \( \triangle \text{POT} \cong \triangle \text{XIF} \)? Why or why not?
   e. Is it correct to say that \( \triangle \text{IFX} \cong \triangle \text{PTO} \)? Why or why not?

In the figure, \( \triangle \text{MNP} \cong \triangle \text{UVW} \). Use the figure for Items 2–4.

2. Write six congruence statements about line segments and angles in the figure.

3. Suppose \( \text{MN} = 14 \text{ cm and } m \angle U = 24^\circ \). What other side lengths or angle measures can you determine? Why?

4. Suppose \( \text{MN} = 2 \text{PN} \) and \( \text{VW} = 8 \text{ in} \). Find \( \text{MN} \).

5. If \( \triangle \text{MAP} \cong \triangle \text{TON}, m \angle M = 80^\circ, \) and \( m \angle N = 50^\circ \), name four congruent angles.

6. \( \triangle \text{ABC} \cong \triangle \text{DEF}, \text{AB} = 15, \text{BC} = 20, \text{AC} = 25, \) and \( \text{FE} = 3x - 7 \). Find \( x \).

7. \( \triangle \text{MNO} \cong \triangle \text{PQR}, m \angle N = 57^\circ, m \angle P = 64^\circ \) and \( m \angle O = 5x + 4 \). Find \( x \) and \( m \angle R \).

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

8.  

9. \( M \) bisects \( \overline{AB} \) and \( \overline{CD} \).

10. 

11. 

Activity 11 • Congruence Transformations and Triangle Congruence 165
In Items 12–15, write each proof as a two-column proof.

12. Given: $EF$ bisects $\angle GEH$; $EF$ bisects $\angle GFH$.
   Prove: $\triangle EGF \cong \triangle EHF$

13. Given: $PQ$ is parallel to $SR$; $PQ \cong SR$.
   Prove: $\triangle PQS \cong \triangle RSQ$

14. Given: $M$ is the midpoint of $AB$; $M$ is the midpoint of $CD$.
   Prove: $\triangle AMD \cong \triangle BMC$

15. Given: $UV$ is parallel to $WX$; $UV \cong WX$.
   Prove: $\triangle UVY \cong \triangle XWY$

16. Draw a figure that includes two triangles. Provide given information that would allow you to prove that the triangles are congruent by the AAS Congruence Postulate.

17. a. Explain why $\triangle ABD \cong \triangle CDB$.
   b. Can you conclude that $\angle A \cong \angle C$? Why or why not?

18. a. Explain why $RS$ and $PQ$ are perpendicular to $QS$.
   b. Explain how to use the SAS Congruence Postulate to show that $\triangle PQT \cong \triangle RST$.

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

19. The opposite sides of a rectangle are congruent. Describe three different ways you could show that a diagonal divides a rectangle into two congruent triangles.