Writing Expressions

ACTIVITY 9

Pebbles in the Sand

Lesson 9-1 Representing Patterns

Learning Targets:

- Identify and represent patterns using models, tables, and expressions.
- Write and evaluate algebraic expressions that represent patterns with constant differences.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Use Manipulatives, Think-Pair-Share

People have been investigating number patterns for thousands of years. ***Legend*** has it that Pythagoras and his students arranged pebbles in the sand to represent number patterns. One pattern they studied is shown below.

Figure 1 Figure 2 Figure 3

1. Draw the fourth, fifth and sixth figures.

2. Organize the number of pebbles in each figure into a table.

Figure Number	Number of Pebbles
1	2
2	4
3	6
4	8
5	10
6	12
7	14

3. Extend the pattern to determine how many pebbles are in the 10th figure.

20 pebbles

4. Describe the patterns you observe in the pebble drawings and the table in words.

Answers will vary. Each figure has 2 more pebbles than the previous one.

5. How many pebbles are in the 53rd figure? Explain your reasoning.

106 pebbles. Sample answer: I multiplied the figure number by 2.

6. Write a numeric expression using the number 3 for the number of pebbles in the third figure.

2(3)

My Notes

ACADEMIC VOCABULARY

A ***legend*** is a story handed down by tradition that is popularly regarded as historical but unverified.

ACTIVITY 9

Investigative

Activity Standards Focus

Unit 2 focuses on linear equations. Activity 9 introduces some of the ideas that will be needed when analyzing equations by first looking at those ideas in the context of *patterns*. Before they see algebraic equations, students will be introduced to algebraic expressions as they use them to identify and represent patterns. They will write and evaluate algebraic expressions that represent patterns—some with constant differences and some without. This idea of *constant difference* will serve as an introduction to the concepts they will come to understand in later activities as *rate of change* and *slope*.

Lesson 9-1

PLAN

Pacing: 2 class periods

Chunking the Lesson

#1–5 #6–8

Check Your Understanding

#14 #15–16 #17

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Give students a few minutes to write down examples of *patterns* in everyday life. Then have volunteers share their ideas with the class. Ask them what makes a sequence a pattern. Explain that this activity will deal with patterns in numbers.

1–5 Look for a Pattern, Use Manipulatives, Create Representations, Think-Pair-Share

Give students a few minutes to use the manipulatives if needed to form figures 4, 5 and 6. Then have them complete the table and extend the pattern by either drawing more figures or adding rows to the table before pairing with a partner to discuss Items 4 and 5. Partners should then share with other pairs to confirm responses and to add to individual patterns observed. Groups should be given an opportunity to share patterns with the entire class. Most students will notice that the number of pebbles increases by 2 each time. Some might say that the numbers of pebbles are the even numbers, 2, 4, 6, and so on. Students who notice that the number of pebbles is twice the figure number are ready to generalize a rule for this pattern.

6–8 Look for a Pattern, Create Representations Depending on how well students make the connection to multiplication, they may work quickly through Items 6 and 7. Use these questions as an opportunity to reinforce the meaning of the word expression, the difference between a numeric and algebraic expression, and how to evaluate expressions. Item 8c emphasizes the importance of checking rules generalized from patterns.

Developing Math Language

This lesson contains multiple vocabulary terms. Use your Word Wall and a vocabulary organizer or notebook to reinforce vocabulary that appears throughout this unit, including the term *expression*, which may have been seen before, and the terms *consecutive terms* and *constant difference*, which may be new.

Students may also want to note the term *evaluate*; be sure they understand that evaluating is something you do to an expression, whereas solving is something you do to an equation.

Also, the word *figure* is used repeatedly in this activity. The *figure number* will be used repeatedly in this activity as the input value when organizing information into tables. Later in this unit, students will see the term *nth figure*. It refers to the general expression for figure number *n*.

A concept map will help students make connections between words and phrases such as *pattern, table, figure number, input, expression, variable, substitute, evaluate,* and *nth figure.*

Encourage students to make notes about new terms and their understanding of what they mean and how to use them to describe precise mathematical concepts and processes.

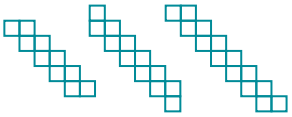
Check Your Understanding

Debrief items 1–8 by having students complete the Check Your Understanding individually before sharing with their groups and revising responses if needed. As part of the whole class discussion, ask students which strategy is more efficient for arriving at the number of pebbles in the 10th, 53rd, or 100th figure, multiplication or the extension of tables.

Answers

9. Figure 1: 4 squares; Figure 2: 6 squares; Figure 3: 8 squares

10.



My Notes

MATH TIP

An *expression* is a mathematical phrase using numbers or variables or both. $1 + 1$ and $3x - 5$ are examples of expressions.

MATH TERMS

You do not solve an expression; you **evaluate** it for a specific value. To do this, substitute a value for the variable and simplify.

7. Write a similar numeric expression using the number 7 for the number of pebbles in the seventh figure.
2(7)

8. Let *n* represent the figure number.
- **a.** Use *n* to write an expression that could be used to determine the number of pebbles in figure *n*.
  2*n*
- **b.** What value would you substitute for *n* to determine the number of pebbles in the third figure?
  3
- **c.** Check to see that your expression from part a is correct by ***evaluating*** it for the value you chose in part b.
  2(3); 6 pebbles
- **d.** Use your expression to determine the number of pebbles in the 100th figure.
  2(100); 200 pebbles

Check Your Understanding

A pattern of small squares is shown. Use the pattern to answer Items 9–13 that follow.

Figure 1 Figure 2 Figure 3

9. How many small squares are in each figure?

10. Draw the fourth, fifth, and sixth figures and determine the number of small squares in each figure.

11. Create a table to organize the number of squares in each figure into a table.

12. Describe in words the patterns you see in the square pattern and in the table.

13. How many squares would be in the 10th figure? Explain your reasoning.

11.

Figure Number	Number of Squares
1	4
2	6
3	8
4	10
5	12
6	14
7	16

12. Answers will vary. The number of squares increases by 2 as the figure number increases by 1.

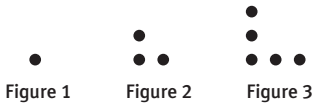
13. 22 squares. Answers may vary. I multiplied 10 by 2 and then added 2.

Lesson 9-1
Representing Patterns

ACTIVITY 9 *continued*

My Notes

14. Another pebble arrangement is shown below.



a. Draw the fourth, fifth, and sixth figures.

b. Organize the number of pebbles in each figure in the table below.

Figure Number	Pebbles	Difference in Number of Pebbles
1	1	----------
2	3	2
3	5	2
4	7	2
5	9	2
6	11	2
7	13	2
8	15	2
9	17	2
10	19	2

c. Describe any patterns you observe in the pebble drawings and in the table above.

Sample answer: The number of pebbles in each figure is 1 less than 2 times the figure number.

15. Subtract ***consecutive terms*** in the pebbles column and record this information in the last column in the table.

See the table in Item 14 part b.

16. Reason quantitatively. How does the ***constant difference*** in the new column relate to the patterns you observed?

Sample answer: Each consecutive figure increases by 2 pebbles. This consistency allows me to observe that the number of pebbles in each figure is one less than twice the figure number.

CONNECT TO AP

The ability to identify patterns allows you to understand and describe different types of functions and provides a foundation for understanding rate of change in calculus.

MATH TERMS

Consecutive terms follow each other directly in a sequence. In the table below, 5 and 8 are *consecutive terms*. There is a **constant difference** between the terms in this table that is equal to 3.

n	term	
1	5	3
2	8	3
3	11	3
4	14	

MINI-LESSON: Evaluating Expressions

For additional student support in evaluating algebraic expressions, a mini-lesson is available. This lesson will feature some simple practice problems in which students evaluate an expression for different values of the same variable, helping them to develop the concept of a function as a set of related input and output values.

See SpringBoard's eBook Teacher Resources for a student page for this mini-lesson.

ACTIVITY 9 Continued

14 Look for a Pattern, Use Manipulatives, Create Representations, Think-Pair-Share, Sharing and Responding Have students work with a partner and discuss their responses with other pairs. If there are groups that have created an algebraic rule for the number of pebbles in the *n*th figure, have them share last.

TEACHER to TEACHER

Spend time during the Sharing and Responding making connections between the number in the table, the arrangement of pebbles and the patterns that students described. Some groups might relate the addition of 2 each time to placing a pebble on the end of the row and the end of the column. Other groups might see the pattern as the sum of n, the number of pebbles along the bottom, and $n - 1$, the number of pebbles on the side; or as two times n, subtracting 1 so as not to double count the corner pebble. If groups do not come up with multiple ways to see the pattern, you may wish to demonstrate alternate methods.

CONNECT TO AP

Observing patterns of change is a great way for students to begin to understand and describe different types of functions. It is also a natural pre-cursor to understanding rate of change in calculus. Only linear patterns exhibit a constant difference between consecutive terms of a sequence. In calculus, the derivative (rate of change) of a linear function is the constant difference.

15–16 Sharing and Responding
Students can look to the Math Terms box for a model on how to complete the third column in their table. Be sure that students understand that the last row in the third column should not be filled in. Students should discuss their observations with peers and make a connection to the pebble figures in Item 16. Each figure has two more pebbles than the previous one.

Teacher to Teacher

Students explore several different pebble patterns in this activity. Be sure to have manipulatives available for groups that need them to build their pebble patterns. One of the primary goals here is for students to connect the constant difference observed in the tables to the coefficient of n in the rules they formulate.

17 Look for a Pattern, Discussion Groups, Create Representations

Using repeated addition, students generalize a rule for the number of pebbles in figure n. Students may struggle with writing $2(n - 1)$ in the expression for the number of pebbles in figure n. Remind them that 2 is added one less time than the figure number.

If time permits, allow students to show how expressions for other patterns observed by groups are equivalent to $2(n - 1) + 1$.

Some groups may use guess and check in order to understand that they have a $2n$ pattern but need to reduce the total by 1 to yield the correct values for this pattern.

My Notes

17. The number of pebbles in a specific figure can be written using repeated additions of the constant difference. For example, the third figure is $1 + 2 + 2$ or $1 + 2(2)$.

Figure Number	Pebbles	Expression Using Repeated Addition
1	1	$1 + 2(0)$
2	$1 + 2$	$1 + 2\ (1)$
3	$1 + 2 + 2$	$1 + 2(2)$
4	$1 + 2 + 2 + 2$	$1 + 2(3)$
5	$1 + 2 + 2 + 2 + 2$	$1 + 2(4)$

a. Write the number of pebbles in the fourth and fifth figure using repeated addition of the constant difference.

fourth: $1 + 2 + 2 + 2$ OR $1 + 2(3)$

fifth: $1 + 2 + 2 + 2 + 2$ OR $1 + 2(4)$

b. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

$1 + 2(n - 1)$

c. What value would you substitute for n to determine the number of pebbles in the third figure?

3

d. Check to see that your expression from part a is correct by evaluating it when $n = 5$.

$1 + 2(5 - 1)$; 9 pebbles

e. Use your expression to determine the number of pebbles in the 100th figure.

$1 + 2(100 - 1)$; 199 pebbles

My Notes

Check Your Understanding

18. A pattern of pebbles is shown. Use the pattern to answer parts a–c.

Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.
b. Create a table to organize the number of pebbles in each figure for the first six figures.
c. What is the constant difference?
d. Write the number of pebbles for each figure using repeated addition of the constant difference.
e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

LESSON 9-1 PRACTICE

19. Check to see that your expression from Item 18 part e is correct by evaluating when $n = 6$.

20. Use your expression from Item 18 part e to determine the number of pebbles in the 51st figure.

21. Critique the reasoning of others. The expression that represents the number of squares in the nth figure of a pattern is given by $2 + 3(n - 1)$. Mia claims the constant difference is 2. Do you agree? Explain.

Answers

18. b.

Figure Number	1	2	3	4	5	6
Pebbles	3	5	7	9	11	13
Expression (part d)	$3 + 2(0)$	$3 + 2(1)$	$3 + 2(2)$	$3 + 2(3)$	$3 + 2(4)$	$3 + 2(5)$

ACTIVITY 9 Continued

Check Your Understanding

Use the Check Your Understanding section to debrief Items 14–17. At this point, students should be comfortable recognizing and identifying expressions for patterns characterized by repeated addition of a constant difference.

Answers

18. a.

Figure 4 Figure 5 Figure 6

b. See table below.
c. 2
d. See table in part b.
e. $3 + 2(n - 1)$

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

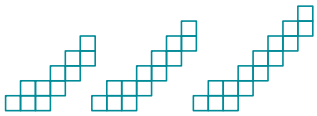
See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Answers

19. $3 + 2(6 - 1) = 13$
20. $3 + 2(51 - 1) = 103$
21. No. Answers may vary. The first 3 terms are 2, 5, and 8, so the constant difference is 3. The first term is 2.

ACTIVITY 9 Continued

22. a.



b.

Figure Number	1	2	3	4	5	6
Number of Squares	5	7	9	11	13	15

c. Answers will vary. Each figure has 2 more squares than the previous one.

d. 2

e. 5

f. $3 + 2(n - 1)$

23. Answers may vary. They both have a constant difference of 2. The number of pebbles in the first figure is 1. The number of squares in the first figure is 5.

ADAPT

This lesson has dealt with recognizing patterns characterized by repeated addition of a constant difference, and deriving expressions for those patterns. Use Activity Practice Items 1–3 to evaluate students' grasp of the concepts involved here. Extra support to help students master these concepts may be provided along with Items 1–2 in the next lesson. These items will review and extend the topic before the lesson moves on to different kinds of patterns.

My Notes

22. A pattern of small squares is shown.

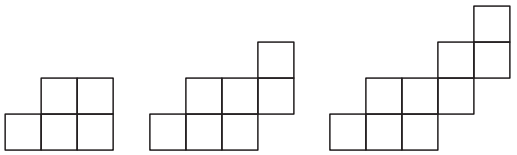


Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.

b. Create a table showing the number of the figure and the number of squares in each figure.

c. Describe the patterns you observe in the square drawings and in your table.

d. What is the constant difference?

e. How many small squares are in the first figure?

f. Let n represent the figure number. Use n to write an expression that could be used to determine the number of squares in the nth figure.

23. Compare the expression you found in Item 22 part f with the expression you found in Item 17 part b. How are they the same? How are they different?

Learning Targets:

- Identify patterns that do not have a constant difference.
- Write and evaluate algebraic expressions that represent patterns that do not have a constant difference.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Group, Group Presentation

1. Four different pebble patterns are shown. Your teacher will assign one to your group. Use your selected pattern to answer parts a–e that follow and then prepare a group presentation of your results.

Pattern A

Figure 1 Figure 2 Figure 3

Pattern B

Figure 1 Figure 2 Figure 3

Pattern C

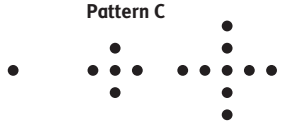


Figure 1 Figure 2 Figure 3

Pattern D

Figure 1 Figure 2 Figure 3

a. Draw a few additional figures and then organize the information in a table. Identify the constant difference.

The next figure in each pattern is shown below.

Pattern A

Pattern B

Pattern C

Pattern D

Figure Number	Pattern A	Pattern B
1	2	1
2	6	4
3	10	7
4	14	10
5	18	13
6	22	16

Figure Number	Pattern C	Pattern D
1	1	2
2	5	6
3	9	10
4	13	14
5	17	18
6	21	22

constant difference: Pattern A: 4, Pattern B: 3, Pattern C: 4, Pattern D: 4

My Notes

Lesson 9-2

PLAN

Pacing: 1–2 class periods

Chunking the Lesson

#1 #2

Check Your Understanding

#7 #8 #9

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

To review the previous lesson, show students a set of figures or a table of values representing a pattern with a simple expression. Ask them to derive the constant difference and the expression representing the pattern. Have volunteers explain how they arrived at their answers.

1 Look for a Pattern, Create Representations, Guess and Check, Discussion Group, Group Presentations To start this process, have students recall how they identified previous patterns. They should recall drawing a few additional figures, organizing the information in a table, describing the pattern in words, writing a general expression, and then using the expression to determine the number of pebbles in a figure number.

As a new challenge, in Part e, groups will need to determine whether or not it is possible to have a figure with 100 pebbles. Honor all solution methods but make sure groups clearly justify their thinking. It is not expected that all groups will write and solve an equation to answer Part e.

1 (continued) The following questions could be used when debriefing the group presentations.

- How did you see the pattern?
- How do you know your pattern works?
- How can two different pebble patterns have the same rule?
- How did you decide on whether a figure could have 100 pebbles?
- Does anyone see the pattern differently?
- How are all these pattern problems alike?
- How are these pattern problems different?

2 Look for Patterns, Graphic Organizer If students have not already made the connection between the constant difference and the coefficient of n in their rule, now is the time to make it clear to students. A graphic organizer like the one shown could be used to help students make this connection.

Figure	Table
Explanation	Rule

Students can fill in their sample problem and identify the constant difference in each pattern.

My Notes

b. Describe the pattern in words.

A and D: Start with 2 pebbles and increase by 4 each time. B: Start with 1 pebble and increase by 3 each time. C: start with 1 pebble and increase by 4 each time.

c. Reason abstractly. Write an expression using the variable n that could be used to determine the number of pebbles in figure n.

A and D: $4(n-1)+2$ OR $4n-2$; B: $3(n-1)+1$ OR $3n-2$; C: $4(n-1)+1$ OR $4n-3$

d. Use your expression to determine the number of pebbles in the 10th, 53rd, and 200th figures.

Figure	10th	53rd	200th
A and D	38	210	798
B	28	157	598
C	37	209	797

e. For the pattern you selected, is it possible to have a figure with 100 pebbles? Explain your reasoning.

For Pattern B it is possible. For Patterns A, C, and D it is not possible. Explanations will vary.

2. Based on the class's work for Item 1, how does the constant difference in a pebble pattern relate to the algebraic expression that can be written to describe the pattern?

The constant difference is the coefficient of n in the expression.

My Notes

Check Your Understanding

Tables representing two pebble patterns are shown below.

Pebble Pattern A					
Figure	1	2	3	4	5
Pebbles	3	7	11	15	19

Pebble Pattern B					
Figure	1	2	3	4	5
Pebbles	2	7	12	17	22

3. What is the constant difference for each pebble pattern shown in the tables?
4. For each pebble pattern, use the variable n to write an expression for the nth figure.
5. How many pebbles are in the 50th figure for each pebble pattern?
6. Both pebble patterns have 7 pebbles in Figure 2. If the patterns continue, will they ever have the same number of pebbles as another figure? Explain your reasoning.

7. The Pythagoreans also studied the following pebble pattern.

Figure 1 Figure 2 Figure 3

a. How many pebbles are there in the fourth, fifth, and sixth figures?
fourth: 16; fifth: 25; sixth: 36

b. Does this pattern have a constant difference? Explain your response.
No. The difference between the number of pebbles in each figure is not constant.

c. Describe the pattern in words.
Sample answer: The number of pebbles in each figure is the figure number times itself, or the figure number squared.

d. How many pebbles are there in the 10th figure? How did you determine your answer?
100 pebbles. Sample answer: I multiplied 10 by itself.

ACTIVITY 9 Continued

Check Your Understanding

Debrief this section of the activity by asking students to explain in their own words how to write an algebraic expression from a visual pattern, a table or a written description of a pattern.

Answers

3. The constant difference for A is 4. The constant difference for B is 5.
4. A: $4n - 1$; B: $5n - 3$
5. A: $4(50) - 1$; 199 pebbles
   B: $5(50) - 3$; 247 pebbles
6. No. Answers will vary. Pattern B increases by 5 pebbles in each additional figure while Pattern A only increases by 4 pebbles in each additional figure. Therefore, Pattern B will always have more pebbles.

Teacher to Teacher

In this portion of the activity, students explore non-linear patterns represented by figurate numbers. Students will observe that while the differences have a pattern, it is not constant. The quadratic patterns come from relating the figure number to the dimensions of the pebble shape and considering the total number of pebbles as an array. For example, in Item 7, the pebbles are arranged in an $n \times n$ square.

7 Look for a Pattern, Think-Pair-Share, Create Representations, Sharing and Responding Students should quickly notice that the difference is not constant but that there is a pattern. Rather than having them extend the pattern of adding the next odd number through the 10^{th} figure, groups should be looking for a way to relate the figure number to the total number of pebbles. If a group is stuck, you might ask them what shape is formed by each pebble arrangement. Alternatively, ask students how many pebbles are in each row and column. As groups share responses, be sure students notice that they should be multiplying two variables together rather than multiplying the variable by a constant.

8 Look for a Pattern, Discussion Groups, Create Representations, Guess and Check, Sharing and Responding, Construct an Argument If students made the connection that the figure number was the length of each side of the squares representing the Square Numbers, they can extend that connection here. Some students may struggle with representing the two consecutive numbers as n and $(n + 1)$ when writing the expression for this pattern. Students who are struggling can be encouraged to think about the *area* of a rectangle. Again, as groups share out, be sure students notice that they should be multiplying two variable expressions together rather than multiplying a variable expression by a constant.

9 Look for a Pattern, Discussion Groups, Create Representations, Guess and Check The pattern for the triangular numbers can be challenging for many students. After groups have struggled with this problem, you may need to direct them back to the rectangular numbers and how to find the area of each corresponding rectangle.

Once students can verbalize that any triangular number is half of the corresponding rectangular number, they will be ready to write the expression.

e. How many pebbles are there in the 40th figure? In the nth figure?
40th: 1,600 pebbles; nth: n^2

f. The Pythagoreans called the numbers in this pattern the square numbers. Why do you think this is so?
Sample answer: The number of pebbles in each figure is the square of the figure number. Also, the pebbles in each figure form a square.

8. The numbers in the pebble pattern shown below are called the rectangular numbers.

Figure 1 Figure 2 Figure 3

a. How many pebbles are in the fourth, fifth, and sixth figures?
fourth: 20; fifth: 30; sixth: 42

b. Describe the pattern in words. Is there a constant difference? Explain your response.
The base and height of each consecutive rectangle both increase by 1 pebble. There is no constant difference. The difference between the number of pebbles in each figure is different.

c. Describe how to find the number of pebbles in the 10th figure.
The base and height of each consecutive rectangle both increase by 1 pebble. So, the base of the 10th rectangle will have 10 pebbles. The rectangle will be 11 pebbles high. The number of pebbles will be 10(11), or 110 pebbles.

d. How many pebbles are there in the 30th figure? In the nth figure?
30th: 930 pebbles; nth: $n(n + 1)$

9. The Pythagoreans called the numbers represented by the pebbles in this pebble pattern the triangular numbers.

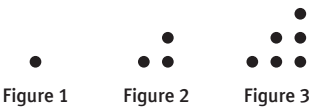


Figure 1 Figure 2 Figure 3

a. Why do you think the Pythagoreans called these numbers triangular?
Sample answer: The pebbles are arranged in a triangular shape.

Lesson 9-2
Using Patterns to Write and Evaluate Expressions

My Notes

b. How is the triangular number pebble pattern related to the pebble pattern of the rectangular numbers?

If each rectangle in the rectangular number pattern is cut in half along the diagonal, it results in two of the same triangular numbers.

c. Use your response to part b to write an algebraic expression for the number of pebbles in the nth triangular number.

$\frac{n(n+1)}{2}$

d. Verify your expression by substituting $n = 4$. Is the result the number of pebbles in the fourth triangular number?

Yes. $\frac{4(4+1)}{2}$; 10 pebbles

e. Use your expression to predict the number of pebbles in the 30th triangular number.

$\frac{30(30+1)}{2}$; 465 pebbles

Check Your Understanding

10. Is the number 72 a square number, rectangular number, or triangular number? Explain your reasoning.

Use the figures below to answer Items 11–15.

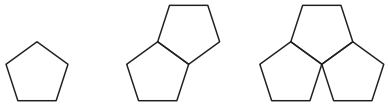


Figure 1 Figure 2 Figure 3

11. Assume each side of each pentagon is 1 cm. What is the perimeter of each figure shown?

12. Draw the next three figures and determine the perimeter of each.

13. Organize the results of Items 11 and 12 in a table. What would be the perimeter of the 10th figure? Explain your reasoning.

14. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.

15. Use your expression to determine the perimeter of the 50th figure.

ACTIVITY 9 Continued

Check Your Understanding

Debrief this section of the activity by having students share within their groups their individual opinions. Groups should then share the consensus with the class. Focus should be on students' explanations.

Answers

10. Rectangular. It would be the 8th rectangular figure. This would give it a base of 8 and a height of 9. 8(9); 72 pebbles.

11. Figure 1: 5 cm; Figure 2: 8 cm; Figure 3: 11 cm

12. 

Figure 4 Figure 5 Figure 6

Perimeter of figure 4: 14 cm;
Perimeter of figure 5: 17 cm;
Perimeter of figure 6: 20 cm

13. 32 cm; Begin with a perimeter of 5 cm in Figure 1 and add 3 nine times.

14. $5 + 3(n - 1)$ OR $3n + 2$

15. $3(50) + 2 = 152$ cm

ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

Answers

16. Answers may vary. Nate does not realize the base and height of a rectangular number differ by 1. He is correct that 56 is a rectangular number. However the reason is the base is 7 and the height is 8.

17. 36; Answers may vary. A square number is the figure number squared. I squared 6.

18. $45 = \frac{9(9+1)}{2}$

19. a. Figure 1: 8 ft; Figure 2: 14 ft; Figure 3: 20 ft

b.

Figure 4: 26 ft

Figure 5: 32 ft

Figure 6: 38 ft

c. $8 + 6(n - 1)$

d. $8 + 6(6 - 1) = 38$

20. $8 + 6(30 - 1) = 182$ ft

ADAPT

Use Items 4–7 in the Activity Practice to evaluate students' ability to work with patterns that are not characterized by a constant difference. Students who need more practice should be given further such patterns to work with, like the quadratic patterns.

My Notes

LESSON 9-2 PRACTICE

16. Critique the reasoning of others. Nate claims 56 is a rectangular number because a rectangle with base 4 and height 14 can be formed. What is his error?

17. What is the sixth square number? How did you get your answer?

18. Use the expression from Item 9 part c to show that 45 is a triangular number.

19. Use the figures below to answer parts a–d. Assume each figure is a regular octagon with sides of 1 foot.

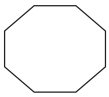


Figure 1

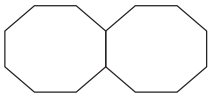


Figure 2

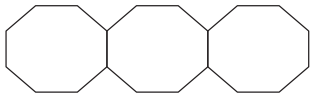


Figure 3

a. What is the perimeter of each figure?

b. Draw the next three figures and determine the perimeter of each.

c. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.

d. Verify the expression in part c by substituting $n = 6$.

20. Model with mathematics. Octagonal blocks are being used to make a walkway along a garden. Use your expression from Item 19 part c to find the perimeter of the walkway if 30 octagonal blocks, each side 1 foot long, are used for the walkway.

ACTIVITY 9 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 9-1

1. Use the figures below to answer parts a–c.

Figure 1 Figure 2 Figure 3

a. What is the perimeter of each figure shown? Assume each side is 1 unit.
b. Draw the next three figures.
c. What would be the perimeter of the 10th figure? Justify your response.

2. Write an expression, using the variable n, that could be used to determine the perimeter of the nth figure in Item 1. Use the expression to determine the perimeter of the 50th figure.

3. A pattern of pebbles is shown.

Figure 1 Figure 2 Figure 3

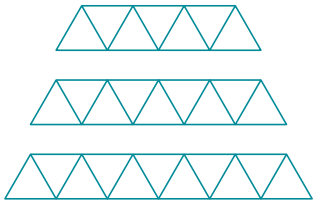
a. Draw the fourth, fifth, and sixth figures.
b. Create a table showing the number of the figure and the number of pebbles in each figure.
c. Describe the patterns you observe in the pebble drawings and the table in words.
d. What is the constant difference?
e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in the nth figure.
f. Use the expression in part e to determine the number of pebbles in the 51st figure.

ACTIVITY 9 Continued

ACTIVITY PRACTICE

1. a. Figure 1: 3 units;
Figure 2: 5 units;
Figure 3: 7 units

b.



c. 21 units. Begin with a perimeter of 3 in figure 1 and add 2 units 9 times.

2. $2n + 1$

3. a.

b.

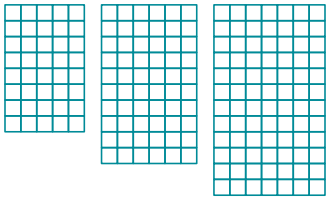
Figure Number	Number of Squares
1	6
2	9
3	12
4	15
5	18
6	21

c. Answers may vary. Each figure has 3 more squares than the previous one.
d. 3
e. $6 + 3(n - 1)$
f. 156

ACTIVITY 9 Continued

4. a. Figure 1: 4 square units;
Figure 2: 12 square units;
Figure 3: 24 square units

b.



c. 220 square units. Figure 10 would be 11 squares by 20 squares

5. $(n + 1)(2n)$; 2520 square units

6. a. Figure 1: 8, Figure 2: 14;
Figure 3: 20; Figure 4: 26;
Figure 5: 32: Figure 6: 38

b. Yes; Answer may vary. The perimeters differ by 6, so the constant difference is 6.

c. $2 + 6n$

d. $2 + 6(35) = 212$

7. a.

b. $\frac{10(10+1)}{2}$; 55 dots

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

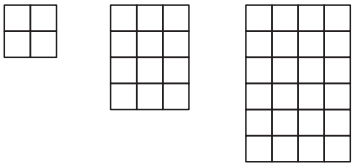
ACTIVITY 9
continued

Writing Expressions
Pebbles in the Sand

Lesson 9-2

4. Use the pattern of unit squares shown to answer parts a–c.

Figure 1 Figure 2 Figure 3



a. What is the area of each figure if each small square has an area of 1 unit?

b. Draw the next three figures in the pattern and determine the area of each.

c. What would be the area of the 10th figure? Justify your response.

5. Write an expression that could be used to determine the area of the nth figure in Item 4. Use the expression to determine the area of the 35th figure.

6. Use the figures from Item 4 to answer parts a–c.

a. What is the perimeter of each of the six figures?

b. Is there a constant difference? Explain.

c. Write an expression that could be used to find the perimeter of the nth figure.

d. Use the expression in part c to find the perimeter of the 35th figure.

MATHEMATICAL PRACTICES
Model with Mathematics

7. Use the expression $\frac{n(n+1)}{2}$.

a. Create a pattern using circles or dots and show the first three figures.

b. Determine the number of circles or dots in the 10th figure. Explain how you determined the number of dots in this figure.

Writing Expressions

Pebbles in the Sand

Lesson 9-1 Representing Patterns

Learning Targets:

- Identify and represent patterns using models, tables, and expressions.
- Write and evaluate algebraic expressions that represent patterns with constant differences.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Use Manipulatives, Think-Pair-Share

People have been investigating number patterns for thousands of years. ***Legend*** has it that Pythagoras and his students arranged pebbles in the sand to represent number patterns. One pattern they studied is shown below.

Figure 1 Figure 2 Figure 3

1. Draw the fourth, fifth and sixth figures.

2. Organize the number of pebbles in each figure into a table.

Figure Number	Number of Pebbles

3. Extend the pattern to determine how many pebbles are in the 10th figure.

4. Describe the patterns you observe in the pebble drawings and the table in words.

5. How many pebbles are in the 53rd figure? Explain your reasoning.

6. Write a numeric expression using the number 3 for the number of pebbles in the third figure.

My Notes

ACADEMIC VOCABULARY

A ***legend*** is a story handed down by tradition that is popularly regarded as historical but unverified.

My Notes

MATH TIP

An *expression* is a mathematical phrase using numbers or variables or both. $1 + 1$ and $3x - 5$ are examples of expressions.

MATH TERMS

You do not solve an expression; you **evaluate** it for a specific value. To do this, substitute a value for the variable and simplify.

7. Write a similar numeric expression using the number 7 for the number of pebbles in the seventh figure.

8. Let n represent the figure number.

a. Use n to write an expression that could be used to determine the number of pebbles in figure n.

b. What value would you substitute for n to determine the number of pebbles in the third figure?

c. Check to see that your expression from part a is correct by ***evaluating*** it for the value you chose in part b.

d. Use your expression to determine the number of pebbles in the 100th figure.

Check Your Understanding

A pattern of small squares is shown. Use the pattern to answer Items 9–13 that follow.

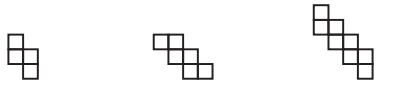


Figure 1 Figure 2 Figure 3

9. How many small squares are in each figure?

10. Draw the fourth, fifth, and sixth figures and determine the number of small squares in each figure.

11. Create a table to organize the number of squares in each figure into a table.

12. Describe in words the patterns you see in the square pattern and in the table.

13. How many squares would be in the 10th figure? Explain your reasoning.

My Notes

14. Another pebble arrangement is shown below.

Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.

b. Organize the number of pebbles in each figure in the table below.

Figure Number	Pebbles	Difference in Number of Pebbles
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

c. Describe any patterns you observe in the pebble drawings and in the table above.

15. Subtract ***consecutive terms*** in the pebbles column and record this information in the last column in the table.

16. Reason quantitatively. How does the ***constant difference*** in the new column relate to the patterns you observed?

CONNECT TO AP

The ability to identify patterns allows you to understand and describe different types of functions and provides a foundation for understanding rate of change in calculus.

MATH TERMS

Consecutive terms follow each other directly in a sequence. In the table below, 5 and 8 are *consecutive terms*. There is a **constant difference** between the terms in this table that is equal to 3.

n	term	
1	5	3
2	8	3
3	11	3
4	14	

My Notes

17. The number of pebbles in a specific figure can be written using repeated additions of the constant difference. For example, the third figure is $1 + 2 + 2$ or $1 + 2(2)$.

Figure Number	Pebbles	Expression Using Repeated Addition
1	1	$1 + 2(0)$
2	$1 + 2$	$1 + 2\ (1)$
3	$1 + 2 + 2$	
4		
5		

a. Write the number of pebbles in the fourth and fifth figure using repeated addition of the constant difference.

b. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

c. What value would you substitute for n to determine the number of pebbles in the third figure?

d. Check to see that your expression from part a is correct by evaluating it when $n = 5$.

e. Use your expression to determine the number of pebbles in the 100th figure.

My Notes

Check Your Understanding

18. A pattern of pebbles is shown. Use the pattern to answer parts a–c.

Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.
b. Create a table to organize the number of pebbles in each figure for the first six figures.
c. What is the constant difference?
d. Write the number of pebbles for each figure using repeated addition of the constant difference.
e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in figure n.

LESSON 9-1 PRACTICE

19. Check to see that your expression from Item 18 part e is correct by evaluating when $n = 6$.

20. Use your expression from Item 18 part e to determine the number of pebbles in the 51st figure.

21. Critique the reasoning of others. The expression that represents the number of squares in the nth figure of a pattern is given by $2 + 3(n - 1)$. Mia claims the constant difference is 2. Do you agree? Explain.

My Notes

22. A pattern of small squares is shown.

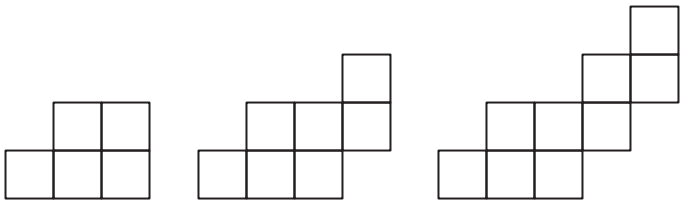


Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.
b. Create a table showing the number of the figure and the number of squares in each figure.
c. Describe the patterns you observe in the square drawings and in your table.
d. What is the constant difference?
e. How many small squares are in the first figure?
f. Let n represent the figure number. Use n to write an expression that could be used to determine the number of squares in the nth figure.

23. Compare the expression you found in Item 22 part f with the expression you found in Item 17 part b. How are they the same? How are they different?

Learning Targets:

- Identify patterns that do not have a constant difference.
- Write and evaluate algebraic expressions that represent patterns that do not have a constant difference.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Group, Group Presentation

1. Four different pebble patterns are shown. Your teacher will assign one to your group. Use your selected pattern to answer parts a–e that follow and then prepare a group presentation of your results.

Pattern A

Figure 1 Figure 2 Figure 3

Pattern B

Figure 1 Figure 2 Figure 3

Pattern C

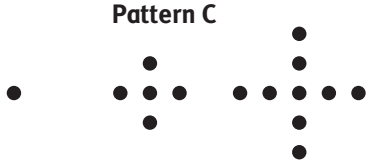


Figure 1 Figure 2 Figure 3

Pattern D

Figure 1 Figure 2 Figure 3

a. Draw a few additional figures and then organize the information in a table. Identify the constant difference.

My Notes

My Notes

b. Describe the pattern in words.

c. **Reason abstractly**. Write an expression using the variable n that could be used to determine the number of pebbles in figure n.

d. Use your expression to determine the number of pebbles in the 10th, 53rd, and 200th figures.

Figure	10th	53rd	200th
A and D			
B			
C			

e. For the pattern you selected, is it possible to have a figure with 100 pebbles? Explain your reasoning.

2. Based on the class's work for Item 1, how does the constant difference in a pebble pattern relate to the algebraic expression that can be written to describe the pattern?

My Notes

Check Your Understanding

Tables representing two pebble patterns are shown below.

Pebble Pattern A					
Figure	1	2	3	4	5
Pebbles	3	7	11	15	19

Pebble Pattern B					
Figure	1	2	3	4	5
Pebbles	2	7	12	17	22

3. What is the constant difference for each pebble pattern shown in the tables?

4. For each pebble pattern, use the variable n to write an expression for the nth figure.

5. How many pebbles are in the 50th figure for each pebble pattern?

6. Both pebble patterns have 7 pebbles in Figure 2. If the patterns continue, will they ever have the same number of pebbles as another figure? Explain your reasoning.

7. The Pythagoreans also studied the following pebble pattern.

Figure 1 Figure 2 Figure 3

a. How many pebbles are there in the fourth, fifth, and sixth figures?

b. Does this pattern have a constant difference? Explain your response.

c. Describe the pattern in words.

d. How many pebbles are there in the 10th figure? How did you determine your answer?

My Notes

e. How many pebbles are there in the 40th figure? In the *n*th figure?

f. The Pythagoreans called the numbers in this pattern the square numbers. Why do you think this is so?

8. The numbers in the pebble pattern shown below are called the rectangular numbers.

Figure 1 Figure 2 Figure 3

a. How many pebbles are in the fourth, fifth, and sixth figures?

b. Describe the pattern in words. Is there a constant difference? Explain your response.

c. Describe how to find the number of pebbles in the 10th figure.

d. How many pebbles are there in the 30th figure? In the nth figure?

9. The Pythagoreans called the numbers represented by the pebbles in this pebble pattern the triangular numbers.

Figure 1 Figure 2 Figure 3

a. Why do you think the Pythagoreans called these numbers triangular?

My Notes

b. How is the triangular number pebble pattern related to the pebble pattern of the rectangular numbers?

c. Use your response to part b to write an algebraic expression for the number of pebbles in the nth triangular number.

d. Verify your expression by substituting $n = 4$. Is the result the number of pebbles in the fourth triangular number?

e. Use your expression to predict the number of pebbles in the 30th triangular number.

Check Your Understanding

10. Is the number 72 a square number, rectangular number, or triangular number? Explain your reasoning.

Use the figures below to answer Items 11–15.

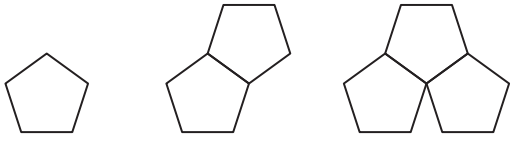


Figure 1 Figure 2 Figure 3

11. Assume each side of each pentagon is 1 cm. What is the perimeter of each figure shown?

12. Draw the next three figures and determine the perimeter of each.

13. Organize the results of Items 11 and 12 in a table. What would be the perimeter of the 10th figure? Explain your reasoning.

14. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.

15. Use your expression to determine the perimeter of the 50th figure.

My Notes

LESSON 9-2 PRACTICE

16. Critique the reasoning of others. Nate claims 56 is a rectangular number because a rectangle with base 4 and height 14 can be formed. What is his error?

17. What is the sixth square number? How did you get your answer?

18. Use the expression from Item 9 part c to show that 45 is a triangular number.

19. Use the figures below to answer parts a–d. Assume each figure is a regular octagon with sides of 1 foot.

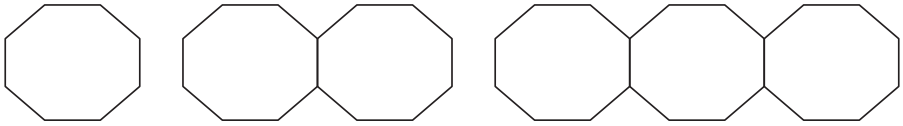


Figure 1 Figure 2 Figure 3

a. What is the perimeter of each figure?
b. Draw the next three figures and determine the perimeter of each.
c. Use n to represent the figure number. Write an expression that could be used to determine the perimeter of the nth figure.
d. Verify the expression in part c by substituting $n = 6$.

20. Model with mathematics. Octagonal blocks are being used to make a walkway along a garden. Use your expression from Item 19 part c to find the perimeter of the walkway if 30 octagonal blocks, each side 1 foot long, are used for the walkway.

ACTIVITY 9 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 9-1

1. Use the figures below to answer parts a–c.

Figure 1 Figure 2 Figure 3

a. What is the perimeter of each figure shown? Assume each side is 1 unit.
b. Draw the next three figures.
c. What would be the perimeter of the 10th figure? Justify your response.

2. Write an expression, using the variable n, that could be used to determine the perimeter of the nth figure in Item 1. Use the expression to determine the perimeter of the 50th figure.

3. A pattern of pebbles is shown.

Figure 1 Figure 2 Figure 3

a. Draw the fourth, fifth, and sixth figures.
b. Create a table showing the number of the figure and the number of pebbles in each figure.
c. Describe the patterns you observe in the pebble drawings and the table in words.
d. What is the constant difference?
e. Let n represent the figure number. Use n to write an expression that could be used to determine the number of pebbles in the nth figure.
f. Use the expression in part e to determine the number of pebbles in the 51st figure.

Lesson 9-2

4. Use the pattern of unit squares shown to answer parts a–c.

Figure 1

Figure 2

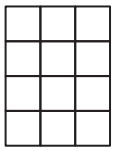
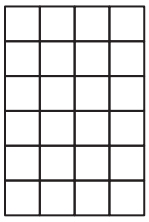


Figure 3



a. What is the area of each figure if each small square has an area of 1 unit?
b. Draw the next three figures in the pattern and determine the area of each.
c. What would be the area of the 10th figure? Justify your response.

5. Write an expression that could be used to determine the area of the *n*th figure in Item 4. Use the expression to determine the area of the 35th figure.

6. Use the figures from Item 4 to answer parts a–c.
a. What is the perimeter of each of the six figures?
b. Is there a constant difference? Explain.
c. Write an expression that could be used to find the perimeter of the *n*th figure.
d. Use the expression in part c to find the perimeter of the 35th figure.

MATHEMATICAL PRACTICES
Model with Mathematics

7. Use the expression $\frac{n(n+1)}{2}$.
a. Create a pattern using circles or dots and show the first three figures.
b. Determine the number of circles or dots in the 10th figure. Explain how you determined the number of dots in this figure.