In 1935, Charles F. Richter developed the Richter magnitude test scale to compare the size of earthquakes. The Richter scale is based on the amplitude of the seismic waves recorded on seismographs at various locations after being adjusted for distance from the epicenter of the earthquake. Richter assigned a magnitude of 0 to an earthquake whose amplitude on a seismograph is 1 micron, or $10^{-4}$ cm. According to the Richter scale, a magnitude 1.0 earthquake causes 10 times the ground motion of a magnitude 0 earthquake. A magnitude 2.0 earthquake causes 10 times the ground motion of a magnitude 1.0 earthquake. This pattern continues as the magnitude of the earthquake increases.

1. **Reason quantitatively.** How does the ground motion caused by earthquakes of these magnitudes compare?
   a. magnitude 5.0 earthquake compared to magnitude 4.0
      A magnitude 5.0 earthquake’s ground motion is 10 times that of a magnitude 4.0 earthquake.
   b. magnitude 4.0 earthquake compared to magnitude 1.0
      A magnitude 4.0 earthquake’s ground motion is 1000 or $10^3$ times that of a magnitude 1.0 earthquake.
   c. magnitude 4.0 earthquake compared to magnitude 0
      A magnitude 4.0 earthquake’s ground motion is 10,000 or $10^4$ times that of a magnitude 0 earthquake.

The list below describes the effects of earthquakes of different magnitudes. Read through this list with your group and identify any words that might be unfamiliar. Find their meanings to aid your understanding.

**Typical Effects of Earthquakes of Various Magnitudes**

1.0 Very weak, no visible damage
2.0 Not felt by humans
3.0 Often felt, usually no damage
4.0 Windows rattle, indoor items shake
5.0 Damage to poorly constructed structures
6.0 Some damage to well-built structures
7.0 Damage to most buildings, some collapse
8.0 Major damage to most buildings, many collapse
9.0 Near total destruction
10.0 Extremely rare, never recorded

**Logarithms and Their Properties**

**Earthquakes and Richter Scale**

**Lesson 14-1 Exponential Data**

**Learning Targets:**
- Complete tables and plot points for exponential data.
- Write and graph an exponential function for a given context.
- Find the domain and range of an exponential function.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Paraphrasing, Create Representations, Quickwrite, Close Reading, Look for a Pattern, Discussion Groups

**My Notes**

**Academic Vocabulary**

**Magnitude** refers to the size or amount of something. It is often used in discussing the size of earthquakes or the brightness of stars.

**Seismic** waves are created by earthquakes, volcanos, or other vibrations of the earth.

**ACTIVITY 14**

**Investigative**

**Activity Standards Focus**

In Activity 14, students examine logarithmic functions and their graphs. They begin reviewing exponential functions. Then they examine the relationship between logarithmic and exponential functions and write equations using both forms. Students discover and use the properties of logarithms and graph logarithmic functions.

**Lesson 14-1**

**Plan**

**Pacing:** 1 class period

**Chunking the Lesson**

1. #1 #2 #3
   - Check Your Understanding
   - #6–7
   - #10
   - Check Your Understanding
   - Lesson Practice

**Teach**

**Bell-Ringer Activity**

Ask students to simplify each expression using properties of exponents.

1. $x^3y^{-3} \cdot xy^2$ $\left(\frac{x^5}{y}\right)$
2. $(2^3)^2$ $[64]$ $\left(\frac{b^6}{a^2}\right)$
3. $\frac{a^5b^{-5}}{a^4b^{-1}}$ $\left(\frac{3^3}{2^2}\right)$ [19,683]

**1 Summarizing, Paraphrasing, Debriefing**

The table of typical effects will help students understand the physical implications of different magnitudes. Lead a class discussion before starting this lesson to make certain that all students are comfortable with earthquake terminology.

Use the table on the next page to assess student understanding of magnitude and ground motion caused by an earthquake.

**Universal Access**

Have students look up all the meanings of the word magnitude to help them understand its meaning with respect to earthquakes. Discuss if and where they have seen or heard the word used before.

**Common Core State Standards for Activity 14**

- **HSA-CED.A.1** Create equations and inequalities in one variable and use them to solve problems.
- **HSA-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **HSF-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **HSF-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
2. Complete the table to show how many times as great the ground motion is when caused by each earthquake as compared to a magnitude 0 earthquake.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Ground Motion Compared to Magnitude 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2.0</td>
<td>100</td>
</tr>
<tr>
<td>3.0</td>
<td>1000</td>
</tr>
<tr>
<td>4.0</td>
<td>10,000</td>
</tr>
<tr>
<td>5.0</td>
<td>100,000</td>
</tr>
<tr>
<td>6.0</td>
<td>1,000,000</td>
</tr>
<tr>
<td>7.0</td>
<td>10,000,000</td>
</tr>
<tr>
<td>8.0</td>
<td>100,000,000</td>
</tr>
<tr>
<td>9.0</td>
<td>1,000,000,000</td>
</tr>
<tr>
<td>10.0</td>
<td>10,000,000,000</td>
</tr>
</tbody>
</table>

3. In parts a–c below, you will graph the data from Item 2. Let the horizontal axis represent the magnitude of the earthquake and the vertical axis represent the amount of ground motion caused by the earthquake as compared to a magnitude 0 earthquake. Alternatively, use technology to perform an exponential regression.

   a. Plot the data using a grid that displays $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. Explain why this grid is or is not a good choice.

   A $[-10, 10] \times [-10, 10]$ window would not be appropriate since only the ordered pair $(1, 10)$ would be plotted on the graph, as shown.

   b. Plot the data using a grid that displays $-10 \leq x \leq 100$ and $-10 \leq y \leq 100$. Explain why this grid is or is not a good choice.

   A $[-10, 100] \times [-10, 100]$ window would not be appropriate since only the ordered pairs $(1, 10)$ and $(2, 100)$ would be plotted, as shown. In addition, the x-axis does not need to be larger than 10 units.

### Common Core State Standards for Activity 14 (continued)

- **HSF-IF.C.7.E**: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **HSF-BF.B.4**: Find inverse functions.
- **HSF-BF.B.4.C (+)**: Read values of an inverse function from a graph or a table, given that the function has an inverse.
- **HSF-BF.B.5**: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- **HSF-LE.A.4**: For exponential models, express as a logarithm the solution $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.
Lesson 14-1
Exponential Data

c. Scales may be easier to choose if only a subset of the data is graphed and if different scales are used for the horizontal and vertical axes. Determine an appropriate subset of the data and a scale for the graph. Plot the data and label and scale the axes. Draw a function that fits the plotted data. 
One possible answer is to choose the subset \{(1, 10), (2, 100), (3, 1000), (4, 10,000)\} and plot those points.

d. Write a function \(G(x)\) for the ground motion caused compared to a magnitude 0 earthquake by a magnitude \(x\) earthquake.
\[ G(x) = 10^x \]

Check Your Understanding

4. What is the domain of the function in Item 3d? Is the graph of the function continuous?
5. Use the graph from Item 3c to estimate how many times greater the ground motion of an earthquake of magnitude 3.5 is than a magnitude 0 earthquake. Solve the equation you wrote in Item 3d to check that your estimate is reasonable.

6. Make sense of problems. In Item 3, the data were plotted so that the ground motion caused by the earthquake was a function of the magnitude of the earthquake.
   a. Is the ground motion a result of the magnitude of an earthquake, or is the magnitude of an earthquake the result of ground motion?
   An earthquake’s magnitude is not assigned until an earthquake actually happens, so an earthquake’s magnitude is a result of ground motion.
   b. Based on your answer to part a, would you choose ground motion or magnitude as the independent variable of a function relating the two quantities? What would you choose as the dependent variable?
   Ground motion should be the independent variable and magnitude should be the dependent variable of a function relating the two quantities.
c. Make a new graph of the data plotted in Item 3c so that the magnitude of the earthquake is a function of the ground motion caused by the earthquake. Scale the axes and draw a function that fits the plotted data.

7. Let the function you graphed in Item 6c be $y = M(x)$, where $M$ is the magnitude of an earthquake for which there is $x$ times as much ground motion as a magnitude 0 earthquake.

a. Identify a reasonable domain and range of the function $y = G(x)$ from Item 3d and the function $y = M(x)$ in this situation. Use interval notation.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = G(x)$</td>
<td>$[0, 10]$</td>
</tr>
<tr>
<td>$y = M(x)$</td>
<td>$(0, 10,000,000,000)$</td>
</tr>
</tbody>
</table>

b. In terms of the problem situation, describe the meaning of an ordered pair on the graphs of $y = G(x)$ and $y = M(x)$.

$y = G(x)$  Richter magnitude  Ground motion compared to magnitude 0 earthquake

$y = M(x)$  Ground motion compared to magnitude 0 earthquake  Richter magnitude
Lesson 14-1
Exponential Data

c. A portion of the graphs of \( y = G(x) \) and \( y = M(x) \) is shown on the same set of axes. Describe any patterns you observe.

Sample answer: The two functions are symmetric about the line \( y = x \). The values of \( x \) and \( y \) in \( M(x) \) are the values of \( y \) and \( x \) in \( G(x) \).

Check Your Understanding

8. How did you choose the scale of the graph you drew in Item 6c?
9. What is the relationship between the functions \( G \) and \( M \)?

10. The table shows the approximate energy released underground from an earthquake of different magnitudes in terms of the explosive force of an amount of TNT in kilograms.

<table>
<thead>
<tr>
<th>Magnitude, ( x )</th>
<th>Energy (kg), ( E(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.048</td>
</tr>
<tr>
<td>2.0</td>
<td>15</td>
</tr>
<tr>
<td>3.0</td>
<td>480</td>
</tr>
<tr>
<td>4.0</td>
<td>15,000</td>
</tr>
<tr>
<td>5.0</td>
<td>480,000</td>
</tr>
<tr>
<td>6.0</td>
<td>15,000,000</td>
</tr>
<tr>
<td>7.0</td>
<td>480,000,000</td>
</tr>
<tr>
<td>8.0</td>
<td>15,000,000,000</td>
</tr>
<tr>
<td>9.0</td>
<td>480,000,000,000</td>
</tr>
<tr>
<td>10.0</td>
<td>15,000,000,000,000</td>
</tr>
</tbody>
</table>

ACTIVITY 14
Continued

Check Your Understanding

Debrief students’ answers to these items to ensure that they understand the functions are inverses. Have them list a few points from each function and reverse the coordinates to help them understand this.

Answers
8. reversed the axes from the graph for Item 3c
9. The functions \( G \) and \( M \) are inverse functions.

10 Create Representations, Discussion Groups, Debriefing

This item allows students to model an exponential function with both a graph and an equation. Have students discuss what makes a scale appropriate for the data. Ask students to decide whether it is feasible to show all of the given data on the graph. You may encourage students to use the graphic organizer Round Table Discussion to help them keep track of other students’ ideas.
Lesson 14-1
Exponential Data

a. Determine an appropriate subset of the data and a scale for the graph. Plot the data and label and scale the axes. Draw a smooth curve through the points.

Possible answer: 
[((1, 0.48), (2, 15), (3, 480), (4, 15,000), (5, 480,000))] 

b. Use a graphing calculator to perform an exponential regression and write a function \( E(x) \) to model the TNT equivalent energy released underground by an earthquake of magnitude \( x \) on the Richter scale. Round each value to the nearest thousandth.

\[ E(x) = 0.006(35.842)^x \]

Check Your Understanding

11. What is a reasonable domain of the function in Item 10b? Use interval notation.

12. Refer to your function from Item 10b. About how many times more energy is released when the magnitude of an earthquake increases by 1?

LESSON 14-1 PRACTICE

13. Magnitude 5.0 is 1000 times greater than magnitude 2.0.

14. Magnitude 7.0 is 10,000,000 times greater than magnitude 0.

15. Magnitude 6.0 is 10 times greater than magnitude 5.0.

16. 100

17. No; the function does not increase linearly. The ground motion of an earthquake of magnitude 6 is 1000 times the ground motion of an earthquake of magnitude 3.

Critique the reasoning of others. Garrett said that the ground motion of an earthquake of magnitude 6 is twice the ground motion of an earthquake of magnitude 3. Is Garrett correct? Explain.
The Richter scale uses a base 10 logarithmic scale. A base 10 logarithmic scale means that when the ground motion is expressed as a power of 10, the magnitude of the earthquake is the exponent. You have seen this function $G(x) = 10^x$, where $x$ is the magnitude, in Item 3d of the previous lesson.

The function $M$ is the inverse of an exponential function $G$ whose base is 10. The algebraic rule for $M$ is a common logarithmic function. Write this function as $M(x) = \log x$, where $x$ is the ground motion compared to a magnitude 0 earthquake.

1. Graph $M(x) = \log x$ on a graphing calculator.
   a. Make a sketch of the calculator graph. Be certain to label and scale each axis.
   b. Use $M$ to estimate the magnitude of an earthquake that causes 120,000 times the ground motion of a magnitude 0 earthquake. Describe what would happen if this earthquake were centered beneath a large city.
   
   $M(120,000) \approx 5.08$. According to the information given in the previous lesson, an earthquake of this magnitude could cause damage to buildings.
   
   c. Use $M$ to determine the amount of ground motion caused by the 2002 magnitude 7.9 Denali earthquake compared to a magnitude 0 earthquake.
   
   79,432,823.47 times as much motion as magnitude 0
   
   d. What is the relationship between the exponential function base 10 and the common logarithmic function? Justify your answer.
   
   Sample answer: They are inverses. The exponential graph contains the point (0, 1) and the logarithmic graph contains the point (1, 0).
2. Complete the tables below to show the relationship between the exponential function base 10 and the common logarithmic function.

<table>
<thead>
<tr>
<th>x</th>
<th>y = 10^x</th>
<th>x</th>
<th>y = log x</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10^0 = 1</td>
<td>1</td>
<td>log 1 = 0</td>
</tr>
<tr>
<td>1</td>
<td>10^1 = 10</td>
<td>10</td>
<td>log (10) = 1</td>
</tr>
<tr>
<td>2</td>
<td>10^2 = 100</td>
<td>100</td>
<td>log (100) = 2</td>
</tr>
<tr>
<td>3</td>
<td>10^3 = 1000</td>
<td>1000</td>
<td>log (1000) = 3</td>
</tr>
<tr>
<td>log x</td>
<td>10^log x = x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Use the data in the table in Item 2 to sketch the function \( y = 10^x \).

4. Use the data in the table in Item 2 to make a sketch of the function \( y = \log x \).

5. Use a graphing calculator to graph \( y = 10^x \) and \( y = \log x \) in the same viewing window. How can you tell that \( y = 10^x \) and \( y = \log x \) are inverses? The graphs are symmetric about the line \( y = x \).
Lesson 14-2
The Common Logarithm Function

6. Write a logarithmic statement for each exponential statement.
   a. \(10^4 = 10,000\)  
      \[\log (10,000) = 4\]
   b. \(10^{-1} = \frac{1}{10}\)  
      \[\log \left(\frac{1}{10}\right) = -1\]

7. Write each logarithmic statement as an exponential statement.
   a. \(\log 100,000 = 5\)  
      \[10^5 = 100,000\]
   b. \(\log \left(\frac{1}{100}\right) = -2\)  
      \[10^{-2} = \frac{1}{100}\]

8. Evaluate each logarithmic expression without using a calculator.
   a. \(\log 1000 \quad 3\)  
      \[\log 10,000,000 = -4\]

Check Your Understanding

9. What function has a graph that is symmetric to the graph of \(y = \log x\) about the line \(y = x\)? Graph both functions and the line \(y = x\).

10. Evaluate \(10^x\) for \(x = 1, 2, 3,\) and \(4\).

11. Let \(f(x) = 10^x\) and let \(g(x) = f^{-1}(x)\). What is the algebraic rule for \(g(x)\)? Describe the relationship between \(f(x)\) and \(g(x)\).

LESSON 14-2 PRACTICE

12. Evaluate without using a calculator.
   a. \(\log 10^6\)  
      \[\log (1,000,000) = \log 100,000\]
   b. \(\log \left(\frac{1}{10}\right)\)  
      \[\log \left(\frac{1}{10}\right) = -1\]

13. Write an exponential statement for each.
   a. \(\log 10 = 1\)  
      \[10^1 = 10\]
   b. \(\log \left(\frac{1}{100}\right) = -6\)  
      \[10^{-6} = \frac{1}{1000,000}\]
   c. \(\log a = b\)

14. Write a logarithmic statement for each.
   a. \(10^3 = 10,000,000\)  
      \[\log 10^3 = 10,000,000\]
   b. \(10^0 = 1\)
   c. \(10^n = n\)

15. Model with mathematics. The number of decibels \(D\) of a sound is modeled with the equation \(D = 10 \log \left(\frac{I}{10^{-12}}\right)\), where \(I\) is the intensity of the sound measured in watts per square meter. Find the number of decibels in each of the following:
   a. whisper with \(I = 10^{-10}\)
   b. normal conversation with \(I = 10^{-6}\)
   c. vacuum cleaner with \(I = 10^{-4}\)
   d. front row of a rock concert with \(I = 10^{-1}\)
   e. military jet takeoff with \(I = 10^7\)

Academic Vocabulary

The range of sound intensities by humans varies over such a large range that it is more convenient to use a logarithmic scale such as the decibel. The decibel is one-tenth of a bel, which is named after Alexander Graham Bell, the inventor of the telephone.

LESSON 14-2 PRACTICE

12. a. 6
    b. 6
    c. -2

13. a. \(10^3 = 10\)
    b. \(10^{-6} = \frac{1}{1,000,000}\)
    c. \(10^0 = a\)

14. a. \(\log 10,000,000 = 7\)
    b. \(\log 1 = 0\)
    c. \(\log n = m\)

ADAPT

Check students’ answers to the Lesson Practice items to ensure that they understand the inverse of an exponential function is a logarithmic function and the inverse of a logarithmic function is an exponential function.

Answers

9. \(y = 10^x\)

Assess

Students’ answers to the Lesson Practice items will provide a formative assessment of their understanding of common logarithms, and of students’ ability to apply their learning. Short-cycle formative assessment items for Lesson 14-2 are also available in the Assessment section on SpringBoard Digital.

Activity 14 • Logarithms and Their Properties 199
You have already learned the properties of exponents. Logarithms also have properties.

1. Complete these three properties of exponents.
   \[ a^m \cdot a^n = a^{m+n} \]
   \[ \frac{a^m}{a^n} = a^{m-n} \]
   \[ (a^m)^n = a^{mn} \]

2. Use appropriate tools strategically. Use a calculator to complete the tables below. Round each answer to the nearest thousandth.

   \[
   \begin{array}{c|c}
   x & y = \log x \\
   \hline
   1 & 0 \\
   2 & 0.301 \\
   3 & 0.477 \\
   4 & 0.602 \\
   5 & 0.699 \\
   \end{array}
   \quad
   \begin{array}{c|c}
   x & y = \log x \\
   \hline
   6 & 0.778 \\
   7 & 0.845 \\
   8 & 0.903 \\
   9 & 0.954 \\
   10 & 1 \\
   \end{array}
   \]

3. Add the logarithms from the tables in Item 2 to see if you can develop a property. Find each sum and round each answer to the nearest thousandth.
   \[ \log 2 + \log 3 \approx 0.778 = \log 6 \]
   \[ \log 2 + \log 4 \approx 0.903 = \log 8 \]
   \[ \log 2 + \log 5 \approx 1 = \log 10 \]
   \[ \log 3 + \log 3 \approx 0.954 = \log 9 \]

MINI-LESSON: Review of Exponent Properties

Students need to be familiar with the properties of exponents in this lesson. If students need to review these properties, a mini-lesson is available to provide practice.

See the Teacher Resources on SpringBoard Digital for a student page for this mini-lesson.
4. Compare the answers in Item 3 to the tables of data in Item 2.
   a. **Express regularity in repeated reasoning.** Is there a pattern or property when these logarithms are added? If yes, explain the pattern that you have found.
   **When two logarithms of like bases are added together, the result is the logarithm of the product of the input values.**

   b. State the property of logarithms that you found by completing the following statement.
   \[ \log m + \log n = \underline{\log (mn)} \]

5. Explain the connection between the property of logarithms stated in Item 4 and the corresponding property of exponents in Item 1.
   **When exponential expressions with like bases are multiplied, the exponents are added. When logarithms with like bases are added, the result is the logarithm of the product of their inputs.**

6. Graph \( y_1 = \log 2 + \log x \) and \( y_2 = \log 2x \) on a graphing calculator. What do you observe? Explain.
   **The two functions are identical because \( y_2 = \log (2x) = \log (2) + \log (x) = y_1 \) by the property in Item 4b.**

### Check Your Understanding

Identify each statement as true or false. Justify your answers.

7. \( \log mn = (\log m)(\log n) \)
   - **False; it is the sum of the logs:** \( \log m + \log n \)

8. \( \log xy = \log x + \log y \)
   - **True by the property in Item 4b**

9. Make a conjecture about the property of logarithms that relates to the property of exponential equations that states the following:
   \[ \frac{a^m}{a^n} = a^{m-n}. \]
   **The conjecture is \( \log (m) - \log (n) = \log \left( \frac{m}{n} \right) \).**

10. Use the information from the tables in Item 2 to provide examples that support your conjecture in Item 9.
    **Sample answers:**
    - \( \log (4) - \log (2) = 0.301 = \log \left( \frac{4}{2} \right) = \log (2) \)
    - \( \log (6) - \log (2) = 0.477 = \log \left( \frac{6}{2} \right) = \log (3) \)

11. Graph \( y_1 = \log x - \log 2 \) and \( y_2 = \log \frac{x}{2} \) on a graphing calculator. What do you observe?
    **The graphs of the two functions are identical.**
Check Your Understanding
Debrief students’ answers to these items to ensure that they can use the Product and Quotient Properties to rewrite logarithms.

Answers
12. Sample answers: log (9 • 4) = log 9 + log 4 = 0.954 + 0.602 = 1.556;
   log (6 • 6) = log 6 + log 6 = 0.778 + 0.778 = 1.556
13. Sample answers: log 10 = log 10 − log 9 = log 10 − 0.699 = 0.301
14. log 7 = 0.845; log 3 = 0.477;
   log 4 = 0.602; 0.477 + 0.602 = 1.079, not 0.845.

LESSON 14-3 PRACTICE
15. a. log \(\frac{8}{3}\) = log 8 − log 3
   = 0.903 − 0.477 = 0.426
   b. log 24 = log (8 • 3) = log 8 + log 3 = 0.903 + 0.477 = 1.38
   c. log 64 = log (8 • 8) = log 8 + log 8 = 0.903 + 0.903 = 1.806
   d. log 27 = log (3 • 3 • 3) = log 3 + log 3 + log 3
      = 0.477 + 0.477 + 0.477 = 1.431
16. a. log \(\frac{4}{9}\) = log 4 − log 9
    = 0.602 − 0.954 = −0.352
   b. log 2.25 = log \(\frac{9}{4}\) = log 9 − log 4
      = 0.954 − 0.602 = 0.352
   c. log 144 = log (4 • 4 • 9) = log 4 + log 4 + log 9
      = 0.602 + 0.602 + 0.954 = 2.158
   d. log 81 = log (9 • 9) = log 9 + log 9 = 0.954 + 0.954 = 1.908
17. Rewrite log 7 + log x − (log 3 + log y) as a single logarithm.
18. Rewrite log \(\frac{8m}{9n}\) as a sum of four logarithmic terms.
19. Make use of structure. Rewrite log 8 + log 2 − log 4 as a single logarithm and evaluate the result using the table at the beginning of the lesson.

ADAPT
Check students’ answers to the Lesson Practice to ensure that they understand how to evaluate a logarithmic expression and rewrite an expression as a single logarithm. As an additional activity, use index cards to create a game for students to match expressions. For example, log \(\frac{4}{9}\) would match the expression log 4 − log 9.

See the Activity Practice on page 205 and the Additional Unit Practice in the Teacher Resources on SpringBoard Digital for additional problems for this lesson.

You may wish to use the Teacher Assessment Builder on SpringBoard Digital to create custom assessments or additional practice.
Lesson 14-4
More Properties of Logarithms

Learning Targets:
- Make conjectures about properties of logarithms.
- Write and apply the Power Property of Logarithms.
- Rewrite logarithmic expressions by using their properties.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations

1. Make a conjecture about the property of logarithms that relates to the property of exponents that states the following: \((a^m)^n = a^{mn}\).
   \[ \log (m^n) = n \log (m) \]

2. Use the information from the tables in Item 2 in the previous lesson and the properties developed in Items 4 and 9 in the previous lesson to support your conjecture in Item 1.
   Possible answers are given below.
   \[ \log (3^2) = \log (3 \cdot 3) = \log (3) + \log (3) = 2 \log (3) \]
   \[ \log (4^2) = \log (4 \cdot 4) = \log (4) + \log (4) = 2 \log (4) \]
   \[ \log (2^2) = \log (2 \cdot 2 \cdot 2) = \log (2) + \log (2) + \log (2) = 3 \log (2) \]

3. Use appropriate tools strategically. Graph \(y_1 = 2 \log x\) and \(y_2 = \log x^2\) on a graphing calculator. What do you observe?
   The graphs of the two functions are identical.

Check Your Understanding

1. Make a conjecture about the property of logarithms that relates to the property of exponents that states the following: \((a^m)^n = a^{mn}\).
   \[ \log (m^n) = n \log (m) \]

2. Use the information from the tables in Item 2 in the previous lesson and the properties developed in Items 4 and 9 in the previous lesson to support your conjecture in Item 1.
   Possible answers are given below.
   \[ \log (3^2) = \log (3 \cdot 3) = \log (3) + \log (3) = 2 \log (3) \]
   \[ \log (4^2) = \log (4 \cdot 4) = \log (4) + \log (4) = 2 \log (4) \]
   \[ \log (2^2) = \log (2 \cdot 2 \cdot 2) = \log (2) + \log (2) + \log (2) = 3 \log (2) \]

3. Use appropriate tools strategically. Graph \(y_1 = 2 \log x\) and \(y_2 = \log x^2\) on a graphing calculator. What do you observe?
   The graphs of the two functions are identical.

Check Your Understanding

Identify each statement as true or false. Justify your answer.

4. \(2 \log \sqrt{m} = \log m\)
5. \(\log 10^2 = \log 2^{10}\)

6. Express regularity in repeated reasoning. The logarithmic properties that you conjectured and then verified in this lesson and the previous lesson are listed below. State each property.

   Product Property: \(\log m + \log n = \log (mn)\)
   Quotient Property: \(\log(m) - \log(n) = \log \left(\frac{m}{n}\right)\)
   Power Property: \(\log (m^n) = n \log (m)\)

6–9 Think-Pair-Share, Create Representations, Debriefing
- Item 6 gives students an opportunity to reflect on the properties derived in the activity. After sharing answers with the entire group to make sure all responses are correct, students can record the information in their math notebooks.
- Monitor group discussions to ensure that all members are participating. Pair or group students carefully to facilitate discussions and understanding of both routine language and mathematical terms.

ACTIVITY 14
Lesson 14-4

PLAN

Pacing: 1 class period

Chunking the Lesson
- #1–3
- #6–9

Check Your Understanding
- #6–9

Check Your Understanding
- #1–3

Pacing:

ACTIVITY 14

ACTIVITY 14 Continued

Check Your Understanding

ACTIVITY 14

Bell-Ringer Activity

Ask students to identify each statement as true or false. If the statement is false, have them correct it.

1. \(\log 3 + \log 5 = \log 8\) [false; \(\log 3 + \log 5 = \log 2 \cdot \log 5\)]
2. \(\log 10 - \log 5 = \log 2\) [true]
3. \(\log 1000 = 3\) [true]

1–3 Guess and Check, Create Representations, Look for a Pattern, Debriefing

Students may need coaching to develop the property in Item 1. There are two ways to approach this problem, following the patterns from Items 2–11. Students could try either \(\log (u^v)\) or \(\log (u^v)\). They may struggle with \(\log (u^v)\) as there is no pattern to be found. However, if students try \(\log (uv)\), they will have more success if they recognize exponentiation as repeated multiplication and apply the Product Property of Logarithms:

\[ \log(u^v) = \log(u \cdot u \cdot u \cdot \ldots \cdot u) = \log(u) + \log(u) + \ldots + \log(u) = v \log(u) \]

In Item 2, students should apply the Product Property of Logarithms to verify their conjecture numerically. For example,

\[ \log(3^2) = \log(3 \cdot 3) = \log 3 + \log 3 = 2 \log 3 \]

In Item 3, students verify their conjecture graphically. A suggested graphing window would be \([0, 10]\) for the \(x\)-axis and \([-2, 2]\) for the \(y\)-axis.

Check Your Understanding

- Debrief students’ answers to these items to ensure that they can correctly use properties of logarithms to justify their answers.

Answers

4. True; by the property in Item 1,
   \[ 2 \log \sqrt{m} = \log \sqrt{m^2} = \log m \]
5. False; \(\log 10^2 = 2\)
7. Use the properties from Item 6 to rewrite each expression as a single logarithm. Assume all variables are positive.
   a. \( \log x - \log 7 \)  
      \( \log \frac{x}{7} \)
   b. \( 2 \log x + \log y \)  
      \( \log (x^2y) \)

8. Use the properties from Item 6 to expand each expression. Assume all variables are positive.
   a. \( \log 5xy^4 \)  
      \( \log 5 + \log x + 4 \log y \)
   b. \( \log x \)  
      \( \log x - 3 \log y \)

9. Rewrite each expression as a single logarithm. Then evaluate.
   a. \( \log 2 + \log 5 \)  
      \( \log 10 = 1 \)
   b. \( \log 5000 - \log 5 \)  
      \( \log 1000 = 3 \)
   c. \( 2 \log 5 + \log 4 \)  
      \( \log 100 = 2 \)

10. Explain why \( \log (a + 10) \) does not equal the sum of the logs; only the product equals the sum of the logs.

11. \( \log (-100) \) is not defined since it represents the exponent you would raise 10 to in order to get \(-100\). A base of 10 will never give a negative value.

12. \( \log 100 = 2 \)
13. \( \log \frac{1}{10} = -1 \)
14. \( \log 10,000 = 4 \)
15. \( \log \frac{1}{100} = -2 \)
16. \( \log 100 \left( \frac{1}{100} \right) = \log 1 = 0 \)
17. \( \log b + 3 \log c + 2 \log d \)

Check Your Understanding

10. Explain why \( \log (a + 10) \) does not equal \( \log a + 1 \).
11. Explain why \( \log (-100) \) is not defined.
ACTIVITY 14 PRACTICE
Answer each item. Show your work.

Lesson 14-1

1. What is the y-intercept of the graph?
2. What is the x-intercept of the graph?
3. Is \( M(x) \) an increasing or decreasing function?
4. Which of these statements are NOT true regarding the graph above?
   a. The graph contains the point (1, 0).
   b. The graph contains the point (10, 1).
   c. The domain is \( x > 0 \).
   d. The x-axis is an asymptote.

Lesson 14-2

5. Use a calculator to find a decimal approximation rounded to three decimal places.
   a. \( \log 47 \)
   b. \( \log 32.013 \)
   c. \( \log \left( \frac{5}{7} \right) \)
   d. \( \log -20 \)
6. A logarithm is (a(n)
   a. variable.
   b. constant.
   c. exponent.
   d. coefficient.

7. Write an exponential statement for each logarithmic statement below.
   a. \( \log 10,000 = 4 \)
   b. \( \log \frac{1}{100,000,000} = -9 \)
   c. \( \log a = 6 \)

8. Write a logarithmic statement for each exponential statement below.
   a. \( 10^{-2} = \frac{1}{100} \)
   b. \( 10^3 = 10 \)
   c. \( 10^4 = n \)

9. Evaluate without using a calculator.
   a. \( \log 10^5 \)
   b. \( \log 100 \)
   c. \( \log \frac{1}{100,000} \)

10. If \( \log a = x \), and \( 10 < a < 100 \), what values are acceptable for \( x \)?
    a. \( 0 < x < 1 \)
    b. \( 1 < x < 2 \)
    c. \( 2 < x < 3 \)
    d. \( 10 < x < 100 \)

Lesson 14-3

11. If \( \log 2 = 0.301 \) and \( \log 3 = 0.447 \), find each of the following using only these values and the properties of logarithms. Show your work.
    a. \( \log 6 \)
    b. \( \log \left( \frac{2}{3} \right) \)
    c. \( \log 1.5 \)
    d. \( \log 18 \)
12. Which expression does NOT equal 3?

A. \( \log 10^4 \)
B. \( \log \frac{10^4}{10} \)
C. \( \log \left( \frac{10^4}{10} \right) \)
D. \( \log 10^{-4} \)

13. Expand each expression.

a. \( \log \left( \frac{3x}{8y} \right) \)

b. \( \log \left( \frac{m+y}{3} \right) \)

c. \( \log \left( \frac{4}{9-u} \right) \)

14. If \( \log 2 = 0.301 \) and \( \log 3 = 0.477 \), find each of the following using the properties of logarithms.

a. \( \log 4 \)

b. \( \log 27 \)

c. \( \log \sqrt{2} \)

d. \( \log \sqrt{12} \)

15. Complete each statement to illustrate a property for logarithms.

a. Product Property \( \log uv = ? \)

b. Quotient Property \( \log \frac{u}{v} = ? \)

c. Power Property \( \log u^x = ? \)

16. Which expression does NOT equal 3?

A. \( \log 10^4 \)
B. \( \log \frac{10^4}{10} \)
C. \( \log \left( \frac{10^4}{10} \right) \)
D. \( \log 10^{-4} \)

17. Rewrite each expression as a single logarithm.

a. \( \log 3 + \log x - (\log 8 + \log y) \)

b. \( \log (m + n) - \log 3 \)

c. \( \log 4 - \log (9 - u) \)

18. Expand each expression.

a. \( \log xy \)

b. \( \log \frac{xy}{z} \)

c. \( \log a^2b^3 \)

19. If \( \log 8 = 0.903 \) and \( \log 3 = 0.477 \), find each of the following using the properties of logarithms.

a. \( \log 3^2 \)

b. \( \log (2^3) \)

c. \( \log 8(3^2) \)

20. Write each expression without using exponents.

a. \( m \log n + \log n^2 \)

b. \( \log (mn)^2 \)

c. \( \log 2^4 + \log 2^3 \)

21. Which of the following statements is TRUE?

A. \( \log \frac{x}{y} = \log x \)

B. \( \log \frac{x}{y} = y \log x \)

C. \( \log (x + y) = \log x + \log y \)

D. \( \log \sqrt{x} = \frac{1}{2} \log x \)

22. Verify using the properties of logarithms that \( \log 10^x = \log 10^y = x \rightarrow 4 \). Then evaluate for \( x = \pi \), using 3.14 for \( \pi \).
Learning Targets:

- Complete tables and plot points for exponential data.
- Write and graph an exponential function for a given context.
- Find the domain and range of an exponential function.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Paraphrasing, Create Representations, Quickwrite, Close Reading, Look for a Pattern, Discussion Groups

In 1935, Charles F. Richter developed the Richter magnitude test scale to compare the size of earthquakes. The Richter scale is based on the amplitude of the seismic waves recorded on seismographs at various locations after being adjusted for distance from the epicenter of the earthquake.

Richter assigned a magnitude of 0 to an earthquake whose amplitude on a seismograph is 1 micron, or $10^{-4}$ cm. According to the Richter scale, a magnitude 1.0 earthquake causes 10 times the ground motion of a magnitude 0 earthquake. A magnitude 2.0 earthquake causes 10 times the ground motion of a magnitude 1.0 earthquake. This pattern continues as the magnitude of the earthquake increases.

1. **Reason quantitatively.** How does the ground motion caused by earthquakes of these magnitudes compare?
   a. magnitude 5.0 earthquake compared to magnitude 4.0
   b. magnitude 4.0 earthquake compared to magnitude 1.0
   c. magnitude 4.0 earthquake compared to magnitude 0

The list below describes the effects of earthquakes of different magnitudes. Read through this list with your group and identify any words that might be unfamiliar. Find their meanings to aid your understanding.

**Typical Effects of Earthquakes of Various Magnitudes**

1.0 Very weak, no visible damage
2.0 Not felt by humans
3.0 Often felt, usually no damage
4.0 Windows rattle, indoor items shake
5.0 Damage to poorly constructed structures
6.0 Some damage to well-built structures
7.0 Damage to most buildings, some collapse
8.0 Major damage to most buildings, many collapse
9.0 Near total destruction
10.0 Extremely rare, never recorded
2. Complete the table to show how many times as great the ground motion is when caused by each earthquake as compared to a magnitude 0 earthquake.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Ground Motion Compared to Magnitude 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2.0</td>
<td>100</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td></td>
</tr>
</tbody>
</table>

3. In parts a–c below, you will graph the data from Item 2. Let the horizontal axis represent the magnitude of the earthquake and the vertical axis represent the amount of ground motion caused by the earthquake as compared to a magnitude 0 earthquake. Alternatively, use technology to perform an exponential regression.

a. Plot the data using a grid that displays $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. Explain why this grid is or is not a good choice.

b. Plot the data using a grid that displays $-10 \leq x \leq 100$ and $-10 \leq y \leq 100$. Explain why this grid is or is not a good choice.
Lesson 14-1
Exponential Data

c. Scales may be easier to choose if only a subset of the data is graphed and if different scales are used for the horizontal and vertical axes. Determine an appropriate subset of the data and a scale for the graph. Plot the data and label and scale the axes. Draw a function that fits the plotted data.

d. Write a function $G(x)$ for the ground motion caused compared to a magnitude 0 earthquake by a magnitude $x$ earthquake.

Check Your Understanding

4. What is the domain of the function in Item 3d? Is the graph of the function continuous?

5. Use the graph from Item 3c to estimate how many times greater the ground motion of an earthquake of magnitude 3.5 is than a magnitude 0 earthquake. Solve the equation you wrote in Item 3d to check that your estimate is reasonable.

6. Make sense of problems. In Item 3, the data were plotted so that the ground motion caused by the earthquake was a function of the magnitude of the earthquake.
   a. Is the ground motion a result of the magnitude of an earthquake, or is the magnitude of an earthquake the result of ground motion?

   b. Based on your answer to part a, would you choose ground motion or magnitude as the independent variable of a function relating the two quantities? What would you choose as the dependent variable?

MATH TIP

The value of the dependent variable is the output of the function and is determined by the input or the value of the independent variable.
c. Make a new graph of the data plotted in Item 3c so that the magnitude of the earthquake is a function of the ground motion caused by the earthquake. Scale the axes and draw a function that fits the plotted data.

7. Let the function you graphed in Item 6c be \( y = M(x) \), where \( M \) is the magnitude of an earthquake for which there is \( x \) times as much ground motion as a magnitude 0 earthquake.
   
a. Identify a reasonable domain and range of the function \( y = G(x) \) from Item 3d and the function \( y = M(x) \) in this situation. Use interval notation.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = G(x) )</td>
<td></td>
</tr>
<tr>
<td>( y = M(x) )</td>
<td></td>
</tr>
</tbody>
</table>

b. In terms of the problem situation, describe the meaning of an ordered pair on the graphs of \( y = G(x) \) and \( y = M(x) \).

\[ y = G(x) \quad \text{__________________________} \]
\[ y = M(x) \quad \text{__________________________} \]
Lesson 14-1
Exponential Data

c. A portion of the graphs of \( y = G(x) \) and \( y = M(x) \) is shown on the same set of axes. Describe any patterns you observe.

Check Your Understanding

8. How did you choose the scale of the graph you drew in Item 6c?
9. What is the relationship between the functions \( G \) and \( M \)?

10. The table shows the approximate energy released underground from an earthquake of different magnitudes in terms of the explosive force of an amount of TNT in kilograms.

<table>
<thead>
<tr>
<th>Magnitude, ( x )</th>
<th>Energy (kg), ( E(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.048</td>
</tr>
<tr>
<td>2.0</td>
<td>15</td>
</tr>
<tr>
<td>3.0</td>
<td>480</td>
</tr>
<tr>
<td>4.0</td>
<td>15,000</td>
</tr>
<tr>
<td>5.0</td>
<td>480,000</td>
</tr>
<tr>
<td>6.0</td>
<td>15,000,000</td>
</tr>
<tr>
<td>7.0</td>
<td>480,000,000</td>
</tr>
<tr>
<td>8.0</td>
<td>15,000,000,000</td>
</tr>
<tr>
<td>9.0</td>
<td>480,000,000,000</td>
</tr>
<tr>
<td>10.0</td>
<td>15,000,000,000,000</td>
</tr>
</tbody>
</table>

Chemistry
TNT is the chemical compound trinitrotoluene. It is a highly flammable compound used in explosives.
Lesson 14-1
Exponential Data

a. Determine an appropriate subset of the data and a scale for the graph. Plot the data and label and scale the axes. Draw a smooth curve through the points.

![Graph](image)

b. Use a graphing calculator to perform an exponential regression and write a function \( E(x) \) to model the TNT equivalent energy released underground by an earthquake of magnitude \( x \) on the Richter scale. Round each value to the nearest thousandth.

Check Your Understanding

11. What is a reasonable domain of the function in Item 10b? Use interval notation.
12. Refer to your function from Item 10b. About how many times more energy is released when the magnitude of an earthquake increases by 1?

LESSON 14-1 PRACTICE

How does the ground motion caused by earthquakes of these magnitudes compare?

13. magnitude 5.0 compared to magnitude 2.0
14. magnitude 7.0 compared to magnitude 0
15. magnitude 6.0 compared to magnitude 5.0
16. A 1933 California earthquake had a Richter scale reading of 6.3. How many times more powerful was the Alaska 1964 earthquake with a reading of 8.3?
17. Critique the reasoning of others. Garrett said that the ground motion of an earthquake of magnitude 6 is twice the ground motion of an earthquake of magnitude 3. Is Garrett correct? Explain.
Lesson 14-2
The Common Logarithm Function

Learning Targets:
- Use technology to graph \( y = \log x \).
- Evaluate a logarithm using technology.
- Rewrite exponential equations as their corresponding logarithmic equations.
- Rewrite logarithmic equations as their corresponding exponential equations.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Vocabulary Organizer, Create Representations, Quickwrite, Think-Pair-Share

The Richter scale uses a base 10 logarithmic scale. A base 10 logarithmic scale means that when the ground motion is expressed as a power of 10, the magnitude of the earthquake is the exponent. You have seen this function \( G(x) = 10^x \), where \( x \) is the magnitude, in Item 3d of the previous lesson.

The function \( M \) is the inverse of an exponential function \( G \) whose base is 10. The algebraic rule for \( M \) is a common logarithmic function. Write this function as \( M(x) = \log x \), where \( x \) is the ground motion compared to a magnitude 0 earthquake.

1. Graph \( M(x) = \log x \) on a graphing calculator.
   a. Make a sketch of the calculator graph. Be certain to label and scale each axis.

   ![Graph of M(x) = log x]

   b. Use \( M \) to estimate the magnitude of an earthquake that causes 120,000 times the ground motion of a magnitude 0 earthquake. Describe what would happen if this earthquake were centered beneath a large city.

   c. Use \( M \) to determine the amount of ground motion caused by the 2002 magnitude 7.9 Denali earthquake compared to a magnitude 0 earthquake.

   d. What is the relationship between the exponential function base 10 and the common logarithmic function? Justify your answer.

**MATH TERMS**
A logarithm is an exponent to which a base is raised that results in a specified value.

A common logarithm is a base 10 logarithm, such as \( \log 100 = 2 \), because \( 10^2 = 100 \).

**TECHNOLOGY TIP**
The \( \log \) key on your calculator is for common, or base 10, logarithms.

**POINT OF INTEGRATION**
Geometry and Algebra
Coordinate geometry is the study of graphs of algebraic equations. Logarithmic functions can be represented graphically or symbolically.
2. Complete the tables below to show the relationship between the exponential function base 10 and the common logarithmic function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 10^x$</th>
<th>$x$</th>
<th>$y = \log x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$10^0 = 1$</td>
<td>1</td>
<td>$10^0 = 1$</td>
</tr>
<tr>
<td>1</td>
<td>$1$</td>
<td>10</td>
<td>$10^1$</td>
</tr>
<tr>
<td>2</td>
<td>$100$</td>
<td>100</td>
<td>$10^2$</td>
</tr>
<tr>
<td>3</td>
<td>$1000$</td>
<td>1000</td>
<td>$10^3$</td>
</tr>
<tr>
<td>$\log x$</td>
<td>$10^x$</td>
<td>$10^x$</td>
<td>$10^x$</td>
</tr>
</tbody>
</table>

3. Use the data in the table in Item 2 to sketch the function $y = 10^x$.

4. Use the data in the table in Item 2 to make a sketch of the function $y = \log x$.

5. Use a graphing calculator to graph $y = 10^x$ and $y = \log x$ in the same viewing window. How can you tell that $y = 10^x$ and $y = \log x$ are inverses?
6. Write a logarithmic statement for each exponential statement.
   a. \(10^4 = 10,000\)  
   b. \(10^{-1} = \frac{1}{10}\)

7. Write each logarithmic statement as an exponential statement.
   a. \(\log 100,000 = 5\)  
   b. \(\log \frac{1}{100} = -2\)

8. Evaluate each logarithmic expression without using a calculator.
   a. \(\log 1000\)  
   b. \(\log \frac{1}{10,000}\)

Check Your Understanding

9. What function has a graph that is symmetric to the graph of \(y = \log x\) about the line \(y = x\)? Graph both functions and the line \(y = x\).

10. Evaluate \(\log 10^x\) for \(x = 1, 2, 3,\) and \(4\).

11. Let \(f(x) = 10^x\) and let \(g(x) = f^{-1}(x)\). What is the algebraic rule for \(g(x)\)? Describe the relationship between \(f(x)\) and \(g(x)\).

LESSON 14-2 PRACTICE

12. Evaluate without using a calculator.
    a. \(\log 10^6\)  
    b. \(\log 1,000,000\)  
    c. \(\log \frac{1}{100}\)

13. Write an exponential statement for each.
    a. \(\log 10 = 1\)  
    b. \(\log \frac{1}{1,000,000} = -6\)  
    c. \(\log a = b\)

14. Write a logarithmic statement for each.
    a. \(10^7 = 10,000,000\)  
    b. \(10^0 = 1\)  
    c. \(10^m = n\)

15. Model with mathematics. The number of decibels \(D\) of a sound is modeled with the equation \(D = 10 \log \left( \frac{I}{10^{-12}} \right)\) where \(I\) is the intensity of the sound measured in watts per square meter. Find the number of decibels in each of the following:
    a. whisper with \(I = 10^{-10}\)  
    b. normal conversation with \(I = 10^{-6}\)  
    c. vacuum cleaner with \(I = 10^{-4}\)  
    d. front row of a rock concert with \(I = 10^{-1}\)  
    e. military jet takeoff with \(I = 10^2\)

MATH TIP

Recall that two functions are inverses when \(f(f^{-1}(x)) = f^{-1}(f(x)) = x\).

The exponent \(x\) in the equation \(y = 10^x\) is the common logarithm of \(y\). This equation can be rewritten as \(\log y = x\).

ACADEMIC VOCABULARY

Activity 14 • Logarithms and Their Properties
Learning Targets:
- Make conjectures about properties of logarithms.
- Write and apply the Product Property and Quotient Property of Logarithms.
- Rewrite logarithmic expressions by using properties.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Create Representations, Look for a Pattern, Quickwrite, Guess and Check

You have already learned the properties of exponents. Logarithms also have properties.

1. Complete these three properties of exponents.
   \[ a^m \cdot a^n = \quad \]
   \[ \frac{a^m}{a^n} = \quad \]
   \[ (a^m)^n = \quad \]

2. Use appropriate tools strategically. Use a calculator to complete the tables below. Round each answer to the nearest thousandth.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \log x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \log x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

3. Add the logarithms from the tables in Item 2 to see if you can develop a property. Find each sum and round each answer to the nearest thousandth.
   \[ \log 2 + \log 3 = \quad \]
   \[ \log 2 + \log 4 = \quad \]
   \[ \log 2 + \log 5 = \quad \]
   \[ \log 3 + \log 3 = \quad \]
Lesson 14-3
Properties of Logarithms

4. Compare the answers in Item 3 to the tables of data in Item 2.
   a. Express regularity in repeated reasoning. Is there a pattern or property when these logarithms are added? If yes, explain the pattern that you have found.

   b. State the property of logarithms that you found by completing the following statement.
      \( \log m + \log n = \) ____________________

5. Explain the connection between the property of logarithms stated in Item 4 and the corresponding property of exponents in Item 1.

6. Graph \( y_1 = \log 2 + \log x \) and \( y_2 = \log 2x \) on a graphing calculator. What do you observe? Explain.

Check Your Understanding

Identify each statement as true or false. Justify your answers.
7. \( \log mn = (\log m)(\log n) \)
8. \( \log xy = \log x + \log y \)

9. Make a conjecture about the property of logarithms that relates to the property of exponential equations that states the following:
   \( \frac{a^m}{a^n} = a^{m-n} \).

10. Use the information from the tables in Item 2 to provide examples that support your conjecture in Item 9.

11. Graph \( y_1 = \log x - \log 2 \) and \( y_2 = \log \frac{x}{2} \) on a graphing calculator. What do you observe?
Lesson 14-3
Properties of Logarithms

Check Your Understanding

Use the information from the tables in Item 2 and the properties in Items 4b and 9.

12. Write two different logarithmic expressions to find a value for \( \log 36 \).
13. Write a logarithmic expression that contains a quotient and simplifies to 0.301.
14. Construct viable arguments. Show that \( \log (3 + 4) \neq \log 3 + \log 4 \).

LESSON 14-3 PRACTICE

Use the table of logarithmic values at the beginning of the lesson to evaluate the logarithms in Items 15 and 16. Do not use a calculator.

15. a. \( \log \left( \frac{8}{3} \right) \)
b. \( \log 24 \)
c. \( \log 64 \)
d. \( \log 27 \)

16. a. \( \log \left( \frac{4}{9} \right) \)
b. \( \log 2.25 \)
c. \( \log 144 \)
d. \( \log 81 \)

17. Rewrite \( \log 7 + \log x - (\log 3 + \log y) \) as a single logarithm.
18. Rewrite \( \log \left( \frac{8m}{9n} \right) \) as a sum of four logarithmic terms.
19. Make use of structure. Rewrite \( \log 8 + \log 2 - \log 4 \) as a single logarithm and evaluate the result using the table at the beginning of the lesson.
Learning Targets:
• Make conjectures about properties of logarithms.
• Write and apply the Power Property of Logarithms.
• Rewrite logarithmic expressions by using their properties.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations

1. Make a conjecture about the property of logarithms that relates to the property of exponents that states the following: \((a^m)^n = a^{mn}\).

2. Use the information from the tables in Item 2 in the previous lesson and the properties developed in Items 4 and 9 in the previous lesson to support your conjecture in Item 1.

3. Use appropriate tools strategically. Graph \(y_1 = 2 \log x\) and \(y_2 = \log x^2\) on a graphing calculator. What do you observe?

Check Your Understanding
Identify each statement as true or false. Justify your answer.

4. \(2 \log \sqrt{m} = \log m\)

5. \(\log 10^2 = \log 2^{10}\)

6. Express regularity in repeated reasoning. The logarithmic properties that you conjectured and then verified in this lesson and the previous lesson are listed below. State each property.

Product Property: ______________________

Quotient Property: ______________________

Power Property: ______________________
Lesson 14-4
More Properties of Logarithms

7. Use the properties from Item 6 to rewrite each expression as a single logarithm. Assume all variables are positive.
   a. \( \log x - \log 7 \)
   
   b. \( 2 \log x + \log y \)

8. Use the properties from Item 6 to expand each expression. Assume all variables are positive.
   a. \( \log 5xy^4 \)
   
   b. \( \log \frac{x}{y^3} \)

9. Rewrite each expression as a single logarithm. Then evaluate.
   a. \( \log 2 + \log 5 \)
   
   b. \( \log 5000 - \log 5 \)
   
   c. \( 2 \log 5 + \log 4 \)

Check Your Understanding

10. Explain why \( \log (a + 10) \) does not equal \( \log a + 1 \).

11. Explain why \( \log (-100) \) is not defined.

LESSON 14-4 PRACTICE

Attend to precision. Rewrite each expression as a single logarithm. Then evaluate the expression without using a calculator.

12. \( \log 5 + \log 20 \)

13. \( \log 3 - \log 30 \)

14. \( 2 \log 400 - \log 16 \)

15. \( \log \frac{1}{400} + 2 \log 2 \)

16. \( \log 100 + \log \left( \frac{1}{100} \right) \)

17. Expand the expression \( \log b^3 d^2 \).
**ACTIVITY 14 PRACTICE**

Answer each item. Show your work.

**Lesson 14-1**

1. What is the $y$-intercept of the graph?

2. What is the $x$-intercept of the graph?

3. Is $M(x)$ an increasing or decreasing function?

4. Which of these statements are NOT true regarding the graph above?
   - A. The graph contains the point (1, 0).
   - B. The graph contains the point (10, 1).
   - C. The domain is $x > 0$.
   - D. The $x$-axis is an asymptote.

**Lesson 14-2**

5. Use a calculator to find a decimal approximation rounded to three decimal places.
   - a. $\log 47$
   - b. $\log 32.013$
   - c. $\log \left(\frac{5}{7}\right)$
   - d. $\log -20$

6. A logarithm is a(n)
   - A. variable.
   - B. constant.
   - C. exponent.
   - D. coefficient.

7. Write an exponential statement for each logarithmic statement below.
   - a. $\log 10,000 = 4$
   - b. $\log \frac{1}{1,000,000,000} = -9$
   - c. $\log a = 6$

8. Write a logarithmic statement for each exponential statement below.
   - a. $10^{-2} = \frac{1}{100}$
   - b. $10^1 = 10$
   - c. $10^4 = n$

9. Evaluate without using a calculator.
   - a. $\log 10^5$
   - b. $\log 100$
   - c. $\log \frac{1}{100,000}$

10. If $\log a = x$, and $10 < a < 100$, what values are acceptable for $x$?
    - A. $0 < x < 1$
    - B. $1 < x < 2$
    - C. $2 < x < 3$
    - D. $10 < x < 100$

**Lesson 14-3**

11. If $\log 2 = 0.301$ and $\log 3 = 0.447$, find each of the following using only these values and the properties of logarithms. Show your work.
    - a. $\log 6$
    - b. $\log \left(\frac{2}{3}\right)$
    - c. $\log 1.5$
    - d. $\log 18$
12. Which expression does NOT equal 3?
   A. \( \log_{10} 3 \)
   B. \( \log_{10} 10 \)
   C. \( \log_{10} \frac{10^7}{10^4} \)
   D. \( \log_{10} 4 - \log_{10} 10 \)

13. Explain the connection between the exponential equation \((10^3 \cdot 10^5 = 10^8)\) and the logarithmic equation \((\log_{10} 3 + \log_{10} 5 = \log_{10} 8)\).

14. Rewrite each expression as a single logarithm.
   a. \( \log_2 x + \log_2 y - (\log_3 z + \log_3 w) \)
   b. \( \log_5 7 - \log_5 3 \)
   c. \( (\log_2 24 + \log_2 12) - \log_2 6 \)

15. Expand each expression.
   a. \( \log \left( \frac{3x}{8y} \right) \)
   b. \( \log \left( \frac{m + v}{3} \right) \)
   c. \( \log \left( \frac{4}{9 - u} \right) \)

16. If \( \log 2 = 0.301 \) and \( \log 3 = 0.477 \), find each of the following using the properties of logarithms.
   a. \( \log 4 \)
   b. \( \log 27 \)
   c. \( \log \sqrt{2} \)
   d. \( \log \sqrt[3]{12} \)

Lesson 14-4

17. Complete each statement to illustrate a property for logarithms.
   a. Product Property \( \log uv = ? \)
   b. Quotient Property \( \log \frac{u}{v} = ? \)
   c. Power Property \( \log u^c = ? \)

18. Rewrite each expression as a single logarithm. Then evaluate without using a calculator.
   a. \( \log 500 + \log 2 \)
   b. \( 2 \log 3 + \log \frac{1}{9} \)
   c. \( \log 80 - 3 \log 2 \)

19. Expand each expression.
   a. \( \log xy^2 \)
   b. \( \log \frac{xy}{z} \)
   c. \( \log a^2b^3 \)

20. If \( \log 8 = 0.903 \) and \( \log 3 = 0.477 \), find each of the following using the properties of logarithms.
   a. \( \log 3^b \)
   b. \( \log (2^3)^3 \)
   c. \( \log 8(3^2) \)

21. Write each expression without using exponents.
   a. \( m \log n + \log n^m \)
   b. \( \log (mn)^0 \)
   c. \( \log 2^4 + \log 2^3 \)

22. Which of the following statements is TRUE?
   A. \( \frac{\log x}{y} = \frac{\log x}{\log y} \)
   B. \( \frac{\log x}{y} = y \log x \)
   C. \( \log (x + y) = \log x + \log y \)
   D. \( \log \sqrt{x} = \frac{1}{2} \log x \)

MATHEMATICAL PRACTICES

23. Verify using the properties of logarithms that \( \log 10^5 - \log 10^4 = x - 4 \). Then evaluate for \( x = \pi \), using 3.14 for \( \pi \).