Ratio and Proportion

ACTIVITY 8

Strange, But True

Lesson 8-1 Ratio and Unit Rates

Learning Targets:

- Express relationships using ratios.
- Find unit rates.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Paraphraing, Note Taking, Sharing and Responding, Discussion Groups

Interesting sports facts and statistics are abundant. For example, there are only *two* days of the year in which there are *no* professional sports games (MLB, NBA, NHL, or NFL): the day before and the day after the Major League Baseball (MLB) All-Star Game.

You can write a ***ratio*** to compare the number of days without any professional sports events each year with the total number of days in a year. There are three ways to write a ratio to express the relationship between two quantities:

a to b $\quad$ $a : b$ $\quad$ $\frac{a}{b}$

1. Write a ratio, in all three forms, to compare the days without games to the days in a year.

   2 to 365; 2 : 365; $\frac{2}{365}$

2. What ratio compares days with games to days in a year? Write it three ways.

   363 to 365; 363 : 365; $\frac{363}{365}$

3. What ratio can you write that compares days with games to days without them? Write this ratio three ways, too.

   363 to 2; 363 : 2; $\frac{363}{2}$

   The ratios above all compare two *like* units—*days* and *days*. A ratio that compares two *different* kinds of units is called a ***rate.*** One common rate in sports is miles per hour (mi/h or mph), as in car racing.

4. List some other sports statistics commonly given as rates.

   Accept all reasonable answers.

   You can use basketball free throws to explore rates. In a group of four, make 12 paper basketballs. Place a wastebasket about six feet from a "free-throw line." Record how many "baskets" each of you makes within the time listed in the table. Have one member of the group keep time. Then work together to answer Items 5-8 on the next page. If you do not know exact words to use during discussion, use synonyms or request assistance from group members. If you need to, use non-verbal cues such as raising your hand for help.

My Notes

CONNECT TO SPORTS

NBA = National Basketball Association

NHL = National Hockey League

NFL = National Football League

MATH TERMS

A **ratio** is a comparison of two quantities. You can write a ratio as a fraction, using the word "to," or using a colon.

A **rate** is a ratio that compares two *different* units, such as distance and time, or a ratio that compares two different things measured with the same unit, such as cups of water and cups of frozen orange juice concentrate.

DISCUSSION GROUP TIPS

As you interact with your group in solving problems, you may hear math terms and other words that may be new to you. Ask for clarification of their meaning, and make notes to help you learn and use vocabulary heard during classroom instruction and interactions.

Common Core State Standards for Activity 8

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

ACTIVITY 8

Investigative

Activity Standards Focus

In previous courses, students have written ratios of two quantities. In Activity 8, students compute the ratio of two quantities with the same units and ratios that compare two different kinds of units, or rates. They also study rates with a denominator of 1, or unit rates.

Lesson 8-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–6 #7–8

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to count the number of girls in the class and the number of boys, and to write a fraction with the number of girls as the numerator and the number of boys as the denominator. Have them predict how the fraction would change if two new girls were added to the class. Discuss with students how they made their predictions.

1–6 Look for a Pattern, Create Representations, Think-Pair-Share, Sharing and Responding As students compare the units used in the free-throw activity, make sure they write the rate correctly as the number of baskets made compared to the time in seconds. It is important for students to have an opportunity to share how they determined the rates before moving on to unit rates. This should provide the teacher with important formative assessment information about the level of student thinking.

In Item 2, students will write ratios to represent the relationship between the number of days with games to the total number of days in a year. Ask students to explain if their answer is a rate (both quantities have units of days).

Developing Math Language

Encourage students to mark the text and add notes explaining the new vocabulary of the lesson. Make sure students understand that *ratios* compare two quantities with like or unlike units, and that if the units are unlike units, then the ratio is also called a *rate*. Monitor the oral and written language of students to be sure that they do not use the word *rate* for like units.

7–8 Debriefing In this set of s, students should use the idea that unit rates are simply rates expressed as a quantity of 1. For Item 7, they may write the rate in fraction form as $\frac{\$24}{1\text{ ticket}}$ or using the word "per" as \$24 per ticket. In Item 8, they compare the made-basket rates in Item 5, by converting the rate to the number of baskets in 1 minute. They will need to understand unit rates in order to do conversions in later problems.

Teacher to Teacher

In Item 8, some students may be confused when they find the unit rate by multiplying rather than by dividing as they did in Item 7. It is important to debrief this part of the lesson to validate why each operation was chosen to find the unit rate. In Item 7, students convert from the price of many tickets to the price of one ticket. Going from a greater quantity to a lesser quantity uses division to find the unit rate. By comparison, in Item 8, students convert from a lesser number of seconds (30) to a greater number of seconds (because 60 seconds equal 1 minute, the desired denominator of the unit rate), so multiplication is used.

Connect to AP

Calculus is often described as the study of how things change, and in so doing, it provides a means to identifying and analyzing the rates at which quantities change to solve problems in the real-world. Students should be able to work with rates in a variety of ways, including rates described verbally, rates given as a pattern in a table, or rates expressed graphically as the slope of a line in the plane.

Developing Math Language

If the rate has a denominator of 1, then the rate is called a *unit* rate. Point out that ratios can be written using the word "to," using a colon, or as fractions, but that ratios in sports are often expressed as decimals. For example, if a baseball player had a batting average of 0.332, it means that he got on base 332 out of 1000 times he batted.

My Notes

Team Member	Baskets Made	Time (in seconds)	Rate
1		30	
2		15	
3		20	
4		10	

5. What units are you comparing in this free-throw activity?
baskets made/time in seconds

6. **Reason quantitatively.** Examine all results. What can you say about the relationship between baskets made and time allowed?
Results will vary; accept all reasonable explanations.

When the second term of a rate is 1, the rate is called a ***unit rate***. Miles *per* hour is a kind of unit rate. So is price *per* pound.

Suppose that you and your friends attend a basketball game. You buy a block of 8 tickets for \$192. You want to know the price per ticket. That price can be expressed as a unit rate.

7. What is that rate? How did you figure it out?
\$24 per ticket; one way: divide both terms by 8.

Now look back at your made-basket rates. To see who the best shooter was, you can express each rate as a unit rate.

Think: 60 seconds = 1 minute. Let made-baskets per minute be your unit rate.

Suppose you made 7 baskets in 30 seconds. Use mental math to find how many you made in 60 seconds.

$$\frac{7}{30} = \frac{7\,(2)}{30\,(2)} = \frac{14\text{ baskets}}{60\text{ seconds}} = \frac{14\text{ baskets}}{1\text{ minute}}$$

So, your unit rate is 14 baskets per minute.

8. a. How can you find the one-minute unit rate for the baskets team member 1 made in 30 seconds? Explain your reasoning.
$30 \times 2 = 60$, so multiply made-baskets by 2.

b. How can you find the unit rate for the baskets that team member 3 made in 20 seconds? Explain your reasoning.
$20 \times 3 = 60$, so multiply the made-baskets by 3.

Math Terms

A **unit rate** is a rate expressed in terms of 1 unit.

In some cases, you find unit rates by *multiplying*. In other cases, you find them by *dividing*.

Lesson 8-1
Ratio and Unit Rates

ACTIVITY 8
continued

My Notes

Check Your Understanding

9. There are nine position players on a baseball team: 3 outfielders, 4 infielders, 1 pitcher, 1 catcher. Write a ratio in simplest form to express each relationship.
- **a.** infielders to number of players
- **b.** outfielders to infielders
- **c.** number of players to number of outfielders
- **d.** number of players other than infielders and outfielders to number of players
- **e.** number of pitchers to number of catchers

10. Write the rate. Then find the unit rate.
- **a.** $162 for 9 tickets
- **b.** 15 baskets in 5 minutes
- **c.** 84 yards in 14 running attempts
- **d.** 24 strikeouts in 12 innings

LESSON 8-1 PRACTICE

11. Jed made 2 free throws in 5 seconds. How many would he make in one minute?

12. Rita ran 5 miles in 48 minutes. What was her time per mile?

13. In a typical Wimbledon tennis tournament, 42,000 balls are used and 650 matches are played. About how many balls are used per match?

14. Reggie Jackson played in major league baseball for 21 years. Although this slugger was known best for his home runs, he holds the major league record for strikeouts: 2,597. What was his approximate rate of strikeouts per year?

15. In skateboarding, an *ollie* is a move in which the athlete pops the skateboard into the air, making it appear that the board and skateboarder are attached. At the 2009 X Games, one skateboarder completed 34 ollies in 30 seconds.
- **a.** If he could keep up that rate, what would be his rate of ollies per minute?
- **b.** What would be his rate per hour?

16. **Reason quantitatively.** A major-league baseball player was able to hit 70 home runs in 162 games. Create a unit rate for this information.

MINI-LESSON: Unit Rates

Explain that a unit rate is a ratio relating a quantity to 1 of another quantity and has a denominator of 1. Then have students identify the unit rate for each situation.

1. 300 miles and 15 gallons of gas
2. 1500 students and 75 classrooms
3. 24 pencils costing a total of $3.00

ACTIVITY 8 Continued

Check Your Understanding

Debrief students' answers to these items to ensure they understand how to write a ratio, a rate, and a unit rate. The questions also assess whether students can convert from a rate to a unit rate.

Answers

9. **a.** 4 : 9 **b.** 3 : 4 **c.** 3 : 1 **d.** 2 : 9 **e.** 1 : 1

10. **a.** 9 for 162; 1 for $18
b. 15 in 5; 3 in 1
c. 84 in 14; 6 in 1
d. 24 in 12; 2 in 1

ASSESS

Use the lesson practice to assess your students' understanding of rates and unit rates. Pay particular attention to Items 11–12, which have students write unit rates.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 8-1 PRACTICE

11. 24 free throws
12. 9.6 min/mi
13. about 65 balls
14. about 124 strike outs per year
15. 68 per minute; 4,080
16. 1 homerun about every 2.3 games

ADAPT

Check students' answers to the Lesson Practice to ensure they understand how to write a rate as a unit rate. Students who need more practice with the concepts in this lesson will have additional opportunities to work with ratios and rates in Lesson 8-2.

Lesson 8-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–5 Example

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity Set a timer for two minutes and ask students to write all the fractions they can think of that are equivalent to $\frac{2}{5}$. Then have partners check each other's work.

Introduction Shared Reading Have students read the introductory paragraphs and highlight the important terms in cooperative learning groups. Point out the definition of proportions and cross-products.

1–3 Create Representations, Visualization Students can use a table to write equal ratios for Ashrita's rate of speed in Items 1–3. Encourage students to use proportions to help them fill in the table. Ask them to discuss with their partners how the table helps them find the length of time it takes Ashrita to run 1, 2, or 3 miles. Doing this will help students understand the pattern in the table.

ELL Support

To help students solve Item 4, suggest that they use Ashrita's record in seconds. Then they can find 30 seconds less than 425 seconds, or 395 seconds. Multiplying 395 seconds by 2 gives the time for 2 miles. As students find the cross-products for this proportion, some will need to verify the steps they use to help them to construct their arguments.

My Notes

Learning Targets:

- Determine whether quantities are in a proportional relationship.
- Solve problems involving proportional relationships.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Predict and Confirm, Note Taking, Create Representations

As you read the following scenario, mark the text to identify key information and parts of sentences that help you make meaning from the text.

The fastest time for running a mile while balancing a baseball bat on a finger is 7 min 5 s. This record was set by Ashrita Furman on June 20, 2009. At that rate of speed, Meg predicts that it would take 1,275 seconds, or 21 min 15 s, for Ashrita to run 3 mi. Is she right?

You can write a proportion to find out. A ***proportion*** is an equation. It consists of two equivalent ratios.

Example: $\frac{2}{5} = \frac{4}{10}$

MATH TERMS

A **proportion** is an equation stating that two ratios are equivalent.

To determine if Meg is correct, let n = the time it will take Ashrita to run 3 miles balancing a baseball bat.

First, convert 7 min 5 seconds to seconds: 425 seconds.

$$\frac{\text{time}}{\text{distance}} \rightarrow = \frac{425}{1} = \frac{1275}{3} \leftarrow \frac{\text{time}}{\text{distance}}$$

When two ratios are equal, their **cross-products** are equal.

For any proportion $\frac{a}{b} = \frac{c}{d}$, $ad = bc$ ← cross-products

1. Using what you know about proportions, use the proportion above involving Ashrita's speed and distance data and find the cross-products. Are the cross products equal?
   yes; 1 × 1,275 = 1,275; 3 × 425 = 1,275; 1,275 = 1,275

READING MATH

Read the proportion $\frac{2}{5} = \frac{4}{10}$ as "the ratio 2 to 5 equals the ratio 4 to 10" or as "2 is to 5 as 4 is to 10."

2. Is Meg's prediction correct?
   Meg is right.

3. **Reason quantitatively.** Are there other ways to determine if Meg is right? Explain.
   Yes; you can use mental math: since 1 × 3 = 3, then 1275 is correct because 3 × 425 = 1275

4. **Construct viable arguments.** Suppose a fast-running juggler beat Ashrita's record by half a minute. Could that person, continuing at that new world-record rate of speed, run 2 mi while juggling in 13 min 10 s? Use a proportion to find out and explain your reasoning.
   Yes; write the proportion $\frac{395}{1} = \frac{790}{2}$, using 395 s for the juggler's one-mile time; the cross-products are equal: 1 × 790 = 2 × 395; 790 s = 13 min 10 s; so the juggler could run 2 mi in 13 min 10 s.

MINI-LESSON: Solving Proportions

Review these three methods for solving proportions.

Method 1: Using equivalent fractions

$\frac{10}{n} = \frac{2}{5}$; so, $\frac{10}{25} = \frac{2 \times 5}{5 \times 5}$; $n = 25$

Method 2: Solving for the variable when it is a numerator

$\frac{n}{9} = \frac{2}{5}$; so, $n = \frac{2}{5} \times 9$, which is $\frac{18}{5}$, or 3.6

Method 3: Cross-products algorithm

$\frac{n}{9} = \frac{2}{5}$; so, $n = 2 \times 9 \div 5 = 3.6$, $n = (2 \times 9) \div 5 = 3.6$

Lesson 8-2
Identifying and Solving Proportions

My Notes

5. A three-toed sloth can cover a mile in 0.15 of an hour. Use proportions and sloth speed to complete the table for the distances shown.

Distance (mi)	1	2	5	12
Time (h)	0.15	0.3	0.75	1.8

You can use proportions to solve problems about ratios and rates.

Example

Roger Bannister was the first person to break the four-minute mile. On May 6, 1954, his time was 3 minutes, 59.4 seconds. Bannister's first quarter-mile time was 57.5 seconds. Use a proportion to find his time if he had kept up this pace.

Step 1: Write a proportion.
Let n = time to run the entire race.
Use 0.25 for the first quarter-mile Bannister ran.

$$\frac{\text{time (sec)}}{\text{distance (mi)}} \rightarrow \frac{57.5}{0.25} = \frac{n}{1} \leftarrow \frac{\text{time (sec)}}{\text{distance (mi)}}$$

Step 2: Solve the proportion using cross-products:
$0.25 \times n = 1 \times 57.5$
$0.25\,n = 57.5$

Step 3: Solve the equation to find n.
$n = 57.5 \div 0.25 \leftarrow$ *Think:* Divide both sides by 0.25.
$n = 230$ s
$n = 3$ min 50 s $\leftarrow$ write as minutes and seconds

Solution: Had Bannister kept up his quarter-mile pace, he would have run the mile in 230 sec, or 3 min 50 sec.

a. Model with mathematics. Why do you divide *both* sides of the equation by 0.25? **to maintain the equality of the equation**

b. Reason abstractly and quantitatively. What other proportions could you have written to solve the problem?
sample answer: $\frac{0.25}{57.5} = \frac{1}{n}$

Try These

Solve each proportion.

a. $\frac{n}{1} = \frac{2.45}{0.35}$ **7** **b.** $\frac{n}{5} = \frac{23.4}{2}$ **58.5** **c.** $\frac{3}{n} = \frac{5.4}{7.2}$ **4**

MATH TIP

Recall that in any proportion, cross-products will be equal. For the proportion

$$\frac{a}{b}\ \frac{c}{d}$$

$ad = bc$

When you write a proportion, be sure to set up the ratios in a consistent way according to the units associated with the numbers.

ACTIVITY 8 Continued

5 Look for a Pattern, Create Representations, Think-Pair-Share Before students complete the table in Item 5, ask them to discuss the pattern in the table with their partners. Have them identify the proportion that would help them fill in the last column. Some students may need to insert additional columns in the table to help them see the pattern in the row for Time. Have students work together to complete the table. Have groups share their tables with the class to check for accuracy. Ask a volunteer to share his/her methods used for the problem.

Developing Math Language

This lesson contains the vocabulary term *proportion.* Encourage students to mark the text by writing the definition near the occurrences of the word in order to internalize the new term and its meaning. Point out that only two ratios that are equal can be expressed as a proportion. Add the word to your class Word Wall.

Example Create Representations Some students may want to analyze this example by writing Roger Bannister's time as a unit rate. Point out that using $\frac{n}{1}$ in the proportion is the same as writing the unit rate because it will give a time in terms of a distance of 1 mile. It is important to discuss the steps for solving an equation for a variable. Explain that students must divide the equation by the coefficient of the variable in order to find the value of the variable. It is important for students to understand these steps before moving on to conversions in Lesson 8-3.

Check Your Understanding

Debrief student answers to these items to ensure that students understand how a proportion represents a situation, and understand how to solve a proportion.

Answers

6. a. Yes
   b. No
   c. No
   d. Yes
   e. Yes
   f. No
   g. Yes
   h. No
7. sample proportions given;
   a. $\frac{336}{1} = \frac{2016}{n}$; $n = 6$ balls
   b. $\frac{3}{2.8} = \frac{33.3}{x}$; $x = 31.08$ min
   c. $\frac{25}{2.5} = \frac{100}{y}$; $y = 10$ min
   d. $\frac{480}{4} = \frac{z}{1}$; $z = 120$ heartbeats
8. 40 mi, 10 mi, 20 mi, 70 mi

Teacher to Teacher

Circulate as students begin calculations to be sure students are writing the proportional relationships correctly.

ASSESS

Use the lesson practice to assess your students' understanding of how to write and solve a proportion.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 8-2 PRACTICE

9. 640 stitches
10. No; compare unit rates: Jed's is 1 in 2.5 s; Alex's is 1 in 3 s
11. 10 girls
12. sample answer: $\frac{4}{180} = \frac{12}{n}$
13. They are the same; 7 : 10 = 21 : 30
14. 20 games
15. Greta, by 20 s

ADAPT

Check students' answers to the Lesson Practice to ensure they understand how to write and solve proportions. Students that have not yet mastered the concepts may need practice with additional situations that require a proportion to solve.

My Notes

Check Your Understanding

6. Use cross products to determine if the ratios are equivalent.
   a. $\frac{3}{4}, \frac{6}{8}$ b. $\frac{8}{5}, \frac{24}{16}$ c. $\frac{70}{60}, \frac{6}{7}$ d. $\frac{1.3}{7.8}, \frac{3}{18}$
   e. $\frac{4}{7}, \frac{10}{17.5}$ f. $\frac{9}{4}, \frac{2.1}{1.4}$ g. $\frac{3}{0.8}, \frac{21}{5.6}$ h. $\frac{0.3}{2}, \frac{0.03}{20}$
7. **Make use of structure.** Write a proportion for each situation. Then solve.
   a. 336 dimples on one golf ball; 2016 dimples in n balls
   b. 3 miles in 2.8 minutes; 33.3 miles in x minutes
   c. 25 yards in $2\frac{1}{2}$ seconds; 100 yards in y seconds
   d. 480 heartbeats in 4 minutes; z heartbeats in 1 minute
8. A zebra can run at a speed of 40 mph. Complete the table using this information.

Time (h)	1	0.25	0.5	1.75
Distance (mi)				

LESSON 8-2 PRACTICE

Solve by writing and solving a proportion.

9. There are 20 stitches per panel on a soccer ball. A soccer ball has 32 leather panels. How many stitches, in all, are on a soccer ball?
10. Jed took 2 free throws in 5 seconds. Alex took 4 free throws in 12 seconds. Did the two shoot free throws at the same rate? Explain.
11. The ratio of girls to boys on a soccer team is 2 to 3. If there are 25 players on the team, how many are girls?
12. **Model with mathematics.** A package of tickets for 4 home games costs \$180. What proportion can you write to find what a 12-game package costs if all individual tickets have the same price?
13. Carlos completed 7 of 10 passes. Ty completed 21 of 30 passes. Compare their pass-completion rates.
14. Carla's team won 3 of its 5 games. Elena's team won games at the same rate and won 12 games. How many games did Elena's team play?
15. Greta completed a mile race in 5 minutes. Inez ran a mile in which each quarter-mile split was 1 min 20 seconds. Which of the two girls had the faster time? How much faster?

Lesson 8-3
Converting Measurements

ACTIVITY 8 *continued*

Learning Targets
- Convert between measurement. Use unit rates and proportions for conversions

SUGGESTED LEARNING STRATEGIES: Visualization, Think Aloud, Discussion Groups, Sharing and Responding, Create a Plan, Identify a Subtask, Note Taking

Some problems involving measurements will require you to convert between customary and metric units of measure.

Example
A tennis court is 78 feet in length and for singles play is 27 feet in width. How many meters wide is the tennis court?

Step 1: Start by converting feet to yards: $\frac{3 \text{ ft}}{1 \text{ yard}} = \frac{27 \text{ feet}}{x \text{ yards}}$

Step 2: Use cross-products to solve the proportion:

$$27 \cdot 1 = 3x$$
$$27 = 3x$$
$$9 = x$$

So, there are 9 yards in 27 feet

Step 3: Next, convert 9 yards to meters:

$$\frac{9 \text{ yd}}{x \text{ m}} = \frac{1 \text{ yd}}{0.9144 \text{ m}}$$
$$1x = 9(0.9144)$$
$$x = 8.2296 \text{ meters}$$
$$x \approx 8.23 \text{ meters}$$

Solution: The tennis court is 8.23 meters wide.

Try These
a. Attend to precision. Find the length of the tennis court above in meters. Be sure to include units. **approximately 23.77 meters**

1. How do can you tell that a proportion involving conversions has been set up correctly?
The ratios are written in a consistent way according to the units associated with the numbers.

2. The conversion factor for converting meters to yards is $\frac{1}{0.9144}$, and the conversion factor for converting yards to meters is $\frac{0.9144}{1}$. Use the conversion chart in the My Notes column to find the conversion factors for converting grams to ounces and converting liters to quarts. $\frac{1}{28.4}$ **and** $\frac{1.06}{1}$

My Notes

MATH TIP

In general, conversions between customary and metric systems result in approximate measurements.

The symbol $\approx$ means "is approximately equal to."

MATH TIP

Conversion factors for some common customary and metric measures:

1 yd $\approx$ 0.9144 m
1 m $\approx$ 1.094 yd
1 in. $\approx$ 2.54 cm
1 mi $\approx$ 1.61 km
1.06 qt $\approx$ 1L
1 oz $\approx$ 28.4 g
1 lb $\approx$ 0.4536 kg
2.2 lb $\approx$ 1 kg
1 cu ft (ft^3) $\approx$ 0.0283m^3

MINI-LESSON: Conversions

Show students that conversion can be done through substitution. For example, to convert 3 gallons to cups follow these steps:

First, list known conversions:
1 pt = 2 c 1 qt = 2 pt 1 gal = 4 qt

Next, substitute for the units, beginning with 1 gal = 4 qt:
- Replace the label of quarts with 2 pints: 1 gal = 4(2 pt)
- Replace the label of pints with 2 cups: 1 gal = 4(2(2 c))
- Simplify to find: 1 gal = 16 c

So, 3 gallons equals 16 × 3 = 48 cups.
Ask students to convert 2 gallons to cups using substitution.

Lesson 8-3

PLAN
Pacing: 1 class period
Chunking the Lesson
Example #1–3
Check Your Understanding
Lesson Practice

TEACH
Bell-Ringer Activity
Set a timer for 3 minutes. Have students write all the measurement conversions they already know on the board. Then have them check their conversions with those in the lesson. Afterwards, discuss the purpose of measurement conversions.

TEACHER to TEACHER

Students may be familiar with converting between measurements in the customary system or between measurements in the metric system, but may not know how to convert between systems. Point out the necessity of using proportions to make either type of conversion, and encourage students to mark the text with additional conversion factors as they learn them or as they become available. Allow students to complete tables similar to those in Lesson 8-2 to help them with conversions between measurements.

Example, 1–2 Create Representations, Visualization
Explain that units must match up in setting up proportions. For example, if yard is the unit in the denominator of the left-hand ratio, then it must be the unit in the denominator of the right-hand ratio. Emphasize in Step 3 that the conversion of 9 yards to meters uses the proportion $\frac{1 \text{ yd}}{0.9144 \text{ meters}} = \frac{9 \text{ yd}}{x \text{ meters}}$. Point out that yard is the unit in the numerator in the left-hand ratio and in the numerator of the right-hand ratio.

Differentiating Instruction

Help students see that a conversion does not change the value of the measurements, but gives an equivalent measurement in another unit. Explain that this is similar to finding equivalent fractions.

3 Discussion Group, Create Representations, Share and Respond Have students discuss in their groups and focus on correctly interpreting the information so that a proportion representing the information can be written and solved.

Check Your Understanding

Debrief student answers to these items to ensure that students understand how to use proportions to convert from one measurement to another.

ASSESS

Use the lesson practice to assess your students' understanding of how to use proportions to convert between customary units of measure and metric units of measure.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 8-3 PRACTICE

6. about 31 mi
7. about 3,703 km
8. 302.7 km/h
9. about 7.1 g; find the difference between both weights expressed in grams
10. about 0.20 km
11. about 16.1 km/h; one way to solve: multiply 880 ft/min by 60 to find ft/h; divide answer by 5,280 to find mi/h; multiply by 1.61 to find km/h
12. Ned; one way to solve: *think:* 6 min 30 s = 6.5 min; multiply 4 by 1.61 to get 6.44, the time it would take Ned to run 1 mi; 6.44 min < 6.5 min, so Ned runs faster

ADAPT

Check students' answers to the Lesson Practice to ensure they understand how to convert between units of measurements. Students that have not yet mastered the concepts may need practice with additional conversions that require a proportion to solve. For example, to convert 500 grams to pounds, students may need to convert to ounces first, and then to pounds.

$\frac{500 \text{ g}}{x \text{ oz}} = \frac{28.4 \text{ g}}{1 \text{ oz}}$ and

$x = 17.606$ oz

$\frac{17.606 \text{ oz}}{x \text{ lb}} = \frac{16 \text{ oz}}{1 \text{ lb}}$

$x = 1.1$ lb

Point out that since 500 g is one-half of 1000 g, or 1 kg, 500 g must equal one-half of 2.2 pounds, the conversion factor for pounds to kilograms.

$\frac{1}{2}(2.2) = 1.1$ pounds.

ACTIVITY 8
continued

My Notes

Lesson 8-3
Converting Measurements

Five hundred years ago, the toy that we now call a yo-yo was bigger and used as a weapon in the Philippines. Each weighed about 4 pounds and was attached to a 20-ft cord.

3. About how much did one of those killer yo-yos weigh, in kilograms? Find out by writing and solving a proportion using a conversion factor as one of the ratios. Use a calculator to speed computation.
$\frac{4 \text{ lb}}{n \text{ kg}} = \frac{1 \text{ lb}}{0.4536 \text{ kg}}$; $4 \times 0.4536 = 1.8144$; So, 4 lb $\approx$ 1.8144, or 1.8 kg

Check Your Understanding

4. **Use appropriate tools strategically.** Convert rounding your answers to the nearest tenth when necessary.
   a. 8 in. ≈ ____ cm b. ____ mi ≈ 20 km c. 16 cm ≈ ____ in.
   d. ____ L ≈ 50 qt e. ____ km ≈ 100 mi f. 60 g ≈ ____ oz
   g. 44 lb ≈ ____ kg h. 500 g ≈ ____ lb i. 1.5 oz ≈ ____ g
5. Write a short note to your teacher explaining how you would estimate the number of kilometers in 19 miles.

LESSON 8-3 PRACTICE

6. The 50-km walk is the longest track event at the Olympics. To the nearest mile, about how long is the race in miles?
7. The Tour de France bicycle race is not only challenging; at 2,300 miles, it is long! In kilometers, about how long is the race?
8. The fastest ball game in the world may well be Jai-Alai. In it, players use a scoop attached to their hand to throw a small hard ball as fast as 188 mph at a granite wall. To the nearest tenth of a kilometer, about how fast is that speed in km/h?
9. **Reason abstractly and quantitatively.** A baseball used in major league games weighs at least 5 oz and not more than 5.25 oz. About what is that range measured in grams? Explain your reasoning.
10. About 50 years ago, the Yankees' Mickey Mantle was one of baseball's great sluggers. He is credited with hitting the longest homerun ever. It traveled a distance of 643 feet. How many kilometers did the ball travel, rounded to the nearest hundredth?
11. **Reason abstractly and quantitatively.** How many km/h equals 880 ft/min? Explain how you solved this problem.
12. **Make sense of problems.** Ed can run a mile in 6 min 30 sec. Ned can run a kilometer in 4 min. Who runs at a faster rate? Explain.

Check Your Understanding

Answers

4. a. 20.32 b. 12.4 c. 6.3
   d. 47.2 e. 161 f. 2.1
   g. 20 h. 1.1 i. 42
5. Sample answer: multiply 19 by 1.61.

Teacher to Teacher

Point out that multiplication is used when converting from a larger unit to a smaller unit. For example, to convert 44 lb to kg, multiply 44 by the conversion factor 2.2. Likewise, divide to convert from a smaller unit to a larger unit. For example, to convert 16 cm to in., divide 16 by the conversion factor 2.54.

ACTIVITY 8 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 8-1

There are five position players on a starting basketball team: 2 guards, 2 forwards, 1 center.

Write a ratio in simplest form to express each relationship.

1. centers to forwards
2. forwards to guards
3. guards to players on the team
4. guards to players on the court
5. players who are not centers to players on the court

For Items 6–9, determine the rate and the unit rate.

6. \$279 for 9 tickets
7. \$18 for 6 volleyballs
8. 4 fouls in 20 minutes
9. 36 strikeouts in 54 innings

A total of 180 students and 35 chaperones are going on a field trip to the Smithsonian Institution in Washington, D.C.

Write each ratio in simplest form.

10. the ratio of students to chaperones
11. the ratio of chaperones to students
12. the ratio of students to people on the trip.

Determine the unit rate. Use mental math when you can.

13. 6 golf balls for \$15
14. 2 dozen tennis balls for \$36
15. 4 lb meat for \$18
16. 24 tickets for \$480

Determine if each pair of ratios are equivalent.

17. $\frac{8}{5}, \frac{24}{20}$ 18. $\frac{0.5}{10}, \frac{5}{100}$ 19. $\frac{1.3}{5.2}, \frac{12}{48}$

Density is the ratio of mass to volume. A 3-liter jug of honey has a mass of 4.5 kg.

20. Write the density of honey as a ratio in three different ways.
21. Write the density of honey as a unit rate.

Lesson 8-2

Solve.

22. In 2002, Takaru Kobyashi ate 50 hot dogs in 12 minutes! At that rate, and assuming that he wouldn't explode, how many dogs could Takaru eat in an hour?
23. If $\frac{3}{4}$-cup of packed brown sugar is needed for one batch of chocolate chip cookies, how much packed brown sugar is needed for five batches?
    A. $\frac{15}{100}$ cup
    B. $3\frac{3}{4}$ cups
    C. 3 cups
    D. $5\frac{3}{4}$ cups
24. **Make use of structure.** If a person walks $\frac{1}{2}$ mile in $\frac{1}{4}$ hour, how far does that person walk in $1\frac{3}{4}$ hours at that rate?
    A. $\frac{1}{8}$ of a mile
    B. $\frac{7}{8}$ of a mile
    C. 5 miles
    D. $3\frac{1}{2}$ miles

ACTIVITY PRACTICE

1. 1 : 2
2. 1 : 1
3. 2 : 5
4. 2 : 5
5. 4 : 5
6. 9 : 279; 1 for \$31
7. 6 : 18; 1 for \$3
8. 4 : 20; 1 every 5 min
9. 36 : 54; 2 every 3 innings
10. 36 to 7
11. 7 to 36
12. 36 to 43
13. \$2.50 each
14. \$1.50 each
15. \$4.50 per lb
16. \$20 per ticket
17. no
18. yes
19. yes
20. 4.5 to 3; 4.5 : 3; $\frac{4.5}{3}$
21. 1.5 : 1
22. 250
23. B
24. D

25. $\frac{1.5}{1} = \frac{n}{7}$; 7.5 c
26. 22
27. 15
28. Jay; one way: express each ratio in simplest form, compare; $\frac{4}{5} > \frac{5}{9}$
29. \$0.69
30. 86.25
31. about 7 more Euros
32. 15.24
33. 17.61
34. 10.89
35. 24.84
36. 6
37. 2.11
38. heavy suitcase
39. 3
40. about 19.39 oz
41. about 9.15 oz
42. about 15.98 lb
43. about 8.70 min
44. 50 min
45. about 13.49 oz
46. 139,764; 2,329.4 h

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Solve by writing and solving a proportion.

25. One recipe for pancakes says to use $1\frac{1}{2}$ cup of mix to make 7 pancakes. How much mix is needed to make 35 pancakes?
26. At the local pizza parlor, game tickets can be traded for small toys. The rate is 10 tickets for 4 small toys. If Meg won 55 tickets playing skeeball, for how many small toys can she trade her tickets?
27. The ratio of boys to girls on a swimming team is 4 to 3. The team has 35 members. How many are girls?
28. Jay made 8 of 10 free throws. Kim made 25 of 45. Who made free throws at the better rate? How do you know?

Troy is going to Spain and needs to convert his dollars to Euros. He knows that when he goes, \$5.00 is equivalent to about 3.45 Euros.

29. Find the unit rate of Euros per dollar
30. How many Euros will he get for \$125?
31. About how many more or fewer Euros would Troy get for \$125 if the exchange rate had changed to 0.75 Euros per dollar?

Lesson 8-3

Convert. Round your answers to the nearest hundredth, as needed.

32. 6 in. ≈ ____ cm
33. 500 g ≈ ____ oz
34. 24 lb ≈ ____ kg
35. ____ mi ≈ 40 km
36. ____ oz = 170.4 g

Solve. Use the conversion factors provided on page 85. As needed, round answers to the nearest hundredth.

37. How many ounces are in 80 grams?
38. What might weigh 20 kg: a small car, a tablet, a heavy suitcase, or a watermelon?
39. A recipe calls for 8 oz of raisins. The raisins come in 100-gram packages. How many packages do you need to buy?
40. A golf ball weighs about 45.9 grams. About how many ounces would a dozen golf balls weigh?
41. A regulation volleyball can weigh anywhere from 260 grams to 280 grams. In ounces, what is the least a volleyball can weigh?
42. The most a bowling ball can weigh is 7,258 grams. What is the most it can weigh when measured in pounds?
43. Lisa can run a mile in 7 minutes. At that rate of speed, how long would it take her to run 2 kilometers?
44. Jen can run a mile in 8 minutes. Which is the most reasonable time for her to run a 10-km race: 1.6 min, 5 min, 50 min, or 500 min?

An official rugby ball can weigh anywhere from 383 grams to 439 grams.

45. What is the least one of these balls can weigh, measured in ounces?

MATHEMATICAL PRACTICES
Make Sense of Problems

46. The record for the most Major League Baseball career innings pitched is held by Cy Young, with 7,356 innings. If the average length of an inning is19 minutes, how many minutes did Young play in Major League games? How many hours is this?

Ratio and Proportion

Strange, But True

Lesson 8-1 Ratio and Unit Rates

Learning Targets:

- Express relationships using ratios.
- Find unit rates.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Paraphraing, Note Taking, Sharing and Responding, Discussion Groups

Interesting sports facts and statistics are abundant. For example, there are only *two* days of the year in which there are *no* professional sports games (MLB, NBA, NHL, or NFL): the day before and the day after the Major League Baseball (MLB) All-Star Game.

You can write a ***ratio*** to compare the number of days without any professional sports events each year with the total number of days in a year. There are three ways to write a ratio to express the relationship between two quantities:

a to b $\qquad$ $a : b$ $\qquad$ $\frac{a}{b}$

1. Write a ratio, in all three forms, to compare the days without games to the days in a year.

2. What ratio compares days with games to days in a year? Write it three ways.

3. What ratio can you write that compares days with games to days without them? Write this ratio three ways, too.

The ratios above all compare two *like* units—*days* and *days*. A ratio that compares two *different* kinds of units is called a ***rate.*** One common rate in sports is miles per hour (mi/h or mph), as in car racing.

4. List some other sports statistics commonly given as rates.

You can use basketball free throws to explore rates. In a group of four, make 12 paper basketballs. Place a wastebasket about six feet from a "free-throw line." Record how many "baskets" each of you makes within the time listed in the table. Have one member of the group keep time. Then work together to answer Items 5-8 on the next page. If you do not know exact words to use during discussion, use synonyms or request assistance from group members. If you need to, use non-verbal cues such as raising your hand for help.

My Notes

CONNECT TO SPORTS

NBA = National Basketball Association

NHL = National Hockey League

NFL = National Football League

MATH TERMS

A **ratio** is a comparison of two quantities. You can write a ratio as a fraction, using the word "to," or using a colon.

A **rate** is a ratio that compares two *different* units, such as distance and time, or a ratio that compares two different things measured with the same unit, such as cups of water and cups of frozen orange juice concentrate.

DISCUSSION GROUP TIPS

As you interact with your group in solving problems, you may hear math terms and other words that may be new to you. Ask for clarification of their meaning, and make notes to help you learn and use vocabulary heard during classroom instruction and interactions.

My Notes

Team Member	Baskets Made	Time (in seconds)	Rate
1		30	
2		15	
3		20	
4		10	

5. What units are you comparing in this free-throw activity?

6. **Reason quantitatively.** Examine all results. What can you say about the relationship between baskets made and time allowed?

When the second term of a rate is 1, the rate is called a ***unit rate*** Miles *per* hour is a kind of unit rate. So is price *per* pound.

MATH TERMS

A **unit rate** is a rate expressed in terms of 1 unit.

In some cases, you find unit rates by *multiplying*. In other cases, you find them by *dividing*.

Suppose that you and your friends attend a basketball game. You buy a block of 8 tickets for \$192. You want to know the price per ticket. That price can be expressed as a unit rate.

7. What is that rate? How did you figure it out?

Now look back at your made-basket rates. To see who the best shooter was, you can express each rate as a unit rate.

Think: 60 seconds = 1 minute. Let made-baskets per minute be your unit rate.

Suppose you made 7 baskets in 30 seconds. Use mental math to find how many you made in 60 seconds.

$$\frac{7}{30} = \frac{7\,(2)}{30\,(2)} = \frac{14 \text{ baskets}}{60 \text{ seconds}} = \frac{14 \text{ baskets}}{1 \text{ minute}}$$

So, your unit rate is 14 baskets per minute.

8. **a.** How can you find the one-minute unit rate for the baskets team member 1 made in 30 seconds? Explain your reasoning.

b. How can you find the unit rate for the baskets that team member 3 made in 20 seconds? Explain your reasoning.

My Notes

Check Your Understanding

9. There are nine position players on a baseball team:
3 outfielders, 4 infielders, 1 pitcher, 1 catcher.
Write a ratio in simplest form to express each relationship.

a. infielders to number of players
b. outfielders to infielders
c. number of players to number of outfielders
d. number of players other than infielders and outfielders to number of players
e. number of pitchers to number of catchers

10. Write the rate. Then find the unit rate.

a. $162 for 9 tickets
b. 15 baskets in 5 minutes
c. 84 yards in 14 running attempts
d. 24 strikeouts in 12 innings

LESSON 8-1 PRACTICE

11. Jed made 2 free throws in 5 seconds. How many would he make in one minute?

12. Rita ran 5 miles in 48 minutes. What was her time per mile?

13. In a typical Wimbledon tennis tournament, 42,000 balls are used and 650 matches are played. About how many balls are used per match?

14. Reggie Jackson played in major league baseball for 21 years. Although this slugger was known best for his home runs, he holds the major league record for strikeouts: 2,597. What was his approximate rate of strikeouts per year?

15. In skateboarding, an *ollie* is a move in which the athlete pops the skateboard into the air, making it appear that the board and skateboarder are attached. At the 2009 X Games, one skateboarder completed 34 ollies in 30 seconds.

a. If he could keep up that rate, what would be his rate of ollies per minute?
b. What would be his rate per hour?

16. **Reason quantitatively.** A major-league baseball player was able to hit 70 home runs in 162 games. Create a unit rate for this information.

My Notes

Learning Targets:

- Determine whether quantities are in a proportional relationship.
- Solve problems involving proportional relationships.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Predict and Confirm, Note Taking, Create Representations

As you read the following scenario, mark the text to identify key information and parts of sentences that help you make meaning from the text.

The fastest time for running a mile while balancing a baseball bat on a finger is 7 min 5 s. This record was set by Ashrita Furman on June 20, 2009. At that rate of speed, Meg predicts that it would take 1,275 seconds, or 21 min 15 s, for Ashrita to run 3 mi. Is she right?

You can write a proportion to find out. A ***proportion*** is an equation. It consists of two equivalent ratios.

Example: $\frac{2}{5} = \frac{4}{10}$

MATH TERMS

A **proportion** is an equation stating that two ratios are equivalent.

To determine if Meg is correct, let n = the time it will take Ashrita to run 3 miles balancing a baseball bat.

First, convert 7 min 5 seconds to seconds: 425 seconds.

$$\frac{\text{time}}{\text{distance}} \rightarrow = \frac{425}{1} = \frac{1275}{3} \leftarrow \frac{\text{time}}{\text{distance}}$$

When two ratios are equal, their **cross-products** are equal.

For any proportion $\frac{a}{b} = \frac{c}{d}$, $ad = bc \leftarrow$ cross-products

READING MATH

Read the proportion $\frac{2}{5} = \frac{4}{10}$ as "the ratio 2 to 5 equals the ratio 4 to 10" or as "2 is to 5 as 4 is to 10."

1. Using what you know about proportions, use the proportion above involving Ashrita's speed and distance data and find the cross-products. Are the cross products equal?

2. Is Meg's prediction correct?

3. **Reason quantitatively.** Are there other ways to determine if Meg is right? Explain.

4. **Construct viable arguments.** Suppose a fast-running juggler beat Ashrita's record by half a minute. Could that person, continuing at that new world-record rate of speed, run 2 mi while juggling in 13 min 10 s? Use a proportion to find out and explain your reasoning.

My Notes

5. A three-toed sloth can cover a mile in 0.15 of an hour. Use proportions and sloth speed to complete the table for the distances shown.

Distance (mi)	1	2	5	12
Time (h)				

You can use proportions to solve problems about ratios and rates.

Example

Roger Bannister was the first person to break the four-minute mile. On May 6, 1954, his time was 3 minutes, 59.4 seconds. Bannister's first quarter-mile time was 57.5 seconds. Use a proportion to find his time if he had kept up this pace.

Step 1: Write a proportion.
Let n = time to run the entire race.
Use 0.25 for the first quarter-mile Bannister ran.

$$\frac{\text{time (sec)}}{\text{distance (mi)}} \rightarrow \frac{57.5}{0.25} = \frac{n}{1} \leftarrow \frac{\text{time (sec)}}{\text{distance (mi)}}$$

Step 2: Solve the proportion using cross-products:
$0.25 \times n = 1 \times 57.5$
$0.25\, n = 57.5$

Step 3: Solve the equation to find n.
$n = 57.5 \div 0.25 \leftarrow$ *Think:* Divide both sides by 0.25.
$n = 230$ s
$n = 3$ min 50 s $\leftarrow$ write as minutes and seconds

Solution: Had Bannister kept up his quarter-mile pace, he would have run the mile in 230 sec, or 3 min 50 sec.

a. Model with mathematics. Why do you divide *both* sides of the equation by 0.25?

b. Reason abstractly and quantitatively. What other proportions could you have written to solve the problem?

MATH TIP

Recall that in any proportion, cross-products will be equal.
For the proportion

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$

When you write a proportion, be sure to set up the ratios in a consistent way according to the units associated with the numbers.

Try These

Solve each proportion.

a. $\frac{n}{1} = \frac{2.45}{0.35}$ **b.** $\frac{n}{5} = \frac{23.4}{2}$ **c.** $\frac{3}{n} = \frac{5.4}{7.2}$

My Notes

Check Your Understanding

6. Use cross products to determine if the ratios are equivalent.
   a. $\frac{3}{4}, \frac{6}{8}$
   b. $\frac{8}{5}, \frac{24}{16}$
   c. $\frac{70}{60}, \frac{6}{7}$
   d. $\frac{1.3}{7.8}, \frac{3}{18}$
   e. $\frac{4}{7}, \frac{10}{17.5}$
   f. $\frac{9}{4}, \frac{2.1}{1.4}$
   g. $\frac{3}{0.8}, \frac{21}{5.6}$
   h. $\frac{0.3}{2}, \frac{0.03}{20}$
7. **Make use of structure.** Write a proportion for each situation. Then solve.
   a. 336 dimples on one golf ball; 2016 dimples in n balls
   b. 3 miles in 2.8 minutes; 33.3 miles in x minutes
   c. 25 yards in $2\frac{1}{2}$ seconds; 100 yards in y seconds
   d. 480 heartbeats in 4 minutes; z heartbeats in 1 minute
8. A zebra can run at a speed of 40 mph. Complete the table using this information.

Time (h)	1	0.25	0.5	1.75
Distance (mi)				

LESSON 8-2 PRACTICE

Solve by writing and solving a proportion.

9. There are 20 stitches per panel on a soccer ball. A soccer ball has 32 leather panels. How many stitches, in all, are on a soccer ball?
10. Jed took 2 free throws in 5 seconds. Alex took 4 free throws in 12 seconds. Did the two shoot free throws at the same rate? Explain.
11. The ratio of girls to boys on a soccer team is 2 to 3. If there are 25 players on the team, how many are girls?
12. **Model with mathematics.** A package of tickets for 4 home games costs $180. What proportion can you write to find what a 12-game package costs if all individual tickets have the same price?
13. Carlos completed 7 of 10 passes. Ty completed 21 of 30 passes. Compare their pass-completion rates.
14. Carla's team won 3 of its 5 games. Elena's team won games at the same rate and won 12 games. How many games did Elena's team play?
15. Greta completed a mile race in 5 minutes. Inez ran a mile in which each quarter-mile split was 1 min 20 seconds. Which of the two girls had the faster time? How much faster?

My Notes

Learning Targets

- Convert between measurement. Use unit rates and proportions for conversions

SUGGESTED LEARNING STRATEGIES: Visualization, Think Aloud, Discussion Groups, Sharing and Responding, Create a Plan, Identify a Subtask, Note Taking

Some problems involving measurements will require you to convert between customary and metric units of measure.

Example

A tennis court is 78 feet in length and for singles play is 27 feet in width. How many meters wide is the tennis court?

Step 1: Start by converting feet to yards: $\frac{3\text{ ft}}{1\text{ yard}} = \frac{27\text{ feet}}{x\text{ yards}}$

Step 2: Use cross-products to solve the proportion:

$27 \cdot 1 = 3x$

$27 = 3x$

$9 = x$

So, there are 9 yards in 27 feet

Step 3: Next, convert 9 yards to meters:

$\frac{9\text{ yd}}{x\text{ m}} = \frac{1\text{ yd}}{0.9144\text{ m}}$

$1x = 9(0.9144)$

$x = 8.2296$ meters

$x \approx 8.23$ meters

Solution: The tennis court is 8.23 meters wide.

Try These

a. Attend to precision. Find the length of the tennis court above in meters. Be sure to include units.

1. How do can you tell that a proportion involving conversions has been set up correctly?

2. The conversion factor for converting meters to yards is $\frac{1}{0.9144}$, and the conversion factor for converting yards to meters is $\frac{0.9144}{1}$. Use the conversion chart in the My Notes column to find the conversion factors for converting grams to ounces and converting liters to quarts.

MATH TIP

In general, conversions between customary and metric systems result in approximate measurements.

The symbol $\approx$ means "is approximately equal to."

MATH TIP

Conversion factors for some common customary and metric measures:

1 yd $\approx$ 0.9144 m
1 m $\approx$ 1.094 yd
1 in. $\approx$ 2.54 cm
1 mi $\approx$1.61 km
1.06 qt $\approx$ 1L
1 oz $\approx$ 28.4 g
1 lb $\approx$ 0.4536 kg
2.2 lb $\approx$ 1 kg
1 cu ft (ft^3) $\approx$ 0.0283m^3

My Notes

Five hundred years ago, the toy that we now call a yo-yo was bigger and used as a weapon in the Philippines. Each weighed about 4 pounds and was attached to a 20-ft cord.

3. About how much did one of those killer yo-yos weigh, in kilograms? Find out by writing and solving a proportion using a conversion factor as one of the ratios. Use a calculator to speed computation.

Check Your Understanding

4. Use appropriate tools strategically. Convert rounding your answers to the nearest tenth when necessary.

a. 8 in. ≈ ____ cm **b.** ____ mi ≈ 20 km **c.** 16 cm ≈ ____ in.
d. ____ L ≈ 50 qt **e.** ____ km ≈ 100 mi **f.** 60 g ≈ ____ oz
g. 44 lb ≈ ____ kg **h.** 500 g ≈ ____ lb **i.** 1.5 oz ≈ ____ g

5. Write a short note to your teacher explaining how you would estimate the number of kilometers in 19 miles.

LESSON 8-3 PRACTICE

6. The 50-km walk is the longest track event at the Olympics. To the nearest mile, about how long is the race in miles?

7. The Tour de France bicycle race is not only challenging; at 2,300 miles, it is long! In kilometers, about how long is the race?

8. The fastest ball game in the world may well be Jai-Alai. In it, players use a scoop attached to their hand to throw a small hard ball as fast as 188 mph at a granite wall. To the nearest tenth of a kilometer, about how fast is that speed in km/h?

9. Reason abstractly and quantitatively. A baseball used in major league games weighs at least 5 oz and not more than 5.25 oz. About what is that range measured in grams? Explain your reasoning.

10. About 50 years ago, the Yankees' Mickey Mantle was one of baseball's great sluggers. He is credited with hitting the longest homerun ever. It traveled a distance of 643 feet. How many kilometers did the ball travel, rounded to the nearest hundredth?

11. Reason abstractly and quantitatively. How many km/h equals 880 ft/min? Explain how you solved this problem.

12. Make sense of problems. Ed can run a mile in 6 min 30 sec. Ned can run a kilometer in 4 min. Who runs at a faster rate? Explain.

ACTIVITY 8 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 8-1

There are five position players on a starting basketball team: 2 guards, 2 forwards, 1 center.

Write a ratio in simplest form to express each relationship.

1. centers to forwards
2. forwards to guards
3. guards to players on the team
4. guards to players on the court
5. players who are not centers to players on the court

For Items 6–9, determine the rate and the unit rate.

6. \$279 for 9 tickets
7. \$18 for 6 volleyballs
8. 4 fouls in 20 minutes
9. 36 strikeouts in 54 innings

A total of 180 students and 35 chaperones are going on a field trip to the Smithsonian Institution in Washington, D.C.

Write each ratio in simplest form.

10. the ratio of students to chaperones
11. the ratio of chaperones to students
12. the ratio of students to people on the trip.

Determine the unit rate. Use mental math when you can.

13. 6 golf balls for \$15
14. 2 dozen tennis balls for \$36
15. 4 lb meat for \$18
16. 24 tickets for \$480

Determine if each pair of ratios are equivalent.

17. $\frac{8}{5}, \frac{24}{20}$ 18. $\frac{0.5}{10}, \frac{5}{100}$ 19. $\frac{1.3}{5.2}, \frac{12}{48}$

Density is the ratio of mass to volume. A 3-liter jug of honey has a mass of 4.5 kg.

20. Write the density of honey as a ratio in three different ways.
21. Write the density of honey as a unit rate.

Lesson 8-2

Solve.

22. In 2002, Takaru Kobyashi ate 50 hot dogs in 12 minutes! At that rate, and assuming that he wouldn't explode, how many dogs could Takaru eat in an hour?
23. If $\frac{3}{4}$-cup of packed brown sugar is needed for one batch of chocolate chip cookies, how much packed brown sugar is needed for five batches?
    A. $\frac{15}{100}$ cup
    B. $3\frac{3}{4}$ cups
    C. 3 cups
    D. $5\frac{3}{4}$ cups
24. **Make use of structure.** If a person walks $\frac{1}{2}$ mile in $\frac{1}{4}$ hour, how far does that person walk in $1\frac{3}{4}$ hours at that rate?
    A. $\frac{1}{8}$ of a mile
    B. $\frac{7}{8}$ of a mile
    C. 5 miles
    D. $3\frac{1}{2}$ miles

Solve by writing and solving a proportion.

25. One recipe for pancakes says to use $1\frac{1}{2}$ cup of mix to make 7 pancakes. How much mix is needed to make 35 pancakes?

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Troy is going to Spain and needs to convert his dollars to Euros. He knows that when he goes, \$5.00 is equivalent to about 3.45 Euros.

29. Find the unit rate of Euros per dollar

30. How many Euros will he get for \$125?

31. About how many more or fewer Euros would Troy get for \$125 if the exchange rate had changed to 0.75 Euros per dollar?

Lesson 8-3

Convert. Round your answers to the nearest hundredth, as needed.

32. 6 in. $\approx$ ____ cm

33. 500 g $\approx$ ____ oz

34. 24 lb $\approx$ ____ kg

35. ____ mi $\approx$ 40 km

36. ____ oz $=$ 170.4 g

Solve. Use the conversion factors provided on page 85. As needed, round answers to the nearest hundredth.

37. How many ounces are in 80 grams?

38. What might weigh 20 kg: a small car, a tablet, a heavy suitcase, or a watermelon?

39. A recipe calls for 8 oz of raisins. The raisins come in 100-gram packages. How many packages do you need to buy?

40. A golf ball weighs about 45.9 grams. About how many ounces would a dozen golf balls weigh?

41. A regulation volleyball can weigh anywhere from 260 grams to 280 grams. In ounces, what is the least a volleyball can weigh?

42. The most a bowling ball can weigh is 7,258 grams. What is the most it can weigh when measured in pounds?

43. Lisa can run a mile in 7 minutes. At that rate of speed, how long would it take her to run 2 kilometers?

44. Jen can run a mile in 8 minutes. Which is the most reasonable time for her to run a 10-km race: 1.6 min, 5 min, 50 min, or 500 min?

An official rugby ball can weigh anywhere from 383 grams to 439 grams.

45. What is the least one of these balls can weigh, measured in ounces?

MATHEMATICAL PRACTICES
Make Sense of Problems

46. The record for the most Major League Baseball career innings pitched is held by Cy Young, with 7,356 innings. If the average length of an inning is19 minutes, how many minutes did Young play in Major League games? How many hours is this?