The Law of Sines

ACTIVITY 25

Got Lost?

Lesson 25-1 Modeling and Applying the Law of Sines

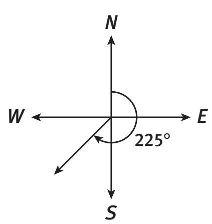
Learning Targets:

- Calculate the bearing of a flight.
- Derive and use the Law of Sines.
- Find unknown sides or angles in oblique triangles.

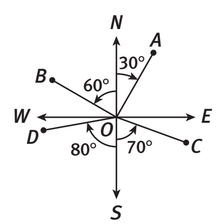
SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Identify a Subtask, Simplify the Problem, Create Representations, Summarizing, Paraphrasing, Work Backward, Look for a Pattern, Quickwrite, Graphic Organizer, Think-Pair-Share, Group Presentation, Guess and Check

In navigation, an object's heading indicates the direction of movement as measured by an angle rotated clockwise from north. A heading of 90° means an object is heading due east. A heading of 225° means an object is heading southwest. The *directional bearing* of a point is stated as the number of degrees east or west of the north-south line. To state the directional bearing of a point, write:

- N or S which is determined by the angle being measured
- the angle between the north or south line and the point, measured in degrees
- E or W which is determined by the location of the point relative to the north-south line



Heading



Directional Bearing

In the figure, A from O is N30°E, B from O is N60°W, C from O is S70°E, and D from O is S80°W.

International Flight 22 was on a course due north from Auckland, New Zealand, to Honolulu, Hawaii. Two thousand miles south of Honolulu, the plane encountered unexpected weather and the pilot changed bearing by 20°, as shown in the figure. The plane traveled on this new course for 1.5 hours, averaging 500 miles per hour.

1. How far did the plane travel during the 1.5 hours it was flying on its new flight path?

750 miles

2. How far was the plane from Honolulu after 1.5 hours?

1320.387 miles

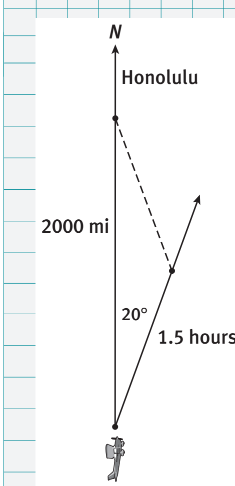
My Notes

CONNECT TO AVIATION

This activity measures speed in miles per hour. However, the speed of commercial jets is typically represented as a Mach number, a percentage of the speed of sound. Mach speed varies as temperature and altitude change.

CONNECT TO NAVIGATION

Bearing is the direction an aircraft is pointing, but the course is the actual direction in which the plane is moving when wind is taken into account. The heading is the clockwise angle in degrees between an aircraft's destination and north.



Common Core State Standards for Activity 25

HSG-SRT.D.10(+) Prove the Laws of Sines and Cosines and use them to solve problems.

ACTIVITY 25

Guided

Activity Standards Focus

In Activity 25, students explore a scenario of an airplane lost over the Pacific Ocean. While students discover mathematical relationships and use the Law of Sines throughout this activity, be sure they take good notes, outlining and diagramming all the cases for which the Law of Sines can be applied.

Lesson 25-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1	#2–3	#4
#5–7	Example A	#8

Check Your Understanding

#12–14

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students use the Law of Cosines to solve $\triangle ABC$ with the given parts.

1. $a = 7, b = 9, c = 10$

[$A = 42.8°$; $B = 61°$; $C = 76.1°$]

2. $C = 116°, a = 25, b = 31$

[$c = 47.6$; $A = 28.2°$; $B = 35.8°$]

CONNECT TO NAVIGATION

Heading is the direction the aircraft is pointing relative to north, measured clockwise from due north. *Bearing* is the angle, measured clockwise, between the aircraft's point of reference and its destination. A flight's *course* refers to the path of the aircraft over the ground, which can be impacted by wind drift.

1–2 Identify a Subtask, Simplify the Problem, Create Representations In Item 2 students should use the Law of Cosines to solve for the length of the dotted line that represents the third side of the triangle shown in the diagram. Note that the aircraft drifted off its desired course due to unexpected weather conditions (i.e., wind drift). The aircraft is on an initial bearing of 360° (since Honolulu bears 360°, or due north, from the aircraft). To fly this actual new course would require the aircraft to fly a heading less than θ to account for wind drift. The amount of correction the pilot needs to apply depends on the aircraft's speed and the wind speed and direction.

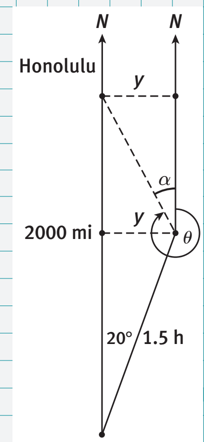
3 Visualization, Create Representations, Identify a Subtask, Simplify the Problem, Debriefing The altitude is equal to 2000 sin 20°. Note that after 1.5 hours of the aircraft flying off its desired course, Honolulu now bears θ from the aircraft, and the aircraft bears $180° - \alpha$ from Honolulu. To get to Honolulu from its new position, the aircraft needs to fly a new course and bearing of θ. Use the inverse sine function to find the measure of α. Students will need to label their solution to Item 2 on the triangle before completing Item 3. Before going on, debrief this item with your class to make sure all students understand the solutions.

4 Identify a Subtask, Simplify the Problem, Debriefing The solution process for this item is similar to that for Item 3. To determine how fast the plane was traveling, divide the distance (length AB) by the time (2.5 hours). When you debrief this item, you might ask students if they would like to learn a more efficient process for finding unknown sides and angles in oblique triangles. Their work on the items so far will indicate their understanding of the Law of Sines, which follows.

TEACHER to TEACHER

The Law of Sines is presented. Students will explain the derivation of this formula by dropping a perpendicular and using it to relate the given sides and angles of the triangle in the next few items. Have students record this formula in their notes after Item 7.

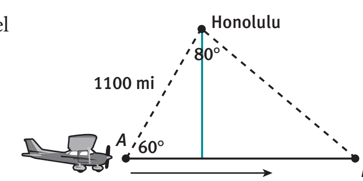
My Notes



3. To adjust the flight path, the pilot changed the course by α degrees, as shown in the figure.
   a. Find the length y of the horizontal dotted lines in the diagram. Then use that length and your answer to Item 2 to find the value of α. What is the directional bearing of the plane?
   $y = 256.515$ miles; $\alpha = 11.2°$; N 11.2°W
   b. Use the value of α to determine the bearing θ of the plane.
   $\theta = 348.8$

Another flight, Flight 33, was 1100 miles southwest of Honolulu when the plane sped up and headed due east. Radio contact with Flight 33 occurred 2.5 hours later, and air traffic controllers placed it somewhere over the Cook Islands at point B.

4. Draw the altitude of the triangle from the point representing Honolulu to the horizontal flight path.
   a. What is length of the altitude?
   952.6 miles
   b. How far did the plane travel in 2.5 hours?
   1685.3 miles
   c. How fast was the plane traveling along the path from point A to point B?
   674 mph

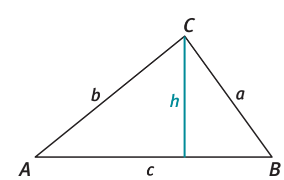


The ***Law of Sines***, shown below, could also be used to solve problems like Items 3 and 4.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The next series of items will show you how your work from the previous items can be generalized to derive the Law of Sines.

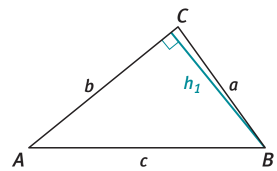
5. **Model with mathematics.** Use the triangle shown below to answer the questions.



   a. Draw the altitude h from vertex C to side c.
   b. What is the measure of h in terms of A and b? **$h = b \sin A$**
   c. What is the measure of h in terms of B and a? **$h = a \sin B$**
   d. Use your work from parts b and c to write an equation relating A, B, a, and b. **$a \sin B = b \sin A$**

My Notes

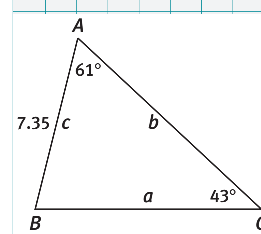
6. **Express regularity in repeated reasoning.** Repeat the process used in Item 5, but draw the altitude from vertex B to side b.
   $a \sin C = c \sin A$



7. How do the equations you wrote in Items 5 and 6 compare to the Law of Sines written before Item 5?
   Sample answer: I can divide by sin C and sin A and get $\frac{a}{\sin A} = \frac{c}{\sin C}$, and I can divide by sin B and sin A and get $\frac{b}{\sin B} = \frac{a}{\sin A}$. Since both equations have $\frac{a}{\sin A}$ in them, all three ratios are equal.

Example A

Use the Law of Cosines and the Law of Sines to find the missing parts of this triangle. Explain which law you used.



Step 1: There are two known angles and one known side. We know c and C, and we know A, so we can use the Law of Sines to find a.

$$\frac{c}{\sin C} = \frac{a}{\sin A};\ a = \frac{(7.35)(\sin 61°)}{\sin 43°} = 9.43$$

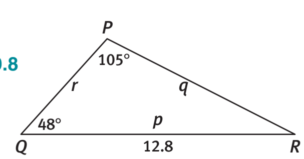
Step 2: Find angle B: $B = 180° - (61° + 43°) = 76°$

Step 3: To find the third side, we can use the Law of Cosines. We can check the answer using the Law of Sines.
$b^2 = a^2 + c^2 - 2ac \cos B = 9.43^2 + 7.35^2 - 2(9.43)(7.35) \cos 76° = 109.41$
$b = 10.46$

$$\text{Check: } \frac{b}{\sin B} = \frac{c}{\sin C};\ b = \frac{(7.35)(\sin 76°)}{\sin 43°} = 10.46$$

Try These A

a. Use the Law of Sines to find q.
   $\frac{q}{\sin Q} = \frac{p}{\sin P};\ q = \frac{(\sin 48°)(12.8)}{\sin 105°} = 9.8$



b. Use the Law of Cosines to find r. Check your value for r using the Law of Sines.
   $R = 27°$, so $r^2 = p^2 + q^2 - 2pq \cos R = 12.8^2 + 9.8^2 - 2(12.8)(9.8) \cos 27° = 36.34$; $r = 6.02$
   Check: $\frac{r}{\sin R} = \frac{p}{\sin P};\ r = \frac{(\sin 27°)(12.8)}{\sin 105°} = 6.02$

ACTIVITY 25 Continued

5–7 Create Representations, Identify a Subtask, Work Backward, Debriefing, Look for a Pattern, Quickwrite In Item 5, the derivation of the Law of Sines is broken down into chunks to help students through the process. As you circulate around the room, observe the progress of your groups. If several groups seem stuck, pause and redirect the entire class. Make sure the process is clear before going on because students will repeat these steps in Item 6. Item 6 asks students to replicate their work from Item 5, but using an altitude drawn from B to side b. In Item 7, students need to realize that all three expressions are equal to each other because the other two equal $c \sin A$. When they compare their results to Items 5 and 6 with the Law of Sines, they should be able to see that they must divide by the sine ratios to algebraically transform their work into the formula as written.

Developing Math Language

To help students learn the Law of Sines, suggest that that they draw and label a triangle similar to the triangle shown in Item 5 each time they use the law to solve a problem. Explain that each part of the law is just a ratio between the sine of a given angle and its opposite side. They may draw a line from that angle to the opposite side. This will help them have a visual reminder of the Law of Sines, and it will help them recall the law more clearly.

Example A Summarizing, Debriefing The main focus of this example is to have the students take the given information and use it to determine which law to apply, Law of Cosines or Law of Sines. Students must begin with Law of Sines, as two angles and only one side length were given. In the example, the length of a is found first; however, you may want to point out to students that it would be possible to find b first, as $m\angle B = 180° - (61° + 43°) = 76°$. Either way, students will have to apply the Law of Sines before applying the Law of Cosines to find the length of the third side.

8 Look for a Pattern, Create Representations, Graphic Organizer Have students record the Law of Sines in their math notebooks at this time along with the table which will help them decide which formula to use when solving oblique triangles, based on the given information.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to modeling real-world problems with trigonometric functions.

Answers

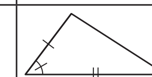
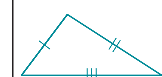
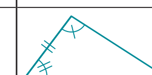
9. We need a side and the angle opposite that side, and then we also need one other side or one other angle.
10. We can apply the Law of Sines, but not the Law of Cosines, if we know any two angles of a triangle and any one side. We can apply the Law of Cosines, but not the Law of Sines, if we know two sides and the included angle or if we know all three sides and no angles.
11. The bearing is 36.87°.

My Notes

Lesson 25-1
Modeling and Applying the Law of Sines

Like the Law of Cosines, the Law of Sines relates the sides and angles in an ***oblique triangle***, and these can be used to find unknown sides or angles given at least three known measures that are not all angle measures.

8. The following table summarizes when each rule should be used to find missing measures in an oblique triangle. For each abbreviation, complete the given information and illustrate the given information by drawing and marking a triangle.

Rule	Given information	Illustration
Law of Cosines	SAS side, included angle, side	
	SSS **side, side, side**	
Law of Sines	ASA **angle, included side, angle**	
	AAS **angle, angle, nonincluded side**	

Check Your Understanding

9. What is the minimum information needed in order to use the Law of Sines?
We need a side and the angle opposite that side, and then we also need 1 other side or 1 other angle.

10. Make use of structure. What information about a triangle is enough to apply the Law of Sines but not the Law of Cosines? What information about a triangle is enough to apply the Law of Cosines but not the Law of Sines?
We can apply the Law of Sines, but not the Law of Cosines, if we know any 2 angles of a triangle and any 1 side. We can apply the Law of Cosines, but not the Law of Sines, if we know 2 sides and the included angle or if we know all 3 sides and no angles.

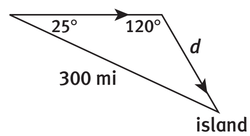
11. In a board game, a plane is flying from the origin of a coordinate plane toward the point (3, 4). What is the directional bearing of the plane? (Assume that north is the direction of the positive y-axis.)
N 36.87°W

Lesson 25-1
Modeling and Applying the Law of Sines

My Notes

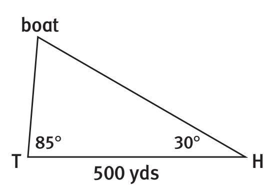
Use the Law of Sines to solve the following problems.

12. The pilots of Flight 33 spotted a deserted island 300 miles from their current location but continued on their course knowing there was another island 500 miles ahead on their current course. After a while they experienced engine trouble and turned to head for the deserted island, hoping to land safely. After making the turn, they estimated the plane could travel another 200 miles. They landed the plane knowing they were very lucky. How many additional miles could they have flown? Explain your reasoning.



$d = \frac{300 \sin 25°}{\sin 120°} = 146$ **miles, so they could have flown an additional 54 miles.**

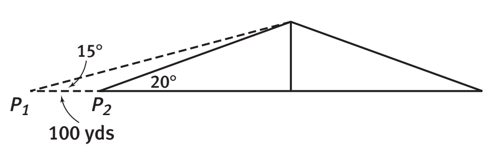
13. Survivors Taylor and Hank are on the beach of the deserted island, 500 yards apart. They spot a boat out at sea and estimate the angles between their positions and the boat as shown below. How far is the boat from Hank? How far is the boat from Taylor?



The distance from the boat to Hank is 543 yards and the distance from the boat to Taylor is 276 yards.

14. Reason quantitatively. Survivors Tariq and Jess were trying to estimate the distance to the top of a mountain they hoped to hike up to get a better view of the landscape. They measured the angles of elevation at points P_1 and P_2 located 100 yards apart. If the two survivors started at point P_2, how far would they have to walk to get to the top of the mountain?

297 yards



ACTIVITY 25 Continued

12–14 Marking the Text, Create Representations, Simplify the Problem, Think-Pair-Share, Group Presentation As students work through these items applying the Law of Sines, emphasize that when students set up these equations, they must be given an angle and opposite-side pair and then one of the other sides or angles. Have students work the first one together with their group members. Use a think-pair-share strategy on the other two, giving individuals a little bit of time to work the problem along, before consulting with group members. Students should be marking the given information shown in the diagrams, where they read it in the text. Have different groups present their solutions to these items to the entire class.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to applying trigonometric functions and their properties to real-world flight path scenarios.

Answers

15. a. The search plane will not intercept the flight path. The closest the plane gets to the tracking station is 819 miles. Drop a perpendicular from the tracking station to the flight path and find this distance.
1000(sin 55°) = 819 miles

b. Yes. Since the plane is 1000 miles away from the tracking station to begin with, a search plane with a 1000-mile radius will intercept the flight path.

c. Yes. The search plane will intercept the flight path in two places. One forms an obtuse triangle with the intersection point close to the plane's starting position, and one forms an acute triangle with the intersection point farther from the plane's starting position to the right of perpendicular drawn in Part a.

16. 819 miles or a search radius greater than or equal to 1000 miles

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

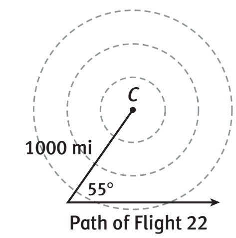
ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to translate from the words to create a diagram they can use to solve the problems. Discuss the terminology of these problems (quarter turn, clockwise, counterclockwise, bearing), and have students present their diagrams to the class.

My Notes

Check Your Understanding

The last known location of Transcontinental Flight 22 was 1000 miles from a tracking station located at point *C*. The plane was flying due east at a 55° angle with the tracking station.



15. Suppose Flight 22 continued to fly due east.
a. If a search plane is sent out from the tracking station and searches a 600-mile radius, will the search plane intercept the flight path? Show your work to explain your reasoning.
b. Would the search plane intercept the flight path if the search radius were 1000 miles? Explain.
c. Would the search plane intercept the flight path if the search radius were 900 miles? Explain.

16. What are some other possible search radii that would intercept the flight path in exactly one point?

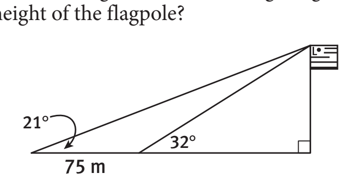
LESSON 25-1 PRACTICE

17. A flight has a heading of 32°, as measured clockwise from north. What is its new heading under the following changes?
a. a quarter turn counterclockwise
b. a quarter turn clockwise
c. a rotation of 55° counterclockwise

18. In triangle *ABC*, angle *A* is 54°, angle *C* is 96°, and $AC = 275$. Find angle *B*, *AB*, and *BC*.

19. In triangle *PQR*, $PR = 14$, $QR = 15$, and $PQ = 13$. Find the three angles of the triangle.

20. In the diagram below, two sightings of the top of a flagpole are taken 75 meters apart on level ground. The two sightings are 21° and 32°. What is the height of the flagpole?



21. Attend to precision. Two groups of students were given a copy of triangle *MNP*. Group 1 measured sides and angles of the triangle and found that $MN = 21$ cm, angle $N = 47°$, and $MP = 37$ cm. Group 2 found that $MN = 20.8$ cm, angle $N = 47.42°$, and $MP = 37.4$ cm. Using the Law of Sines, what values will Group 1 and Group 2 get for angle *P*, for angle *M*, and for *NP*?

LESSON 25-1 PRACTICE

17. a. 302°
b. 122°
c. 337°

18. $B = 30°$, $AB = 547$, $BC = 445$

19. angle $P = 67.4°$, angle $R = 53.1°$, angle $Q = 59.5°$

20. 74.6 m

21. Group 1: angle $P = 24.5°$, angle $M = 108.5°$, $NP = 48.0$
Group 2: angle $P = 24.17°$, angle $M = 108.41°$, $NP = 48.19$

Learning Targets:

- Determine the number of distinct triangles given certain criteria.
- Use the Law of Sines to solve triangles with unknown sides or angles.

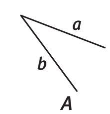
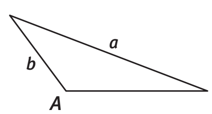
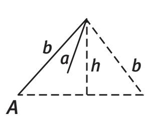
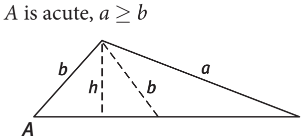
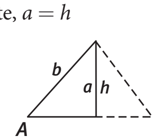
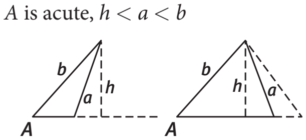
SUGGESTED LEARNING STRATEGIES: Note Taking, Interactive Word Wall, Graphic Organizer, Quick Write, Look for a Pattern, Think-Pair-Share, Create Representations, Guess and Check, Role Play, Identify a Subtask, Simplify the Problem, Group Presentation

My Notes

When you are given two sides and the opposite angle, it is possible to have zero, one, or two distinct triangles depending on the given information. This situation is known as the ***ambiguous case (SSA)*** and is summarized below.

The Ambiguous Case (SSA)

Given a, b, and A with $h = b \sin A$, where b is adjacent to and a is opposite angle A, and h is the altitude of the potential triangle.

A is obtuse, $a \leq b$  no triangle	A is obtuse, $a > b$  one triangle
A is acute, $a < h$  no triangle	A is acute, $a \geq b$  one triangle
A is acute, $a = h$  one triangle	A is acute, $h < a < b$  two triangles

ACADEMIC VOCABULARY

The word ***ambiguous*** is used for things that are open to interpretation, or can have two meanings.

1. How can the value of $b \sin A$ help you determine the number of solutions given the ambiguous case?
   Sample answer: Since $b \sin A$ is the height of the triangle, you can compare the value of a to this measure to determine whether or not 0, 1, or 2 triangles are possible.

2. How would you use this table to interpret the number of possible triangles for the ambiguous case if you were given angle C, side b, and side c?
   Sample answer: It is basically the same, except you use angle C instead of angle A and determine the height using $b \sin C$. Then compare side c to that height and side b.

Lesson 25-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–2 #3 #4
#5 Example A
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Have students use the Law of Sines to solve $\triangle ABC$ with the given parts.

1. $A = 40°$, $C = 60°$, $a = 10$
   [$B = 80°$; $b = 15.3$; $c = 13.5$]
2. $A = 41°$, $C = 52°$, $a = 6.5$
   [$B = 87°$; $b = 9.9$; $c = 7.8$]
3. $B = 5$, $C = 116$, $b = 11$
   [$A = 59°$; $a = 108.2$; $c = 113.4$]

1–2 Quickwrite, Look for a Pattern

Item 1 gets students to reflect on the importance of computing the altitude of the possible triangle.

The point of this item is to get students to understand that the relationships will hold even if the given angle is named with a different variable. The important information is that you are presented with the SSA configuration. For Item 2, students can copy all or part of this table into their notes. At the very least, they should mark the text and add comments to make it meaningful if they need to refer to it when solving problems.

TEACHER TO TEACHER

The table presented on this page could be a bit overwhelming to students. However, making connections back to the items they just solved will help ease the transition. You might ask them to identify which parts of Items 18 and 19 in Lesson 25-1 match each case.

3 Debriefing, Visualization, Graphic Organizer Remind students that the SSA relationship will hold true if the given angle is named with another variable other than A. There are six possible SSA cases from which to choose. If the students copied the table into their notes, have them use it as a reference for this item. However, if the students did not copy the table, refer them to the one in the textbook. The first thing the students should ask is whether the given angle is acute or obtuse. Then, based upon the answer to that question, students proceed to look at how the angle's opposite and adjacent sides are related to each other and/or to the altitude of the triangle. Note: It will only be necessary for students to consider and compute the altitude if the given angle is acute. Emphasize the importance of a sketch to visualize which of the six cases each item fits.

4 Look for a Pattern, Role Play Give each student a card. Students form into groups so that each group holds either a one-solution case or a two-solution case (comprising angle, opposite side, and adjacent side cards).

5 Look for a Pattern, Role Play Students regroup themselves so that each group holds a no-solution case using *any* combination of one angle and two sides. Do not specify which cards must be adjacent and which ones opposite.

My Notes

3. Construct viable arguments. Determine how many triangles are possible with the given information. Draw a sketch and show any calculations you used.

a. $A = 30°$, $b = 10$, $a = 5$ **1 triangle**

b. $C = 75°$, $c = 18$, $a = 7$ **1 triangle**

c. $B = 100°$, $b = 50$, $c = 75$ **no triangle**

d. $C = 40°$, $b = 25$, $c = 21$ **2 triangles**

e. $A = 63°$, $a = 10$, $c = 45$ **no triangle**

4. The Ambiguous Case Game 1: Use the information given to you by your teacher. With your classmates, organize into groups so you have the three groups with one solution and two groups with two solutions. Record your results.

Other solutions possible. One is shown below.

Angle	Adjacent	Opposite
40°	12	11
50°	15	18
80°	5	14
110°	16	17
20°	10	8

5. The Ambiguous Case Game 2: Use the information given to you by your teacher. With your classmates, organize into groups so that every group of three people forms no triangle. Record your results.

Other solutions possible. One is shown below.

Angle	Adjacent	Opposite
40°	18	8
50°	16	10
80°	15	14
110°	12	11
20°	17	5

Lesson 25-2
The Ambiguous Case (SSA)

Example A

Solving the two-solution SSA situation: Use the Law of Sines to solve a triangle given $A = 42°$, $a = 18$, $b = 22$.

Step 1: Determine the number of solutions. There are two solutions since the measure of side a is between $b \sin A$ and b.
$b \sin A = 22 \sin 42° \approx 14.720 < 18$

Step 2: Solve the first acute triangle using the Law of Sines.
$$\frac{18}{\sin 42°} = \frac{22}{\sin B_1}$$
$$\sin B_1 = \frac{22 \sin 42°}{18} \approx 0.8178$$

Find B_1. $B_1 = \sin^{-1}\left(\frac{22 \sin 42°}{18}\right) \approx 54.9°$

Find C_1. $C_1 = 180° - (42° + 54.9°) = 83.1°$

Find c_1. $c_1 = \frac{18 \sin(83.1°)}{\sin 42°} \approx 26.7$

Step 3: Solve the second obtuse triangle. The angles B_1 and B_2 opposite the given adjacent side are supplementary.
$$B_2 = 180 - B = 180 - 54.9 = 125.1$$
$$C_2 = 180 - (42 + 125.1) = 12.9$$
$$c_2 = \frac{18 \sin 12.9}{\sin 42} \approx 6.0$$

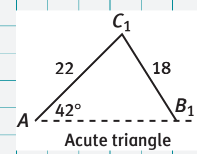
Try These A

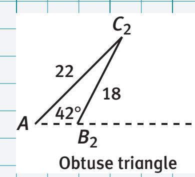
Each of these triangles has two possible solutions. Find them both.

a. $A = 55°$, $b = 40$, $a = 35$
$B_1 = 69.4°$, $C_1 = 55.6°$, $c_1 = 35.2$ and $B_2 = 110.6°$, $C_2 = 14.4°$, $c_2 = 10.6$

b. $C = 20°$, $c = 6$, $b = 12$
$B_1 = 43.2°$, $A_1 = 116.8°$, $a_1 = 15.7$ and $B_2 = 136.8°$, $A_2 = 23.2°$, $a_2 = 6.9$

My Notes

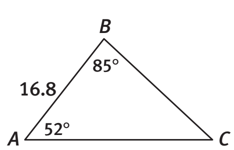




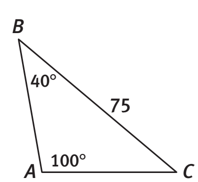
Check Your Understanding

6. Solve each triangle using the Law of Sines.

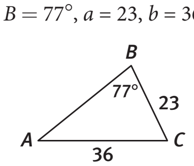
a. $A = 52°$, $B = 85°$, $c = 16.8$



b. $A = 100°$, $B = 40°$, $a = 75$



c. $B = 77°$, $a = 23$, $b = 36$



ACTIVITY 25 Continued

Example A Visualization, Debriefing, Identify a Subtask, Discussion Groups As you work through this ambiguous SSA example with students, be sure to sketch both triangles on the board for the students to see. Emphasize the fact that you are solving two different triangles; the only parts of the triangle that will remain the same are the given measures of A, a, and b. Be sure students understand they will be finding two different sets of B, C, and c. (You may want to use subscripts of 1 and 2 to differentiate between the two different sets of solutions). On the Try These items, have students work independently at first and then collaborate with their groups. You should circulate around the room monitoring group progress on these items. If you see certain mistakes being made consistently across several groups, debrief the item with the class as a whole before sending them back to work in their groups.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to using the Law of Sines.

Answers

6. a. $C = 43°$, $a = 19.4$, $b = 24.5$
b. $C = 40°$, $c = b = 49$
c. $A = 38.5°$, $C = 64.5°$, $c = 33.3$

Check Your Understanding

Answers

7. The firefighters are 20.3 miles from the fire.

8. a. no triangle
b. 2 triangles
c. no triangle
d. 1 triangle

9. Sample answer: The term *ambiguous* refers to the fact that there may be 0, 1, or 2 triangles for given information about the triangle. As one way to test for the ambiguous case, if we are given sides a and b and angle A, calculate $b \sin A$ as the length of an altitude of a triangle. Then compare that length with side a. If a is less than, greater than, or equal to $b \sin A$, then there are respectively 0, 1, or 2 possible triangles.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 25-2 PRACTICE

10. 62 feet and 94.9 feet
11. The billboard is 84.2 feet away.
12. $B_1 = 66.8°, A_1 = 63.2°, a_1 = 116.5$
$B_2 = 113.2°, A_2 = 16.8°, a_2 = 37.7$
13. a. $C = 123°, a = 19.207, b = 29.408$
b. $C = 40°, b = 4.257, c = 5.472$
c. $A = 21.311°, C = 101.689°, c = 35.029$
14. 21.926 yd
15. 1243.7 yd
16. Angle $C = 98°$, $AC = 8.11$, $CB = 8.45$; Students' actual measurements may vary.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand concepts related to solving oblique triangles by using the Law of Sines. Prepare students to have the ability to determine and differentiate the cases for which the Law of Sines applies, as well as those for which the Law of Cosines applies.

My Notes

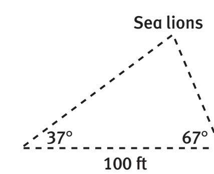
7. A lookout tower, firefighters located 25 miles from the tower, and a forest fire form three vertices of a triangle. At the lookout tower, the angle between the forest fire and the firefighters is 35°. At the firefighters' location, the angle between the lookout tower and the fire is 100°. How far are the firefighters from the fire?

8. **Make sense of problems.** Determine the number of possible triangles for each given situation.
a. $A = 45°, c = 100, a = 25$
b. $B = 70°, c = 90, b = 85$
c. $C = 100°, c = 6, a = 7.5$
d. $A = 60°, b = 4, a = 2\sqrt{3}$

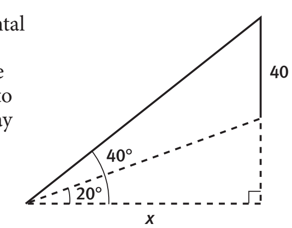
9. Why is the term *ambiguous case* used in this lesson? Explain how you know the situation is ambiguous. Describe how to solve an "ambiguous case" situation without using a formula.

LESSON 25-2 PRACTICE

10. Two marine biologists spotted some sea lions in the bay. The biologists were located on a beach about 100 feet apart. The angle between the shore and the sea lions for each biologist is shown below. How far were the sea lions from each biologist?



11. A billboard is 40 feet tall. At a horizontal distance x feet from the billboard, the angle of elevation to the bottom of the sign is 20° and the angle of elevation to the top of the sign is 40°. How far away is the billboard?



12. Solve the two-solution ambiguous case situation given $C = 50°$, $b = 120$, $c = 100$.

13. Solve each triangle using the Law of Sines.
a. $A = 22°, B = 35°, c = 43$
b. $A = 110°, B = 30°, a = 8$
c. $B = 57°, a = 13, b = 30$

14. The angle of elevation from a point 50 yards from a tree to the top of the tree is 23°. The tree leans 4° away from vertical in the direction opposite the point 50 yards away. How tall is the tree?

15. Joaquin is fencing in a triangular pasture. Two posts are located 300 yards apart, and the angles from the posts to the third one are 75° and 68°, respectively. About how much fencing does Joaquin need?

16. **Use appropriate tools strategically.** Use a ruler and protractor to construct triangle ABC with $AB = 12.5$ cm, angle $A = 42°$, and angle $B = 40°$. Use your ruler and protractor to measure AC, CB, and angle C. Then calculate the size of angle C and use the Law of Sines to find AC and CB. How close were your measurements to your calculated values?

ACTIVITY 25 PRACTICE

Write your answers on notebook paper.
Show your work.

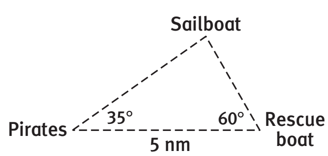
Lesson 25-1

1. Use the Law of Sines to solve triangle ABC with the following measures.

   angle $A = 150°$, angle $C = 20°$, $a = 200$

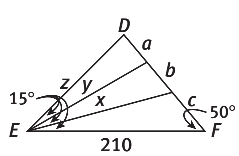
2. Two points, A and B, are 6 miles apart on level ground. An airplane is flying between A and B. The angle of elevation to the plane from point A is 51° and from point B is 68°. What is the altitude of the airplane?

3. A rescue boat and a pirate ship located 5 nautical miles apart both spotted a stranded sailboat at the same time. The rescue boat had a maximum speed of 18 knots (nautical miles per hour), and the pirate ship was capable of 22 knots. The angle between boats is shown below. If both ships set off at their top speed, which one will get to the stranded sailboat first, and how long will it take?

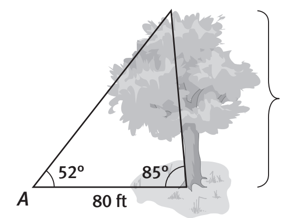


4. The angle of elevation from a point P 65 yards from a tree to the top of the tree is 31°. The tree leans 7 degrees away from P. How tall is the tree?

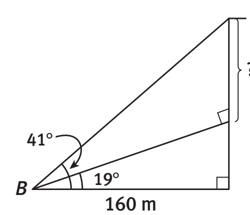
5. In triangle DEF below, angle DEF is divided into three angles, each of 15°, and angle F is 50°. If $XZ = 210$, find the values of x, y, z, a, b, and c.



6. Joanna is interested in determining the height of a tree. She is at a point A, 80 feet from the base of the tree, and she notices that the angle of elevation to the top of the tree is 52°. The tree is leaning toward her and is growing at an angle of 85° with respect to the ground. What is the height of the tree?



7. From a point B on the ground that is level with the base of a building and is 160 meters from the building, the angle of elevation to the top of the building is 41°. From point B, the angle of elevation to a ledge on the side of the building is 19°. What is the distance between the ledge and the top of the building?



ACTIVITY 25 Continued

ACTIVITY PRACTICE

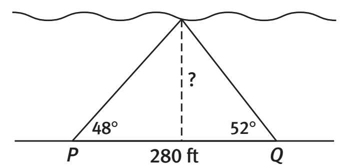
1. $B = 10°$, $b = 69.5$, $c = 136.8$
2. 4.94 miles
3. Sample explanation: The rescue boat will arrive first because it takes 0.1599 hour to get to the sailboat but it takes the pirates 0.1976 hour to get there.
4. 127.45 ft
5. $x = 177.5$, $y = 163.4$, $z = 161.5$, $a = 42.4$, $b = 46.6$, $c = 60.0$
6. 92.4 ft
7. 84.0 ft

8. 166.5 ft
9. B
10. a. 1 triangle
    b. no triangle
    c. 1 triangle
    d. 1 triangle
11. a. $A_1 = 62.438°$, $C_1 = 65.562°$, $c_1 = 9.243$; $A_2 = 117.562°$, $C_2 = 10.438°$, $c_2 = 1.839$
    b. $B_1 = 56.443°$, $A_1 = 93.557°$, $a_1 = 23.954$; $B_2 = 123.557°$, $A_2 = 26.443°$, $a_2 = 10.687$
12. Sample explanation: Using the Law of Sines, $\sin B = 0.4$, so angle $B = 24°$ or $156°$. But angle B cannot be $156°$ because the sum $A + B + C = 30 + 156 + C > 180$. So only one value is possible for angle B and only one distinct triangle has the given measurements.
13. Sample explanation: Using the Law of Sines, $\frac{m}{\sin M} = \frac{p}{\sin P}$ so $\frac{7}{\sin 30°} = \frac{16}{\sin P}$. Then $\sin P = \frac{16 \sin 30°}{7}$, which is greater than 1. Since $\sin P$ cannot be greater than 1, there is no triangle that has the given measurements.
14. Sample explanation: Using the Law of Sines, $\frac{x}{\sin X} = \frac{y}{\sin Y}$, so $\frac{10}{\sin 30°} = \frac{16}{\sin Y}$. Then $\sin Y = 0.8$ and $Y = 53°$ or $127°$. If $Y = 53°$, then the triangle can have angles of $53°$, $30°$, and $97°$. If $Y = 127°$, then the triangle can have angles of $127°$, $30°$, and $23°$.
15. This is the ambiguous case. Angle 1 $= 41.9°$, angle 2 $= 110.1°$, angle 3 $= 69.9°$, angle 4 $= 69.9°$, angle 5 $= 40.2°$, $QR = 15.6$, $RS = 7.6$, $QS = 23.2$
16. angle $Z = 43.5°$, angle 1 $= 78.9°$, angle 2 $= 101.1°$, angle 3 $= 35.4°$, angle 4 $= 54.6°$, $XW = 7.64$, $WZ = 5.72$, $YZ = 9.7$, $XZ = 13.3$
17. Sample answer: The statement is not correct. The ambiguous case means that two distinct triangles are possible from the given information about the triangle, one an obtuse triangle and one an acute triangle.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

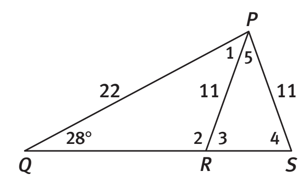
8. An explorer wants to know the width of a river. She starts by establishing two points, P and Q, on one side of the river that are 280 feet apart. She notices that a particular tree on the far side of the river forms an angle of 48° with side PQ when sighted from point P, and forms an angle of 52° with side PQ when sighted from point Q. How wide is the river? Show your work.



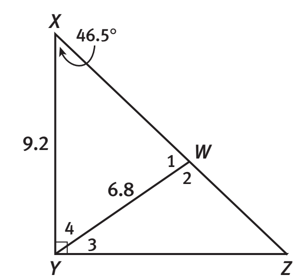
9. Which of the following statements is NOT true?
   A. You can use the Law of Sines if you know any two angles and any one side of a triangle.
   B. You can use the Law of Sines if you know the three sides of a triangle.
   C. You can use the Law of Cosines if you know any two sides and any one angle of a triangle.
   D. You can use the Law of Cosines if you know the three sides of a triangle.

Lesson 25-2

10. Determine the number of possible triangles for each situation:
    a. $A = 30°$, $c = 10$, $a = 5$
    b. $B = 63°$, $c = 90$, $b = 75$
    c. $C = 110°$, $c = 60$, $a = 47$
    d. $A = 60°$, $b = 9$, $a = 9$
11. Solve the two-solution ambiguous case situation.
    a. $B = 52°$, $a = 9$, $b = 8$
    b. $C = 30°$, $b = 20$, $c = 12$
12. Explain why only one triangle ABC is possible if $a = 20$, $b = 16$, and angle $A = 30°$.
13. Explain why no triangle MNP is possible if $m = 7$, $p = 16$, and angle $M = 30°$.
14. Explain why two triangles are possible if $x = 10$, $y = 16$, and angle $X = 30°$.
15. For the figure below, find angles 1, 2, 3, 4, and 5, and find QR, RS, and QS.



16. For the figure below, find angles Z, 1, 2, 3, and 4, and find XW, WX, YZ, and XZ. If necessary, round values to the nearest tenth.



MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

17. A student claims that the ambiguous case means you cannot tell whether 0, 1, or 2 triangles are possible given information about the triangle. Is that statement correct? Explain.

The Law of Sines

ACTIVITY 25

Got Lost?

Lesson 25-1 Modeling and Applying the Law of Sines

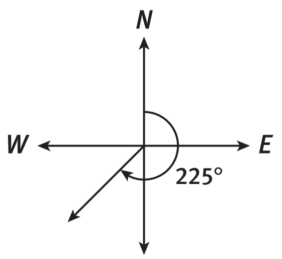
Learning Targets:

- Calculate the bearing of a flight.
- Derive and use the Law of Sines.
- Find unknown sides or angles in oblique triangles.

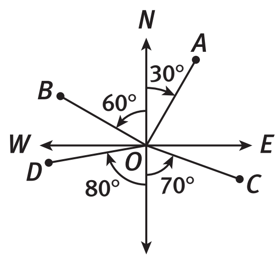
SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Identify a Subtask, Simplify the Problem, Create Representations, Summarizing, Paraphrasing, Work Backward, Look for a Pattern, Quickwrite, Graphic Organizer, Think-Pair-Share, Group Presentation, Guess and Check

In navigation, an object's heading indicates the direction of movement as measured by an angle rotated clockwise from north. A heading of 90° means an object is heading due east. A heading of 225° means an object is heading southwest. The *directional bearing* of a point is stated as the number of degrees east or west of the north-south line. To state the directional bearing of a point, write:

- N or S which is determined by the angle being measured
- the angle between the north or south line and the point, measured in degrees
- E or W which is determined by the location of the point relative to the north-south line



Heading



Directional Bearing

In the figure, *A* from *O* is N30°E, *B* from *O* is N60°W, *C* from *O* is S70°E, and *D* from *O* is S80°W.

International Flight 22 was on a course due north from Auckland, New Zealand, to Honolulu, Hawaii. Two thousand miles south of Honolulu, the plane encountered unexpected weather and the pilot changed bearing by 20°, as shown in the figure. The plane traveled on this new course for 1.5 hours, averaging 500 miles per hour.

1. How far did the plane travel during the 1.5 hours it was flying on its new flight path?

2. How far was the plane from Honolulu after 1.5 hours?

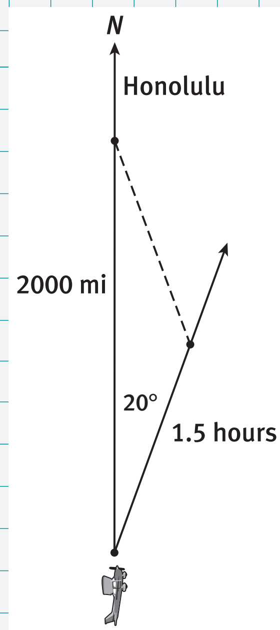
My Notes

CONNECT TO AVIATION

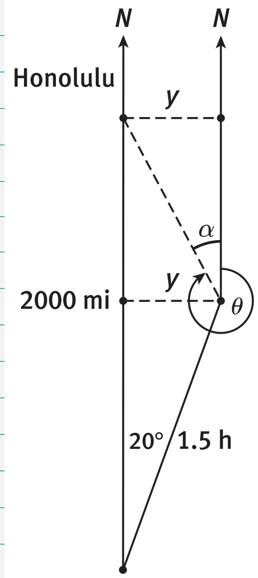
This activity measures speed in miles per hour. However, the speed of commercial jets is typically represented as a Mach number, a percentage of the speed of sound. Mach speed varies as temperature and altitude change.

CONNECT TO NAVIGATION

Bearing is the direction an aircraft is pointing, but the course is the actual direction in which the plane is moving when wind is taken into account. The heading is the clockwise angle in degrees between an aircraft's destination and north.



My Notes



3. To adjust the flight path, the pilot changed the course by α degrees, as shown in the figure.

a. Find the length y of the horizontal dotted lines in the diagram. Then use that length and your answer to Item 2 to find the value of α. What is the directional bearing of the plane?

b. Use the value of α to determine the bearing θ of the plane.

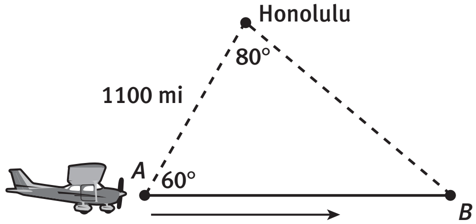
Another flight, Flight 33, was 1100 miles southwest of Honolulu when the plane sped up and headed due east. Radio contact with Flight 33 occurred 2.5 hours later, and air traffic controllers placed it somewhere over the Cook Islands at point B.

4. Draw the altitude of the triangle from the point representing Honolulu to the horizontal flight path.

a. What is length of the altitude?

b. How far did the plane travel in 2.5 hours?

c. How fast was the plane traveling along the path from point A to point B?

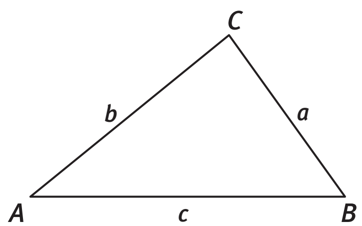


The ***Law of Sines***, shown below, could also be used to solve problems like Items 3 and 4.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The next series of items will show you how your work from the previous items can be generalized to derive the Law of Sines.

5. Model with mathematics. Use the triangle shown below to answer the questions.



a. Draw the altitude h from vertex C to side c.

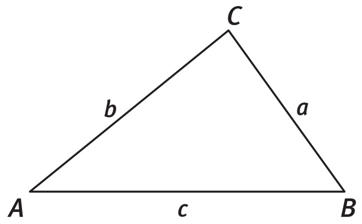
b. What is the measure of h in terms of A and b?

c. What is the measure of h in terms of B and a?

d. Use your work from parts b and c to write an equation relating A, B, a, and b.

My Notes

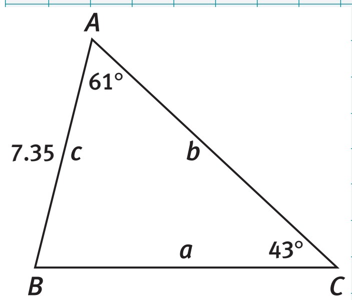
6. Express regularity in repeated reasoning. Repeat the process used in Item 5, but draw the altitude from vertex B to side b.



7. How do the equations you wrote in Items 5 and 6 compare to the Law of Sines written before Item 5?

Example A

Use the Law of Cosines and the Law of Sines to find the missing parts of this triangle. Explain which law you used.



Step 1: There are two known angles and one known side. We know c and C, and we know A, so we can use the Law of Sines to find a.

$$\frac{c}{\sin C} = \frac{a}{\sin A};\ a = \frac{(7.35)(\sin 61^\circ)}{\sin 43^\circ} = 9.43$$

Step 2: Find angle B: $B = 180^\circ - (61^\circ + 43^\circ) = 76^\circ$

Step 3: To find the third side, we can use the Law of Cosines. We can check the answer using the Law of Sines.

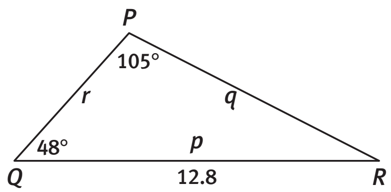
$b^2 = a^2 + c^2 - 2ac\cos B = 9.43^2 + 7.35^2 - 2(9.43)(7.35)\cos 76^\circ = 109.41$

$b = 10.46$

Check: $\dfrac{b}{\sin B} = \dfrac{c}{\sin C};\ b = \dfrac{(7.35)(\sin 76^\circ)}{\sin 43^\circ} = 10.46$

Try These A

a. Use the Law of Sines to find q.

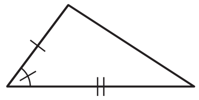


b. Use the Law of Cosines to find r. Check your value for r using the Law of Sines.

My Notes

Like the Law of Cosines, the Law of Sines relates the sides and angles in an ***oblique triangle***, and these can be used to find unknown sides or angles given at least three known measures that are not all angle measures.

8. The following table summarizes when each rule should be used to find missing measures in an oblique triangle. For each abbreviation, complete the given information and illustrate the given information by drawing and marking a triangle.

Rule	**Given information**	**Illustration**
Law of Cosines	SAS side, included angle, side SSS	
Law of Sines	ASA AAS	

Check Your Understanding

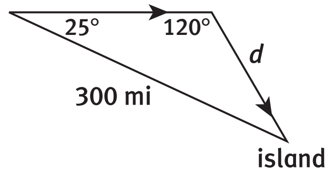
9. What is the minimum information needed in order to use the Law of Sines?

10. Make use of structure. What information about a triangle is enough to apply the Law of Sines but not the Law of Cosines? What information about a triangle is enough to apply the Law of Cosines but not the Law of Sines?

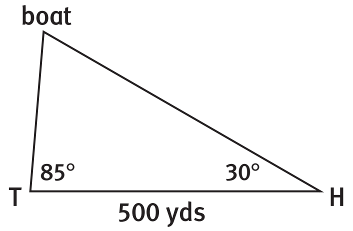
11. In a board game, a plane is flying from the origin of a coordinate plane toward the point (3, 4). What is the directional bearing of the plane? (Assume that north is the direction of the positive y-axis.)

Use the Law of Sines to solve the following problems.

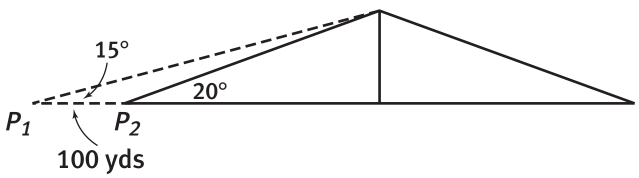
12. The pilots of Flight 33 spotted a deserted island 300 miles from their current location but continued on their course knowing there was another island 500 miles ahead on their current course. After a while they experienced engine trouble and turned to head for the deserted island, hoping to land safely. After making the turn, they estimated the plane could travel another 200 miles. They landed the plane knowing they were very lucky. How many additional miles could they have flown? Explain your reasoning.



13. Survivors Taylor and Hank are on the beach of the deserted island, 500 yards apart. They spot a boat out at sea and estimate the angles between their positions and the boat as shown below. How far is the boat from Hank? How far is the boat from Taylor?



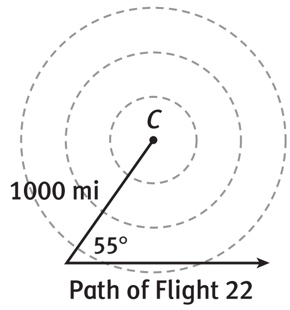
14. Reason quantitatively. Survivors Tariq and Jess were trying to estimate the distance to the top of a mountain they hoped to hike up to get a better view of the landscape. They measured the angles of elevation at points P_1 and P_2 located 100 yards apart. If the two survivors started at point P_2, how far would they have to walk to get to the top of the mountain?



My Notes

Check Your Understanding

The last known location of Transcontinental Flight 22 was 1000 miles from a tracking station located at point *C*. The plane was flying due east at a 55° angle with the tracking station.



15. Suppose Flight 22 continued to fly due east.

a. If a search plane is sent out from the tracking station and searches a 600-mile radius, will the search plane intercept the flight path? Show your work to explain your reasoning.

b. Would the search plane intercept the flight path if the search radius were 1000 miles? Explain.

c. Would the search plane intercept the flight path if the search radius were 900 miles? Explain.

16. What are some other possible search radii that would intercept the flight path in exactly one point?

LESSON 25-1 PRACTICE

17. A flight has a heading of 32°, as measured clockwise from north. What is its new heading under the following changes?

a. a quarter turn counterclockwise

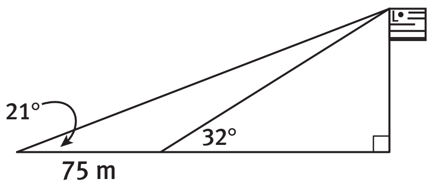
b. a quarter turn clockwise

c. a rotation of 55° counterclockwise

18. In triangle *ABC*, angle *A* is 54°, angle *C* is 96°, and $AC = 275$. Find angle *B*, *AB*, and *BC*.

19. In triangle *PQR*, $PR = 14$, $QR = 15$, and $PQ = 13$. Find the three angles of the triangle.

20. In the diagram below, two sightings of the top of a flagpole are taken 75 meters apart on level ground. The two sightings are 21° and 32°. What is the height of the flagpole?



21. **Attend to precision.** Two groups of students were given a copy of triangle *MNP*. Group 1 measured sides and angles of the triangle and found that $MN = 21$ cm, angle $N = 47°$, and $MP = 37$ cm. Group 2 found that $MN = 20.8$ cm, angle $N = 47.42°$, and $MP = 37.4$ cm. Using the Law of Sines, what values will Group 1 and Group 2 get for angle *P*, for angle *M*, and for *NP*?

Lesson 25-2
The Ambiguous Case (SSA)

Learning Targets:
- Determine the number of distinct triangles given certain criteria.
- Use the Law of Sines to solve triangles with unknown sides or angles.

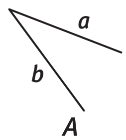
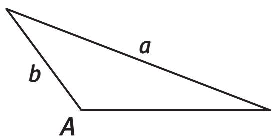
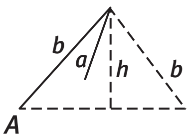
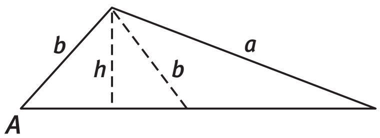
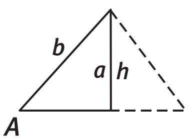
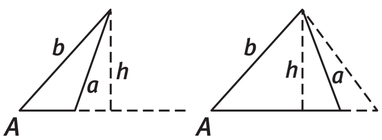
SUGGESTED LEARNING STRATEGIES: Note Taking, Interactive Word Wall, Graphic Organizer, Quick Write, Look for a Pattern, Think-Pair-Share, Create Representations, Guess and Check, Role Play, Identify a Subtask, Simplify the Problem, Group Presentation

My Notes

When you are given two sides and the opposite angle, it is possible to have zero, one, or two distinct triangles depending on the given information. This situation is known as the ***ambiguous case (SSA)*** and is summarized below.

The Ambiguous Case (SSA)

Given a, b, and A with $h = b \sin A$, where b is adjacent to and a is opposite angle A, and h is the altitude of the potential triangle.

A is obtuse, $a \le b$  no triangle	A is obtuse, $a > b$  one triangle
A is acute, $a < h$  no triangle	A is acute, $a \ge b$  one triangle
A is acute, $a = h$  one triangle	A is acute, $h < a < b$  two triangles

1. How can the value of $b \sin A$ help you determine the number of solutions given the ambiguous case?

2. How would you use this table to interpret the number of possible triangles for the ambiguous case if you were given angle C, side b, and side c?

ACADEMIC VOCABULARY

The word ***ambiguous*** is used for things that are open to interpretation, or can have two meanings.

My Notes

3. Construct viable arguments. Determine how many triangles are possible with the given information. Draw a sketch and show any calculations you used.

a. $A = 30°$, $b = 10$, $a = 5$

b. $C = 75°$, $c = 18$, $a = 7$

c. $B = 100°$, $b = 50$, $c = 75$

d. $C = 40°$, $b = 25$, $c = 21$

e. $A = 63°$, $a = 10$, $c = 45$

4. The Ambiguous Case Game 1: Use the information given to you by your teacher. With your classmates, organize into groups so you have the three groups with one solution and two groups with two solutions. Record your results.

5. The Ambiguous Case Game 2: Use the information given to you by your teacher. With your classmates, organize into groups so that every group of three people forms no triangle. Record your results.

My Notes

Example A

Solving the two-solution SSA situation: Use the Law of Sines to solve a triangle given $A = 42°$, $a = 18$, $b = 22$.

Step 1: Determine the number of solutions. There are two solutions since the measure of side a is between $b \sin A$ and b.

$$b \sin A = 22 \sin 42° \approx 14.720 < 18$$

Step 2: Solve the first acute triangle using the Law of Sines.

$$\frac{18}{\sin 42°} = \frac{22}{\sin B_1}$$

$$\sin B_1 = \frac{22 \sin 42°}{18} \approx 0.8178$$

Find B_1.

$$B_1 = \sin^{-1}\left(\frac{22 \sin 42°}{18}\right) \approx 54.9°$$

Find C_1.

$$C_1 = 180° - (42° + 54.9°) = 83.1°$$

Find c_1.

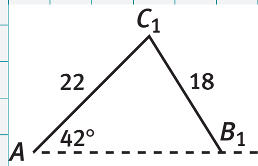
$$c_1 = \frac{18 \sin(83.1°)}{\sin 42°} \approx 26.7$$

Step 3: Solve the second obtuse triangle. The angles B_1 and B_2 opposite the given adjacent side are supplementary.

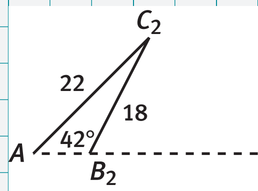
$$B_2 = 180 - B = 180 - 54.9 = 125.1$$

$$C_2 = 180 - (42 + 125.1) = 12.9$$

$$c_2 = \frac{18 \sin 12.9}{\sin 42} \approx 6.0$$



Acute triangle



Obtuse triangle

Try These A

Each of these triangles has two possible solutions. Find them both.

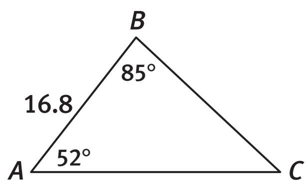
a. $A = 55°$, $b = 40$, $a = 35$

b. $C = 20°$, $c = 6$, $b = 12$

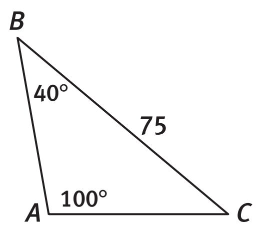
Check Your Understanding

6. Solve each triangle using the Law of Sines.

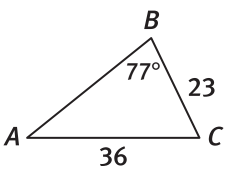
a. $A = 52°$, $B = 85°$, $c = 16.8$



b. $A = 100°$, $B = 40°$, $a = 75$



c. $B = 77°$, $a = 23$, $b = 36$



My Notes

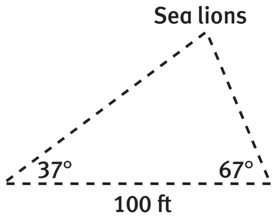
7. A lookout tower, firefighters located 25 miles from the tower, and a forest fire form three vertices of a triangle. At the lookout tower, the angle between the forest fire and the firefighters is 35°. At the firefighters' location, the angle between the lookout tower and the fire is 100°. How far are the firefighters from the fire?

8. **Make sense of problems.** Determine the number of possible triangles for each given situation.
a. $A = 45°$, $c = 100$, $a = 25$
b. $B = 70°$, $c = 90$, $b = 85$
c. $C = 100°$, $c = 6$, $a = 7.5$
d. $A = 60°$, $b = 4$, $a = 2\sqrt{3}$

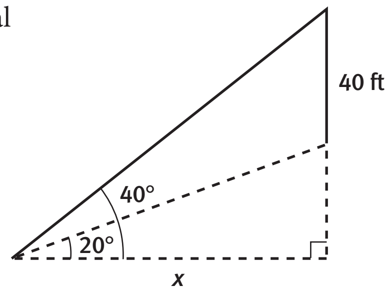
9. Why is the term *ambiguous case* used in this lesson? Explain how you know the situation is ambiguous. Describe how to solve an "ambiguous case" situation without using a formula.

LESSON 25-2 PRACTICE

10. Two marine biologists spotted some sea lions in the bay. The biologists were located on a beach about 100 feet apart. The angle between the shore and the sea lions for each biologist is shown below. How far were the sea lions from each biologist?



11. A billboard is 40 feet tall. At a horizontal distance x feet from the billboard, the angle of elevation to the bottom of the sign is 20° and the angle of elevation to the top of the sign is 40°. How far away is the billboard?



12. Solve the two-solution ambiguous case situation given $C = 50°$, $b = 120$, $c = 100$.

13. Solve each triangle using the Law of Sines.
a. $A = 22°$, $B = 35°$, $c = 43$
b. $A = 110°$, $B = 30°$, $a = 8$
c. $B = 57°$, $a = 13$, $b = 30$

14. The angle of elevation from a point 50 yards from a tree to the top of the tree is 23°. The tree leans 4° away from vertical in the direction opposite the point 50 yards away. How tall is the tree?

15. Joaquin is fencing in a triangular pasture. Two posts are located 300 yards apart, and the angles from the posts to the third one are 75° and 68°, respectively. About how much fencing does Joaquin need?

16. **Use appropriate tools strategically.** Use a ruler and protractor to construct triangle ABC with $AB = 12.5$ cm, angle $A = 42°$, and angle $B = 40°$. Use your ruler and protractor to measure AC, CB, and angle C. Then calculate the size of angle C and use the Law of Sines to find AC and CB. How close were your measurements to your calculated values?

ACTIVITY 25 PRACTICE

Write your answers on notebook paper.
Show your work.

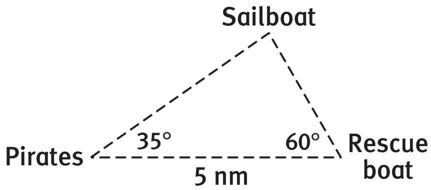
Lesson 25-1

1. Use the Law of Sines to solve triangle ABC with the following measures.

   angle $A = 150°$, angle $C = 20°$, $a = 200$

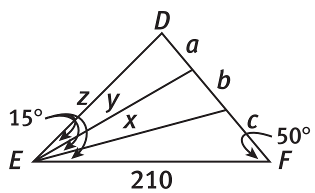
2. Two points, A and B, are 6 miles apart on level ground. An airplane is flying between A and B. The angle of elevation to the plane from point A is 51° and from point B is 68°. What is the altitude of the airplane?

3. A rescue boat and a pirate ship located 5 nautical miles apart both spotted a stranded sailboat at the same time. The rescue boat had a maximum speed of 18 knots (nautical miles per hour), and the pirate ship was capable of 22 knots. The angle between boats is shown below. If both ships set off at their top speed, which one will get to the stranded sailboat first, and how long will it take?

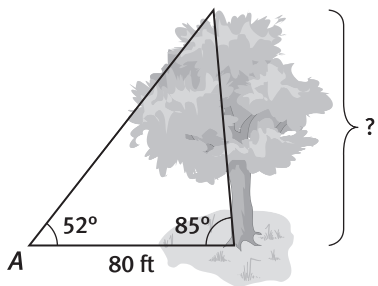


4. The angle of elevation from a point P 65 yards from a tree to the top of the tree is 31°. The tree leans 7 degrees away from P. How tall is the tree?

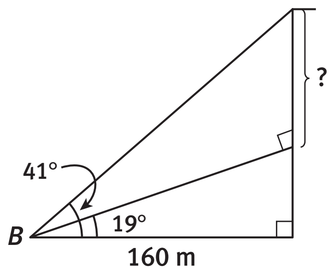
5. In triangle DEF below, angle DEF is divided into three angles, each of 15°, and angle F is 50°. If $XZ = 210$, find the values of x, y, z, a, b, and c.



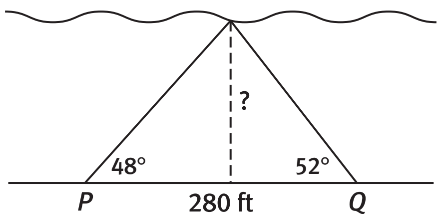
6. Joanna is interested in determining the height of a tree. She is at a point A, 80 feet from the base of the tree, and she notices that the angle of elevation to the top of the tree is 52°. The tree is leaning toward her and is growing at an angle of 85° with respect to the ground. What is the height of the tree?



7. From a point B on the ground that is level with the base of a building and is 160 meters from the building, the angle of elevation to the top of the building is 41°. From point B, the angle of elevation to a ledge on the side of the building is 19°. What is the distance between the ledge and the top of the building?



8. An explorer wants to know the width of a river. She starts by establishing two points, P and Q, on one side of the river that are 280 feet apart. She notices that a particular tree on the far side of the river forms an angle of 48° with side PQ when sighted from point P, and forms an angle of 52° with side PQ when sighted from point Q. How wide is the river? Show your work.



9. Which of the following statements is NOT true?
A. You can use the Law of Sines if you know any two angles and any one side of a triangle.
B. You can use the Law of Sines if you know the three sides of a triangle.
C. You can use the Law of Cosines if you know any two sides and any one angle of a triangle.
D. You can use the Law of Cosines if you know the three sides of a triangle.

Lesson 25-2

10. Determine the number of possible triangles for each situation:
a. $A = 30°$, $c = 10$, $a = 5$
b. $B = 63°$, $c = 90$, $b = 75$
c. $C = 110°$, $c = 60$, $a = 47$
d. $A = 60°$, $b = 9$, $a = 9$

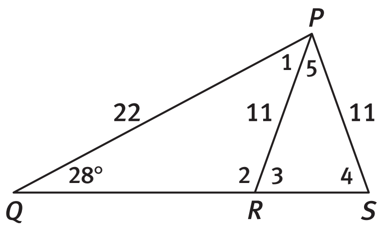
11. Solve the two-solution ambiguous case situation.
a. $B = 52°$, $a = 9$, $b = 8$
b. $C = 30°$, $b = 20$, $c = 12$

12. Explain why only one triangle ABC is possible if $a = 20$, $b = 16$, and angle $A = 30°$.

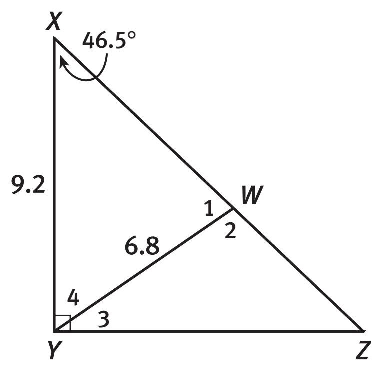
13. Explain why no triangle MNP is possible if $m = 7$, $p = 16$, and angle $M = 30°$.

14. Explain why two triangles are possible if $x = 10$, $y = 16$, and angle $X = 30°$.

15. For the figure below, find angles 1, 2, 3, 4, and 5, and find QR, RS, and QS.



16. For the figure below, find angles Z, 1, 2, 3, and 4, and find XW, WX, YZ, and XZ. If necessary, round values to the nearest tenth.



MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

17. A student claims that the ambiguous case means you cannot tell whether 0, 1, or 2 triangles are possible given information about the triangle. Is that statement correct? Explain.