Understanding Ratios

ACTIVITY 17

All About Pets

Lesson 17-1 Understanding Ratios

Learning Targets:

- Understand the concept of a ratio and use ratio language.
- Represent ratios with concrete models, fractions, and decimals.
- Give examples of ratios as multiplicative comparisons of two quantities describing the same attribute.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Look for a Pattern

A ***ratio*** is a comparison of two quantities. Ratios can represent a comparison of part-to-part, part-to-whole, or whole-to-part. Ratios can be written as fractions, or using the word "to" or a colon.

Example A

Use the tags below. Find each of these ratios:

a. stars to bones

b. stars to total number of tags

c. total number of tags to bones

Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

Solution:

a. stars to bones

part-to-part: $\frac{\text{number of stars}}{\text{number of bones}} = \frac{4}{8}$; 4 to 8, 4:8

b. stars to total number of tags

part-to-whole: $\frac{\text{number of stars}}{\text{number of tags}} = \frac{4}{12}$; 4 to 12; 4:12

c. total number of tags to bones

whole-to-part: $\frac{\text{number of tags}}{\text{number of bones}} = \frac{12}{8}$; 12 to 8; 12:8

Try These A

Use ratios to compare the pet toys shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

a. balls of yarn to mice $\frac{4}{5}$, 4:5, 4 to 5; part-to-part

b. white balls of yarn to total number of toys $\frac{3}{9}$, 3:9, 3 to 9 or $\frac{1}{3}$, 1:3, 1 to 3; part-to-whole

c. gray mice to white mice $\frac{4}{1}$, 4:1, 4 to 1; part-to-part

My Notes

MATH TERMS

Each part of a **ratio** is called a term. Terms can be:

- Numbers, such as 4 and 8: $\frac{4}{8}$, 4 to 8, 4:8
- Variables, such as x and y: $\frac{x}{y}$, x to y, x:y
- The product of a number and a variable, such as $3x$ and $9y$: $\frac{3x}{9y}$, $3x$ to $9y$, $3x$:$9y$

MATH TIP

Like fractions, ratios can sometimes be rewritten in lowest terms.

$\frac{4}{8} = \frac{1}{2}$, 1 to 2, or 1:2

$\frac{4}{12} = \frac{1}{3}$, 1 to 3, or 1:3

$\frac{12}{8} = \frac{3}{2}$, 3 to 2, or 3:2

Common Core State Standards for Activity 17

6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.A.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

ACTIVITY 17

Directed

Activity Standards Focus

In this activity, students learn that a ratio is a comparison of two quantities, and can be written as a fraction, using the word "to", or using a colon. They also learn the terminology associated with ratios, and apply ratios in real-life situations to find missing values in a table and represent the table as a graph in the coordinate plane determining if the relationship is proportional.

Lesson 17-1

PLAN

Pacing: 1 class period

Chunking the Lesson

Example A Example B

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to count the number of girls in the class and the number of boys, and to write a fraction with the number of girls as the numerator and the number of boys as the denominator. Have them show how the fraction would change if two girls moved away. Discuss with students how they rewrote their fractions to represent this new situation.

Example A Note taking, Shared Reading Students explore ratios as a comparison of part-to-part, part-to-whole, and whole-to-part. As students compare the numbers of stars, bones, and tags by writing various ratios, make sure they write the ratio correctly by identifying each quantity as a part or as a whole. It is important for students to have an opportunity to share how they expressed the ratios before moving on to calculating with ratios in Lesson 17-2. This should provide the teacher with an important formative assessment.

Students may mention that the ratios in the example have not been simplified. Take time to read the Math Tip signal box so that students understand that they often are simplified.

Developing Math Language

Encourage students to mark the text and take notes explaining the new vocabulary. Make sure students understand that *ratios* compare two quantities, and that each part of a ratio is called a *term*.

Paragraphs Have students read and discuss the text above Example B. Students may need to extend the pattern to understand what is meant by the text.

Example B Activating Prior Knowledge, Think-Pair-Share, Debriefing In this example, students will use the idea that a ratio is also a multiplicative comparison between two quantities. They will need to multiply both terms in the ratio by a factor that will give an equivalent ratio. Have students read the steps together and identify the equivalent fractions as ratios. Make sure that debriefing the example includes checking the solution by using the inverse process of dividing both terms of the ratio by 4 to get the original reduced ratio of cats to dogs. It is important to validate the purpose of each step in finding the number of cats adopted when 16 dogs were adopted.

TEACHER to TEACHER

If students need more review for finding unknown quantities in ratios, there is a Mini-Lesson: Using Ratios in the eBook Teacher Resources.

CONNECT TO AP

Emphasize the importance of ratios in algebra and future math courses. For example, the ratio of the vertical change to the horizontal change of the graph of a linear function gives the slope of the line, and this ratio is constant for all pairs of points on the line. It is the rate of change for the linear function. Nonlinear functions do not have this property, but in calculus, the ratio of the "rise" to the "run" is still used to describe the rate of change quickly and efficiently.

My Notes

MATH TIP

Like fractions, ratios can be written as decimals. The ratio $\frac{3}{4}$ is the quotient of $3 \div 4$ or 0.75.

A ratio is also a multiplicative comparison of two quantities.

The ratio of circles to the total number of shapes below is $\frac{2}{5}$.

This means $\frac{2}{5}$ of all the shapes are circles and that for every 2 circles added, a total of 5 shapes will be added.

Suppose a set of shapes with the pattern above includes 8 circles. You know that $2 \times 4 = 8$, so multiply the number of shapes in the repeating part of the set (2 circles + 3 squares = 5 shapes) by 4 to find the total number of shapes when there are 8 circles: $5 \times 4 = 20$ total shapes.

Example B

Make sense of problems. In January, for every 3 cats adopted, 4 dogs were adopted. A total of 16 dogs were adopted. How many cats were adopted?

Step 1: Write a ratio comparing the number of cats to the number of dogs adopted.

$$\frac{\text{number of cats}}{\text{number of dogs}} = \frac{3}{4}$$

The number of cats adopted is $\frac{3}{4}$ times the number of dogs adopted.

Step 2: Multiply the ratio times the number needed to create an equivalent ratio showing 16 dogs.

$$\frac{3}{4} \times \frac{4}{4} = \frac{12 \text{ cats}}{16 \text{ dogs}}$$

Solution: 12 cats were adopted.

Check: Does the ratio of 12 cats to 16 dogs equal $\frac{3}{4}$?

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

Try These B

At the dog park on Monday, 2 dogs out of every 5 were terriers. A total of 20 dogs were at the park.

a. How many terriers were there? Explain how you got your answer.

8 terriers; the ratio of terriers to the total number of dogs is $\frac{2}{5} \cdot \frac{4}{4} = \frac{8 \text{ terriers}}{20 \text{ dogs}}$.

b. The ratio of Irish terriers to the total number of terriers was 1:4. How many of the terriers were Irish terriers? Explain how you got your answer.

2 Irish terriers; the ratio of Irish terriers to the total number of terriers is $\frac{1}{4} \cdot \frac{8}{1} = 2$ Irish terriers.

Common Core State Standards for Activity 17 *(continued)*

6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

My Notes

Check Your Understanding

1. For a given ratio, how many equivalent ratios can be written? Explain your reasoning.
2. How can you check to see if the ratio 1:2 is equivalent to another ratio?
3. Find as many whole-number ratios equal to 50:100 as you can, using division.

LESSON 17-1 PRACTICE

4. Use ratios to compare the dog bowls shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

   a. white bowls to total number of bowls
   b. black bowls to gray bowls
   c. all bowls to bowls that are not gray

5. At the veterinarian's office, 4 animals out of every 5 seen were cats. A total of 35 animals were seen.
   a. How many cats were seen?
   b. The ratio of male cats to all cats seen was 6:7. How many of the cats seen were males?
6. There are twelve rabbits in a cage. The ratio of white rabbits to all rabbits is 3:4. How many white rabbits are in the cage?
7. **Make sense of problems.** Each veterinarian has seen 40 animals today. Two out of every 5 animals Vet A has seen have been dogs. Three out of every 8 animals Vet B has seen have been dogs. Which vet saw more dogs today? Explain your reasoning.
8. **Reason abstractly.** The ratio of red collars to black collars sold at one store is 9 to 10. In one month 30 black collars are sold. Is 57 a reasonable number for the total number of red and black collars sold that month? Explain your reasoning.
9. There are 15 black mice in a cage. The ratio of all mice to black mice is 5:1. How many mice are in the cage?

Teacher to Teacher

Students should understand that when they find a ratio equivalent to another ratio, they are using the Identity Property of Multiplication. In Example B, since they are multiplying a ratio (a real number) by a form of 1, the product is an equivalent ratio (also equal to the same real number).

Check Your Understanding

Debrief students' answers to these items to ensure they understand how to write ratios equivalent to a given ratio.

Answers

1. Infinitely many because you can multiply both terms in the ratio by any number.
2. Multiply both terms in 1:2 by a factor that will give the terms in the second ratio.
3. Five; 50:100, 25:50, 10:20, 5:10, 1:2.

ASSESS

Use the lesson practice to assess students' understanding of how to write ratios and how to use a ratio as a multiplicative comparison of two quantities.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 17-1 PRACTICE

4. a. $\frac{3}{6} = \frac{1}{2}$, 1:2, 1 to 2; part-to-whole
   b. $\frac{2}{1}$, 2:1, 2 to 1; part-to-part
   c. $\frac{6}{5}$, 6:5, 6 to 5; whole-to-part
5. a. 28 cats; $\frac{4}{5} = \frac{28}{35}$
   b. 24 male cats: $\frac{6}{7} = \frac{24}{28}$
6. 9 white rabbits
7. Vet A saw 16 dogs: $\frac{2}{5} = \frac{16}{40}$, Vet B saw 15 dogs: $\frac{3}{8} = \frac{15}{40}$, $16 > 15$. Vet A saw more dogs.
8. Yes. In the ratios $\frac{9}{10} = \frac{?}{30}$, 30 is 3×10, so ? is 3×9, or 27. $27 + 30 = 57$
9. 75 mice

ADAPT

Check students' answers to the Lesson Practice to ensure they understand how to use a ratio as a multiplicative comparison. Review this concept with students and provide additional practice as needed.

Lesson 17-2

PLAN

Pacing: 1 class period

Chunking the Lesson

Example A Example B
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Give students 2 minutes to write all the fractions they can that are equivalent to $\frac{3}{4}$. Have them compare their list with a partner.

Introduction Shared Reading, Note Taking, Developing Math Language Have students read the introductory paragraph and mark the text. Encourage them to mark the text and add notes in their own words about the definition and usage of *equivalent ratios*, *proportional relationships*, and *ratio table*. They will use these concepts as they do the examples and items, so have them revisit and update their notes to enhance their understanding.

Example A Create Representations, Think-Pair-Share Encourage students to study the table to make sense of the information given in the problem and to see how division and multiplication were used to fill in the table and determine the ratios needed to answer the questions posed. Point out the hints at the top of the tables if students need help to begin. Have them discuss the example questions with their group members. Point out that they can use the pattern in the table to help them find the number of ounces of oats for a given number of ounces of liver. Ask students to describe patterns that they notice in the table and encourage them to extend the table using these patterns. Then ask them to apply what they have learned to Try These A problems. Encourage precision and accuracy in mathematical language.

My Notes

MATH TERMS

Equivalent ratios are ratios that name the same number, just as equivalent fractions do.

Equivalent ratios are found by multiplying or dividing both terms of a ratio by the same number.

Learning Targets:

- Make tables of equivalent ratios relating quantities.
- Use tables to compare ratios.
- Plot the pairs of values on the coordinate plane and describe the relationship.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Identify a Subtask

Relationships that have ***equivalent ratios*** are called **proportional relationships**. All the columns in a **ratio table** show equivalent ratios.

Example A

Reason quantitatively. A recipe for a homemade dog treat calls for a mixture of 8 ounces of oats to 12 ounces of finely chopped liver. Complete the ratio table.

	8 ÷ 4	8 ÷ 2		8 × 2	8 × 10
Oats (oz)	2	4	8	16	80
Liver (oz)	3	6	12	24	120
	12 ÷ 4	12 ÷ 2		12 × 2	12 × 10

a. How many ounces of liver are needed with 16 oz of oats?

Solution: 24 oz of liver are needed with 16 oz of oats.

b. How many ounces of oats are needed with 120 oz of liver?

Solution: 80 oz of oats are needed with 120 oz of liver.

c. Use the table to name four ratios equivalent to $\frac{8}{12}$.

Solution: The ratios $\frac{2}{3}$, $\frac{4}{6}$, $\frac{16}{24}$, and $\frac{80}{120}$ are equivalent to $\frac{8}{12}$.

Try These A

a. In one recipe for dog biscuits, the ratio of cups of water to cups of flour used is 3:9. Complete the ratio table.

	3 ÷ 3		3 × 2	3 × 4	3 × 6	3 × 9
Water (c)	1	3	6	12	18	27
Flour (c)	3	9	18	36	54	81
	9 ÷ 3		9 × 2	9 × 4	9 × 6	9 × 9

b. How many cups of water are needed with 81 cups of flour? 27

c. How many cups of flour are needed with 12 cups of water? 36

d. Use the table to name five ratios equivalent to 3:9.

Sample answer: 1:3, 6:18, 12:36, 18:54, and 27:81.

Lesson 17-2
Ratios in Proportional Relationships

My Notes

A relationship is proportional if the graph of the relationship is a set of points through which a straight line can be drawn and the straight line passes through the point (0, 0).

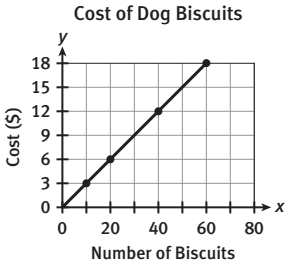
Example B

At the animal food store, 20 dog biscuits cost $6. Is the relationship between the number of biscuits and the cost proportional?

Step 1: Make a ratio table.

Number of Biscuits, x	10	20	40	60
Total Cost ($), y	3	6	12	18

Step 2: Graph the relationship between the number of biscuits x and the cost y.
Plot the ordered pairs (x, y) from the table: (10, 3), (20, 6), (40, 12), and (60, 18).



Solution: A line passes through all the points and through (0, 0). This means that the relationship is proportional.

Try These B

Graph each relationship in the My Notes section to the right. Determine if the relationship is proportional or not proportional. Explain your reasoning

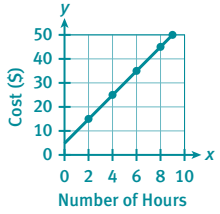
a.

Number of Hours, x	2	4	6	8	9
Total Cost ($), y	15	25	35	45	50

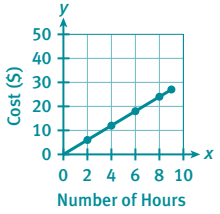
b.

Number of Hours, x	2	4	6	8	9
Total Cost ($), y	6	12	18	24	27

a. Not proportional; the line does not pass through (0, 0).



b. Proportional; the line passes through (0, 0) and all the points.



Example B Look for a Pattern, Create Representations, Think-Pair-Share
Ask students to discuss the pattern in the table with their partners and how the pattern translates into a graph that appears to be points that lie in line. Point out that if a line does pass through all of the points *and through (0, 0)*, then the relationship is proportional. Some students may need to insert additional columns in the table and then graph the points to help them see the pattern.

Teacher to Teacher

Noticing whether and how students label the graph and create the scale in the Try These B items provides a formative assessment regarding student knowledge of these skills. Note that some students may reverse the independent and dependent variables when they graph the data from the table. If necessary, discuss proper labeling and where the dependent and independent variables should be graphed. Have students explain their reasoning for describing the relationships as proportional or not proportional. Make sure they consider whether (0, 0) is a point of the graph before they decide.

CONNECT TO AP

Equivalent fractions in the ratio table will translate into a constant ratio between points of the line, and that constant ratio will be known as the *slope* of the line in future lessons and mathematics courses. Modeling slope as the rate of change is an important connection to more advanced mathematics classes like calculus.

As noted in the AP Calculus Course Description, "students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical and verbal."

Developing Math Language

This lesson contains the idea of a *proportional relationship*. Encourage students to return to where they first marked this term in the text and revise or add to the explanation they wrote at the beginning of the lesson.

Check Your Understanding

Debrief student answers to these items to ensure that students understand how a write a ratio table and then graph the table.

Answers

1. Since $2 \times 10 = 20$, multiply 5×10 to find x: $x = 5 \times 10$, or 50.
2. Graph the ordered pairs, (x, y). If a line can be drawn connecting the points and the line goes through (0, 0), the values have a proportional relationship. If the x- and y-values in the table simplify to the same ratio, then the values are proportional.

ASSESS

Use the lesson practice to assess students' understanding of how to write and graph a ratio table. In Item 3, have partners discuss how they found the values for the ratio table. This will ensure that students understand how to write equivalent ratios. Then debrief the answers to parts b and c as a class. The whole group debriefing should focus on how the ratios in the table are interpreted as a real-life use of the ratios. Circulate as students begin calculations to be sure students are graphing the ratio table data in Item 4b correctly.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 17-2 PRACTICE

3. a.

Number of Eggs	1	**2**	6	10	16
Cups of Flour	4	**8**	24	40	64

   b. 10 eggs
   c. 24 eggs
   d. $\frac{1}{4}, \frac{6}{24}, \frac{10}{40}, \frac{16}{64}$
   e. 1:4

4. a.

Number of Days, x	1	2	**4**	8
Total Cost ($), y	5	10	**20**	40

ADAPT

Check students' answers to the Lesson Practice to ensure they understand how to write and graph a ratio table. Review these concepts and provide additional practice as needed.

My Notes

Check Your Understanding

1. How can you use a ratio table to find the value of x in the ratio x:20 if the ratio is equivalent to 5:2? Explain your reasoning.
2. Name two ways to determine if the x- and y-values in a table have a proportional relationship.

LESSON 17-2 PRACTICE

3. **Reason quantitatively.** The recipe for a homemade dog treat calls for a mixture of 2 eggs for every 8 cups of flour.
   a. Complete the ratio table.

Number of Eggs	1	**2**	6		
Cups of Flour		**8**		40	64

   b. How many eggs are needed with 40 cups of flour?
   c. How many cups of flour are needed with 6 eggs?
   d. Use the table to name four ratios equivalent to $\frac{2}{8}$.
   e. Which ratio is equivalent to 2:8 in lowest terms?

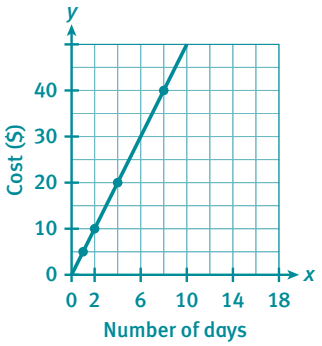
4. **Model with mathematics.** For every 4 days of dog sitting Julie charges $20.
   a. Complete the table to find the amount Julie should charge for 1, 2, and 8 days of dog sitting.

Number of Days, x	1	2	**4**	8
Total Cost ($), y			**20**	

   b. Graph the relationship between the number of days x and the cost y.
   c. Is the relationship between the number of days and the cost proportional? Justify your answer.
   d. Use your graph to determine how much Julie should charge for 6 days of dog sitting.
   e. Is 4:20 equivalent to 10:50? Explain using the graph.

5. Are $\frac{2}{3}$ and $\frac{5}{6}$ equivalent ratios? Justify your answer.
6. Are $\frac{2}{7}$ and $\frac{6}{21}$ equivalent ratios? Justify your answer.
7. Are $\frac{2}{4}$ and $\frac{3}{6}$ equivalent ratios? Justify your answer.

b.



   c. Yes; a line can be drawn connecting the points, and the line goes through (0, 0).
   d. $30
   e. Yes; (10, 50) is a point on the line, so 10:50 is equivalent to 4:20.

5. No, $3 \times 2 = 6$, but $2 \times 2 = 4$, not 5. Therefore $\frac{2}{3}$ and $\frac{5}{6}$ are not equivalent ratios.
6. Yes, $7 \times 3 = 21$, and $2 \times 3 = 6$. Therefore $\frac{2}{7}$ and $\frac{6}{21}$ are equivalent ratios.
7. Yes, $\frac{2}{4} = \frac{1}{2}$ and $\frac{3}{6} = \frac{1}{2}$, so they are equivalent ratios. Also, both ratios, when graphed, are on a line that goes through (0, 0).

ACTIVITY 17 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 17-1

1. Write a ratio in three different ways to represent the number of boys to the number of girls in the class.

Girls	Boys
12	15

2. Write a ratio for each situation.
   a. 310 heartbeats per 5 minutes
   b. $68 for 8 hours of work
   c. Work 40 hours in 5 days

3. A recent study shows that out of 100 pieces of a popular multicolored snack, there will usually be the following number of pieces of each color.

Brown	Yellow	Red	Blue	Orange	Green
13	14	13	24	20	16

   a. The numbers for two colors form a ratio that is equal to $\frac{7}{12}$. What are the colors? What is their ratio?
   b. If there were 500 pieces, about how many would be red?

4. Katie is making lemonade from a powder mix. The ratio of scoops of powder mix to water is 4 scoops to 1 gallon.
   a. How much water should Katie mix if she uses 12 scoops of mix?
   b. How much powder mix should Katie use if she plans to use 5 gallons of water?

5. There are a total of 60 plastic blocks. Three out of every 5 blocks are red. Is it reasonable for Briana to think there are enough red blocks to make a design that uses 32 red blocks? Explain your reasoning.

6. Which of the following expressions is not a ratio?
   A. $\frac{2}{3}$ B. 2:3
   C. 2 to 3 D. $2 + 3$

7. Which of the following compares the number of stars to the number of circles?

   A. $\frac{6}{8}$ B. 4:3
   C. 3:4 D. 8 to 14

8. How does a ratio comparing the number of squares to the total number of shapes compare to a ratio comparing the number of arrows to the total number of shapes?

9. There are three types of animals in the pictures in Mica's album: horses, cows, and sheep. The ratio of horses to total number of animals in the pictures is 2:8. The ratio of cows to total number of animals in the pictures is 1:4.
   a. What can you conclude about the number of horses and the number of cows in the pictures?
   b. There are 40 animals pictured in Mica's album. How many are either horses or cows?

10. Write a ratio in lowest terms for each type of relationship for the following shapes.

   a. part-to-whole
   b. part-to-part
   c. whole-to-part

ACTIVITY 17 Continued

ACTIVITY PRACTICE

1. 15:12, 15 to 12, $\frac{15}{12}$
2. a. 310:5 (or $\frac{310}{5}$ or 310 to 5)
   b. 68:8 (or $\frac{68}{8}$ or 68 to 8)
   c. 40:5 (or $\frac{40}{5}$ or 40 to 5)
3. a. yellow and blue; 14:24, which simplifies to 7:12
   b. about 65; $\frac{13}{100} = \frac{?}{500}$, $5 \times 13 = 65$
4. a. 3 gallons
   b. 20 scoops
5. Yes. The ratio of red to all blocks is 3 to 5. $\frac{3}{5} = \frac{?}{60}$. $5 \times 12 = 60$, so $3 \times 12 = 36$. So there are more than enough red blocks.
6. D
7. B
8. Both ratios are 6:12, so they are equivalent ratios.
9. a. There is the same number of horses as there are cows because the ratios are equivalent.
   b. 20, $10 + 10 = 20$
10. a. Sample answer: circles to all shapes; 3:4
    b. Sample answer: circles to rectangles; 3:1
    c. Sample answer: all shapes to rectangles; 4:1

11.

45	3	90	36	63
165	11	330	132	231

12. C

13. D

14. a. Yes. The graph is a line that passes through (0, 0), so the relationship is proportional.

b. 150 miles

c. $x = 5$

15. Check students' graphs. Not proportional; The graph is a line but it does not pass through (0, 0), so the relationship is not proportional.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 17-2

11. Complete the ratio table to show ratios equivalent to 9:33.

45	3			63
		330	132	

12. Which of the following ratios is not equivalent to 9:33?

A. $\frac{54}{198}$ **B.** $\frac{18}{66}$

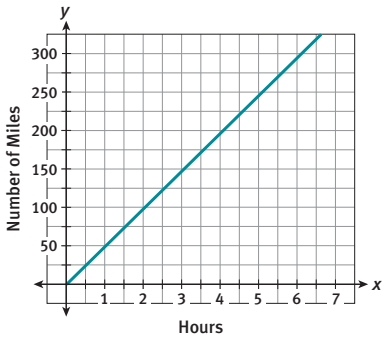
C. $\frac{1}{25}$ **D.** $\frac{6}{22}$

13. The ratios 4:5 and x:80 have a proportional relationship. What is the value of x?

A. 79 **B.** 100

C. 81 **D.** 64

14. The following is a graph of the number of hours driven versus the number of miles traveled. Use the graph to answer parts a–c.



a. Is the relationship between the number of hours driven and the number of miles traveled proportional? Explain your reasoning.

b. After 3 hours of driving, how many miles would be traveled?

c. Find the value of x.

$$\frac{2}{100} = \frac{x}{250}$$

MATHEMATICAL PRACTICES

Construct Viable Arguments

15. Graph the following relationship. Determine if the relationship is proportional or not proportional. Explain your reasoning

Number of Pens, y	2	4	8	12
Total Cost ($), x	10	12	16	20

Understanding Ratios

All About Pets

Lesson 17-1 Understanding Ratios

Learning Targets:

- Understand the concept of a ratio and use ratio language.
- Represent ratios with concrete models, fractions, and decimals.
- Give examples of ratios as multiplicative comparisons of two quantities describing the same attribute.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Look for a Pattern

A ***ratio*** is a comparison of two quantities. Ratios can represent a comparison of part-to-part, part-to-whole, or whole-to-part. Ratios can be written as fractions, or using the word "to" or a colon.

Example A

Use the tags below. Find each of these ratios:

a. stars to bones

b. stars to total number of tags

c. total number of tags to bones

Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

Solution:

a. stars to bones

part-to-part: $\frac{\text{number of stars}}{\text{number of bones}} = \frac{4}{8}$; 4 to 8, 4:8

b. stars to total number of tags

part-to-whole: $\frac{\text{number of stars}}{\text{number of tags}} = \frac{4}{12}$; 4 to 12; 4:12

c. total number of tags to bones

whole-to-part: $\frac{\text{number of tags}}{\text{number of bones}} = \frac{12}{8}$; 12 to 8; 12:8

Try These A

Use ratios to compare the pet toys shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

a. balls of yarn to mice

b. white balls of yarn to total number of toys

c. gray mice to white mice

My Notes

MATH TERMS

Each part of a **ratio** is called a term. Terms can be:

- Numbers, such as 4 and 8: $\frac{4}{8}$, 4 to 8, 4:8
- Variables, such as x and y: $\frac{x}{y}$, x to y, x:y
- The product of a number and a variable, such as $3x$ and $9y$: $\frac{3x}{9y}$, $3x$ to $9y$, $3x$:$9y$

MATH TIP

Like fractions, ratios can sometimes be rewritten in lowest terms.

$\frac{4}{8} = \frac{1}{2}$, 1 to 2, or 1:2

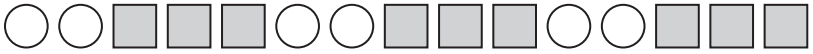
$\frac{4}{12} = \frac{1}{3}$, 1 to 3, or 1:3

$\frac{12}{8} = \frac{3}{2}$, 3 to 2, or 3:2

My Notes

A ratio is also a multiplicative comparison of two quantities.

The ratio of circles to the total number of shapes below is $\frac{2}{5}$.



This means $\frac{2}{5}$ of all the shapes are circles and that for every 2 circles added, a total of 5 shapes will be added.

Suppose a set of shapes with the pattern above includes 8 circles. You know that $2 \times 4 = 8$, so multiply the number of shapes in the repeating part of the set (2 circles + 3 squares = 5 shapes) by 4 to find the total number of shapes when there are 8 circles: $5 \times 4 = 20$ total shapes.

MATH TIP

Like fractions, ratios can be written as decimals. The ratio $\frac{3}{4}$ is the quotient of $3 \div 4$ or 0.75.

Example B

Make sense of problems. In January, for every 3 cats adopted, 4 dogs were adopted. A total of 16 dogs were adopted. How many cats were adopted?

Step 1: Write a ratio comparing the number of cats to the number of dogs adopted.

$$\frac{\text{number of cats}}{\text{number of dogs}} = \frac{3}{4}$$

The number of cats adopted is $\frac{3}{4}$ times the number of dogs adopted.

Step 2: Multiply the ratio times the number needed to create an equivalent ratio showing 16 dogs.

$$\frac{3}{4} \times \frac{4}{4} = \frac{12 \text{ cats}}{16 \text{ dogs}}$$

Solution: 12 cats were adopted.

Check: Does the ratio of 12 cats to 16 dogs equal $\frac{3}{4}$?

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

Try These B

At the dog park on Monday, 2 dogs out of every 5 were terriers. A total of 20 dogs were at the park.

a. How many terriers were there? Explain how you got your answer.

b. The ratio of Irish terriers to the total number of terriers was 1:4. How many of the terriers were Irish terriers? Explain how you got your answer.

My Notes

Check Your Understanding

1. For a given ratio, how many equivalent ratios can be written? Explain your reasoning.
2. How can you check to see if the ratio 1:2 is equivalent to another ratio?
3. Find as many whole-number ratios equal to 50:100 as you can, using division.

LESSON 17-1 PRACTICE

4. Use ratios to compare the dog bowls shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

   a. white bowls to total number of bowls
   b. black bowls to gray bowls
   c. all bowls to bowls that are not gray

5. At the veterinarian's office, 4 animals out of every 5 seen were cats. A total of 35 animals were seen.
   a. How many cats were seen?
   b. The ratio of male cats to all cats seen was 6:7. How many of the cats seen were males?

6. There are twelve rabbits in a cage. The ratio of white rabbits to all rabbits is 3:4. How many white rabbits are in the cage?

7. **Make sense of problems.** Each veterinarian has seen 40 animals today. Two out of every 5 animals Vet A has seen have been dogs. Three out of every 8 animals Vet B has seen have been dogs. Which vet saw more dogs today? Explain your reasoning.

8. **Reason abstractly.** The ratio of red collars to black collars sold at one store is 9 to 10. In one month 30 black collars are sold. Is 57 a reasonable number for the total number of red and black collars sold that month? Explain your reasoning.

9. There are 15 black mice in a cage. The ratio of all mice to black mice is 5:1. How many mice are in the cage?

My Notes

Learning Targets:

- Make tables of equivalent ratios relating quantities.
- Use tables to compare ratios.
- Plot the pairs of values on the coordinate plane and describe the relationship.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Identify a Subtask

Relationships that have ***equivalent ratios*** are called **proportional relationships**. All the columns in a **ratio table** show equivalent ratios.

MATH TERMS

Equivalent ratios are ratios that name the same number, just as equivalent fractions do.

Equivalent ratios are found by multiplying or dividing both terms of a ratio by the same number.

Example A

Reason quantitatively. A recipe for a homemade dog treat calls for a mixture of 8 ounces of oats to 12 ounces of finely chopped liver. Complete the ratio table.

	8 ÷ 4	**8 ÷ 2**		**8 × 2**	**8 × 10**
Oats (oz)			**8**	16	
Liver (oz)	**3**	6	**12**		120
	12 ÷ 4	**12 ÷ 2**		**12 × 2**	**12 × 10**

a. How many ounces of liver are needed with 16 oz of oats?

Solution: 24 oz of liver are needed with 16 oz of oats.

b. How many ounces of oats are needed with 120 oz of liver?

Solution: 80 oz of oats are needed with 120 oz of liver.

c. Use the table to name four ratios equivalent to $\frac{8}{12}$.

Solution: The ratios $\frac{2}{3}$, $\frac{4}{6}$, $\frac{16}{24}$, and $\frac{80}{120}$ are equivalent to $\frac{8}{12}$.

Try These A

a. In one recipe for dog biscuits, the ratio of cups of water to cups of flour used is 3:9. Complete the ratio table.

	3 ÷ 3		**3 × 2**	**3 × 4**	**3 × 6**	**3 × 9**
Water (c)		**3**	6	12		
Flour (c)	3	**9**			54	81
	9 ÷ 3		**9 × 2**	**9 × 4**	**9 × 6**	**9 × 9**

b. How many cups of water are needed with 81 cups of flour?

c. How many cups of flour are needed with 12 cups of water?

d. Use the table to name five ratios equivalent to 3:9.

My Notes

A relationship is proportional if the graph of the relationship is a set of points through which a straight line can be drawn and the straight line passes through the point (0, 0).

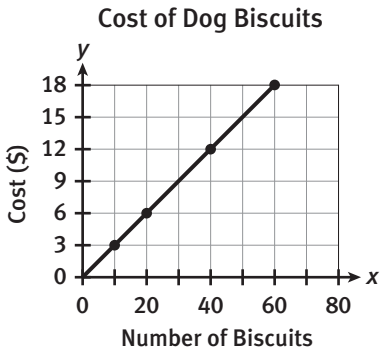
Example B

At the animal food store, 20 dog biscuits cost $6. Is the relationship between the number of biscuits and the cost proportional?

Step 1: Make a ratio table.

Number of Biscuits, x	10	**20**	40	60
Total Cost ($), y	3	**6**	12	18

Step 2: Graph the relationship between the number of biscuits x and the cost y.
Plot the ordered pairs (x, y) from the table: (10, 3), (20, 6), (40, 12), and (60, 18).



Solution: A line passes through all the points and through (0, 0). This means that the relationship is proportional.

Try These B

Graph each relationship in the My Notes section to the right. Determine if the relationship is proportional or not proportional. Explain your reasoning

a.

Number of Hours, x	2	4	6	8	9
Total Cost ($), y	15	25	35	45	50

b.

Number of Hours, x	2	4	6	8	9
Total Cost ($), y	6	12	18	24	27

My Notes

Check Your Understanding

1. How can you use a ratio table to find the value of x in the ratio x:20 if the ratio is equivalent to 5:2? Explain your reasoning.
2. Name two ways to determine if the x- and y-values in a table have a proportional relationship.

LESSON 17-2 PRACTICE

3. **Reason quantitatively.** The recipe for a homemade dog treat calls for a mixture of 2 eggs for every 8 cups of flour.
   a. Complete the ratio table.

Number of Eggs	1	2	6		
Cups of Flour		8		40	64

   b. How many eggs are needed with 40 cups of flour?
   c. How many cups of flour are needed with 6 eggs?
   d. Use the table to name four ratios equivalent to $\frac{2}{8}$.
   e. Which ratio is equivalent to 2:8 in lowest terms?

4. **Model with mathematics.** For every 4 days of dog sitting Julie charges $20.
   a. Complete the table to find the amount Julie should charge for 1, 2, and 8 days of dog sitting.

Number of Days, x	1	2	4	8
Total Cost ($), y			20	

   b. Graph the relationship between the number of days x and the cost y.
   c. Is the relationship between the number of days and the cost proportional? Justify your answer.
   d. Use your graph to determine how much Julie should charge for 6 days of dog sitting.
   e. Is 4:20 equivalent to 10:50? Explain using the graph.

5. Are $\frac{2}{3}$ and $\frac{5}{6}$ equivalent ratios? Justify your answer.
6. Are $\frac{2}{7}$ and $\frac{6}{21}$ equivalent ratios? Justify your answer.
7. Are $\frac{2}{4}$ and $\frac{3}{6}$ equivalent ratios? Justify your answer.

ACTIVITY 17 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 17-1

1. Write a ratio in three different ways to represent the number of boys to the number of girls in the class.

Girls	Boys
12	15

2. Write a ratio for each situation.
- **a.** 310 heartbeats per 5 minutes
- **b.** $68 for 8 hours of work
- **c.** Work 40 hours in 5 days

3. A recent study shows that out of 100 pieces of a popular multicolored snack, there will usually be the following number of pieces of each color.

Brown	Yellow	Red	Blue	Orange	Green
13	14	13	24	20	16

- **a.** The numbers for two colors form a ratio that is equal to $\frac{7}{12}$. What are the colors? What is their ratio?
- **b.** If there were 500 pieces, about how many would be red?

4. Katie is making lemonade from a powder mix. The ratio of scoops of powder mix to water is 4 scoops to 1 gallon.
- **a.** How much water should Katie mix if she uses 12 scoops of mix?
- **b.** How much powder mix should Katie use if she plans to use 5 gallons of water?

5. There are a total of 60 plastic blocks. Three out of every 5 blocks are red. Is it reasonable for Briana to think there are enough red blocks to make a design that uses 32 red blocks? Explain your reasoning.

6. Which of the following expressions is not a ratio?
- **A.** $\frac{2}{3}$
- **B.** 2:3
- **C.** 2 to 3
- **D.** $2 + 3$

7. Which of the following compares the number of stars to the number of circles?

- **A.** $\frac{6}{8}$
- **B.** 4:3
- **C.** 3:4
- **D.** 8 to 14

8. How does a ratio comparing the number of squares to the total number of shapes compare to a ratio comparing the number of arrows to the total number of shapes?

9. There are three types of animals in the pictures in Mica's album: horses, cows, and sheep. The ratio of horses to total number of animals in the pictures is 2:8. The ratio of cows to total number of animals in the pictures is 1:4.
- **a.** What can you conclude about the number of horses and the number of cows in the pictures?
- **b.** There are 40 animals pictured in Mica's album. How many are either horses or cows?

10. Write a ratio in lowest terms for each type of relationship for the following shapes.

- **a.** part-to-whole
- **b.** part-to-part
- **c.** whole-to-part

Lesson 17-2

11. Complete the ratio table to show ratios equivalent to 9:33.

45	3			63
		330	132	

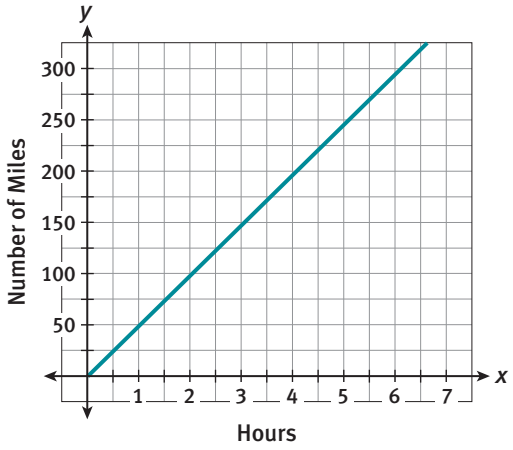
12. Which of the following ratios is not equivalent to 9:33?

A. $\frac{54}{198}$ **B.** $\frac{18}{66}$
C. $\frac{1}{25}$ **D.** $\frac{6}{22}$

13. The ratios 4:5 and x:80 have a proportional relationship. What is the value of x?

A. 79 **B.** 100
C. 81 **D.** 64

14. The following is a graph of the number of hours driven versus the number of miles traveled. Use the graph to answer parts a–c.



a. Is the relationship between the number of hours driven and the number of miles traveled proportional? Explain your reasoning.

b. After 3 hours of driving, how many miles would be traveled?

c. Find the value of x.

$$\frac{2}{100} = \frac{x}{250}$$

MATHEMATICAL PRACTICES
Construct Viable Arguments

15. Graph the following relationship. Determine if the relationship is proportional or not proportional. Explain your reasoning

Number of Pens, *y*	2	4	8	12
Total Cost ($), *x*	10	12	16	20