Linear Models

Stacking Boxes
Lesson 10-1 Direct Variation

ACTIVITY 10

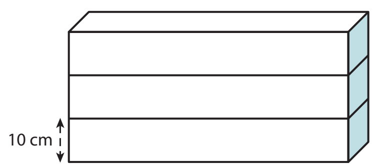
My Notes

Learning Targets:

- Write and graph direct variation.
- Identify the constant of variation.

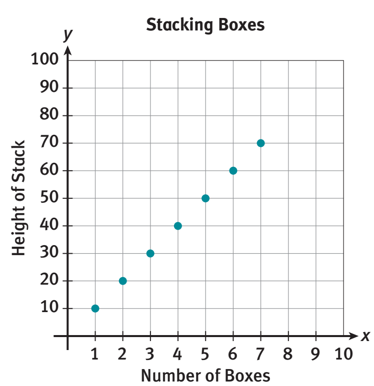
SUGGESTED LEARNING STRATEGIES: Create Representations, Interactive Word Wall, Marking the Text, Sharing and Responding, Discussion Groups

You work for a packaging and shipping company. As part of your job there, you are part of a package design team deciding how to stack boxes for packaging and shipping. Each box is 10 cm high.



1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

Number of Boxes	Height of the Stack (cm)
0	0
1	10
2	20
3	30
4	40
5	50
6	60
7	70



2. Write a function to represent the data in the table and graph above.

$f(x) = 10x$, or $y = 10x$

WRITING MATH

Either y or $f(x)$ can be used to represent the output of a function.

3. What is a reasonable and realistic domain for the function? Explain.

The domain is the set of integers that are greater than or equal to 0. The domain cannot include values less than 0 because there is no such thing as fewer than 0 boxes.

4. What is a reasonable and realistic range for the function? Explain.

The range is 0 and positive multiples of 10. The range cannot include values less than 0 because the height of a stack cannot be less than 0. You cannot have a fraction of a box, so the range cannot include values that are not multiples of 10.

Common Core State Standards for Activity 10

HSN-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
HSA-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
HSF-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
HSF-BF.A.1	Write a function that describes a relationship between two quantities.
HSF-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.

ACTIVITY 10
Guided

Activity Standards Focus

In this activity students solve problems by gathering real-world data, recording the results in tables, representing results with graphs, and writing function equations. They also learn to write and use inverse functions. These concepts and skills will be important to students as they use functions in increasingly complex mathematical contexts.

Lesson 10-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–6 #7–8 #9–10 #11–12
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Show students the ratios $\frac{2}{3}$, $\frac{9}{15}$, $\frac{15}{21}$ and $\frac{15}{25}$. Ask them to identify the two equivalent ratios. Students should then write five other ratios that are equivalent to those two ratios.

1–6 Marking the Text, Discussion Groups, Create Representations, Look for a Pattern, Sharing and Responding Ordered pairs in the table should correspond to discrete points on students' graphs since they are being used to represent the number of and heights of boxes. As students respond to Items 3 and 4, be sure students are choosing appropriate values for the given problem situation. As students share responses with the class, focus the discussion on why there are no negative values for x or y. Students should understand that there cannot be a negative number of boxes nor a negative height for the stack. Students should identify patterns focusing on the constant difference of 10 as it appears in both the table and the graph.

Developing Math Language

Refer to the table as you discuss the term *directly proportional*. Discuss with students that when quantities have *direct variation,* the two values are directly proportional. All the ratios of $y : x$ are equivalent in a situation. Relate direct variations to the concept of slope in which the ratios along a line of $\Delta y{:}\Delta x$ are equivalent. Note that the *constant of variation* is the ratio $\frac{y}{x}$ and is represented by k. Demonstrate how to rewrite the equation $\frac{y}{x} = k$ as $y = kx$.

7–8 Marking the Text, Activating Prior Knowledge, Discussion Groups, Interactive Word Wall Monitor student discussions carefully to be sure students understand the new vocabulary and how the words are interrelated. Direct proportionality is a concept that is familiar to students. By connecting to prior knowledge, students will better understand the new concept. Add the terms *directly proportional* and *direct variation* to the classroom word wall.

9–10 Think-Pair-Share, Construct an Argument, Interactive Word Wall, Sharing and Responding In these items students derive the equation for determining the constant of variation. Add new vocabulary to the classroom word wall. Be sure students understand before they begin that the constant of variation, k, represents a ratio, and that the ratio may be a whole number or a fraction.

Teacher to Teacher

The variable k is also sometimes called the *constant of proportionality*. Students should understand that they may see it called this as well as *constant of variation*.

My Notes

MATH TERMS

A **direct proportion** is a relationship in which the ratio of one quantity to another remains constant.

5. What do $f(x)$, or y, and x represent in your equation from Item 2?
$f(x)$ represents the height of the stack and x represents the number of boxes.

6. Describe any patterns that you notice in the table and graph representing your function.
Answers may vary. Each time a box is added to the stack, the height increases by 10 cm.

7. The number of boxes is ***directly proportional*** to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.
$\frac{1 \text{ box}}{10 \text{ cm}} = \frac{12 \text{ boxes}}{120 \text{ cm}}$; **the height is 120 cm.**

When two values are directly proportional, there is a ***direct variation***. In terms of stacking boxes, the height of the stack *varies directly* as the number of boxes.

8. Using variables x and y to represent the two values, you can say that y varies directly as x. Use your answer to Item 6 to explain this statement.
Answers may vary. y is the height of the stack and x is the number of boxes, so "y varies directly as x" means that the height of the stack varies as the number of boxes changes.

9. Direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient k is the ***constant of variation***.

a. Consider your answer to Item 2. What is the constant of variation in your function?
10

b. Why do you think the coefficient is called the constant of variation?
The height *constantly varies* by 10 with each addition of another box.

c. **Reason quantitatively.** Explain why the value of k cannot be equal to 0.
Answers may vary. If $k = 0$, then $y = 0x = 0$, which means that the value of y will always be 0 and will not vary.

d. Write an equation for finding the constant of variation by solving the equation $y = kx$ for k.
$k = \frac{y}{x}$

Lesson 10-1
Direct Variation

ACTIVITY 10
continued

My Notes

10. **a.** Interpret the meaning of the point (0, 0) in your table and graph.

A stack of 0 boxes has no height.

b. True or False? Explain your answer. "The graphs of all direct variations are lines that pass through the point (0, 0)."

True; explanations may vary. In an equation of the form $y = kx$, when $x = 0$ then $y = 0$.

c. Identify the slope and y-intercept in the graph of the stacking boxes.

$m = 10$; y-intercept $= (0, 0)$

d. Describe the relationship between the constant of variation and the slope.

The constant of variation and the slope are equal.

Direct variation can be used to answer questions about stacking and shipping your boxes.

11. The height y of a different stack of boxes varies directly as the number of boxes x. For this type of box, 25 boxes are 500 cm high.

a. Find the value of k. Explain how you found your answer.

$k = 20$; use the equation $k = \frac{y}{x}$ to find the constant of variation;

$$k = \frac{y}{x} = \frac{500 \text{ cm}}{25 \text{ boxes}} = 20$$

b. Write a direct variation equation that relates y, the height of the stack, to x, the number of boxes in the stack.

$y = 20x$

c. How high is a stack of 20 boxes? Explain how you would use your direct variation equation to find the height of the stack.

400 cm; replace x with 20 in the equation, $y = 20x$, and solve for y.
$y = 20(20) = 400$

ACTIVITY 10 Continued

9–10 (continued) It is important for students to understand that the graphs of all direct variations pass through the origin and that the constant of variation is the same as the slope. Students may need to graph multiple situations involving direct variation to confirm that (0,0) will always be a point on the line.

11–12 Discussion Groups, Debriefing It is important for students to understand that y, or $f(x)$, represents the height of the stack, and that x represents the number of boxes. The constant of variation represents the rate at which the stack height increases with the addition of each box. In this case, it is also the height of each box.

Differentiating Instruction

Extend students' understanding of direct variations by having them examine a linear function that is not a direct variation. Change the problem about the boxes by saying that the boxes are stacked on a platform that is 5 centimeters high. Have them generate an equation that gives the height of the stack of boxes including the platform. Instruct them to make a table and a graph to show the height with various numbers of boxes. Then have them analyze the table and graph to determine whether or not it is a direct variation. They should consider both the ratio of height to boxes as well as the y-intercept of the equation.

Check Your Understanding

As you debrief the lesson, continue to emphasize that, in a linear function, the ratio of $y : x$ is constant, and the graph passes through the point (0, 0). Use the vocabulary introduced in this lesson as you discuss the problems.

Answers

13. a. No; the graph is not a line through the origin.
b. Yes; the graph is a line through the origin.
c. Yes; the values can be described by the equation $y = 4x$, which is in the form $y = kx$, where $k = 4$; also, the graph is a line through the origin.
d. No; the graph is not a line.
e. Yes; the equation is in the form $y = kx$.
f. No; the equation cannot be written in the form $y = kx$.

My Notes

12. At the packaging and shipping company, you get paid each week. One week you earned \$48 for 8 hours of work. Another week you earned \$30 for 5 hours of work.

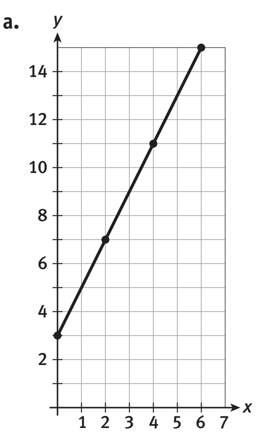
a. Write a direct variation equation that relates your wages to the number of hours you worked each week. Explain the meaning of each variable and identify the constant of variation.
$y = 6x$ where y equals your wages, x is the number of hours you work in a week, and the constant of variation is 6.

b. How much would you earn if you worked 3.5 hours in one week?
\$21.00

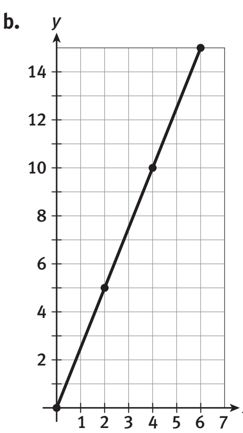
Check Your Understanding

13. Tell whether the tables, graphs, and equations below represent direct variations. Justify your answers.

a.



b.



c.

x	y
2	12
4	24
6	36

d.

x	y
2	8
4	12
6	16

e. $y = 20x$

f. $y = 3x + 2$

My Notes

LESSON 10-1 PRACTICE

14. In the equation $y = 15x$, what is the constant of variation?

15. In the equation $y = 8x$, what is the constant of variation?

16. The value of y varies directly with x and the constant of variation is 7. What is the value of x when $y = 63$?

17. The value of y varies directly with x and the constant of variation is 12. What is the value of y when $x = 5$?

18. **Model with mathematics.** The height of a stack of boxes varies directly with the number of boxes. A stack of 12 boxes is 15 feet high. How tall is a stack of 16 boxes?

19. Jan's pay is in direct variation to the hours she works. Jan earns $54 for 12 hours of work. How much will she earn for 18 hours work?

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 10-1 PRACTICE

14. 15
15. 8
16. 9
17. 60
18. 20 feet
19. $81

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to write and use equations of direct variation. Reinforce the correct use of *directly proportional*, *direct variation*, and *constant of variation* as you discuss the problems.

Lesson 10-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–4 #5 #6–7
8-9 #10–11
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to determine the volumes of rectangular prisms with these dimensions: 3 in. × 4 in. × 5 in.; 15 cm × 7 cm × 11 cm; 2 ft × 4 ft × 8 ft. Challenge them to find all possible whole-number dimensions for rectangular prisms with volumes of 12 ft³, 30 in.³, and 60 cm³.

1–4 Marking the Text, Create Representations, Discussion Groups, Look for a Pattern, Sharing and Responding Students should observe that the product of the length and width is 40. Students should also notice that the graph is decreasing but not linear. This chunk of items provides the opportunity for rich discussion about distinguishing between a constant rate of change and a nonconstant rate of change. In addition, it is important for students to understand that both x and y can come very close to zero, but they can never be equal to it. Have students discuss any patterns they see in the table and graph.

My Notes

MATH TIP

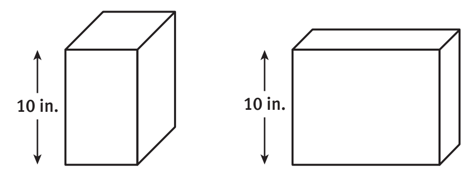
The volume of a rectangular prism is found by multiplying length, width, and height: $V = lwh$.

Learning Targets:

- Write and graph indirect variations.
- Distinguish between direct and indirect variation.

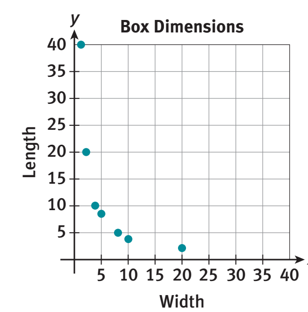
SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Sharing and Responding, Think-Pair-Share, Discussion Groups

When packaging a different product, your team at the packaging and shipping company determines that all boxes for this product will have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.



1. To explore the relationship between length and width, complete the table and make a graph of the points.

Width (x)	Length (y)
1	40
2	20
4	10
5	8
8	5
10	4
20	2



2. How are the lengths and widths in Item 1 related? Write an equation that shows this relationship.

Answers may vary. The product of the length and width must be 40, so look for pairs of factors that have a product of 40; $xy = 40$.

3. Use the equation you wrote in Item 2 to write a function to represent the data in the table and graph above.

$f(x) = \frac{40}{x}$, or $y = \frac{40}{x}$

4. Describe any patterns that you notice in the table and graph representing your function.

Answers may vary. As the width increases, the length decreases.

Lesson 10-2
Indirect Variation

My Notes

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an ***indirect variation***.

MATH TIP

Indirect variation is also known as **inverse variation.**

5. Recall that direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient k is the constant of variation.

a. How would you define indirect variation in terms of y, k, and x?

$y = \frac{k}{x}$

b. Are there any limitations on these variables as there are on k in direct variation? Explain.

$k \neq 0$, $x \neq 0$, $y \neq 0$. Answers may vary. If $k = 0$, then x can be any number except 0 and y will always be 0. From the graph in Item 1, the values of x and y can get closer and closer to 0 but never equal 0.

c. Write an equation for finding the constant of variation by solving for k in your answer to Part (a).

$k = xy$

6. Reason abstractly. Compare and contrast the equations of direct and indirect variation.

Answers may vary. With direct variation you multiply k by x to find y. With indirect variation you divide k by x to find y.

7. Compare and contrast the graphs of direct and indirect variation.

Answers may vary. With direct variation, as x increases, y also increases by a given constant of variation, k. With indirect variation, as x increases, y decreases because the constant of variation, k, is divided by x.

8. Use your function in Item 3 to determine the following measurements for your company.

a. Find the length of a box whose width is 80 inches.

0.5 inches

b. Find the length of a box whose width is 0.4 inches.

100 inches

ACTIVITY 10 Continued

5 Marking the Text, Interactive Word Wall, Think-Pair-Share, Sharing and Responding It is important for students to understand that as the width of the box increases, the length of the box decreases. Monitor pair discussions carefully and refer students who are having difficulty back to their tables and graphs in Item 1. Note that in an indirect variation, the product of the two quantities represented by the variables is always a constant. As students share responses to part b with the class, be sure the reasons that x, y and k cannot equal 0 are highlighted.

6–7 Quickwrite Students should be able to use real-world language to compare and contrast these new terms. Provide real-world contexts for students who are struggling.

Developing Math Language

Review the mathematical definition of an *indirect variation* as a relationship between two quantities in which the value of one decreases as a result of an increase in the other. Ask students to provide real-world examples of indirect variation relationships. A possible example of direct variation is the ratio of minutes a person types to the number of pages typed. An example of indirect variation is the ratio of the height of a candle to the length of time it has burned.

8–9 Discussion Groups, Create Representations Monitor group discussions carefully to be sure students understand that, in Item 9, the constant of variation is found by solving the inverse variation equation for k and then substituting a known ordered pair (x,y).

10–11 Think-Pair-Share, Construct an Argument, Debriefing At this point, students should begin to make to make the connection between linear functions and direct variations and be able to identify an inverse variation as a nonlinear function. Encourage the use of graphs if students are struggling, but allow students to make determinations based on the equations if they are ready. Debriefing should focus on the fact that in an indirect variation, as one variable increases, the other decreases proportionally, while in a direct variation both quantities either increase or decrease proportionally.

My Notes

9. The time, y, needed to load the boxes on a truck for shipping varies indirectly as the number of people, x, working. If 10 people work, the job is completed in 20 hours.

a. Explain how to find the constant of variation. Then find it.

To find the constant of variation multiply the number of people working by the number of hours worked; $k = 10(20) = 200$.

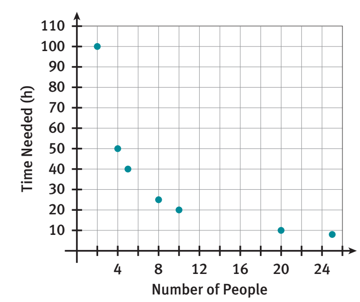
b. Write an indirect variation equation that relates the time to load the boxes to the number of people working.

$y = \frac{200}{x}$

c. How long does it take 8 people to finish loading the boxes? Use your equation to answer this question.

$y = \frac{200}{8} = 25$ **hours**

d. On the grid below, make a graph to show the time needed for 2, 4, 5, 8, 10, and 25 people to load the boxes on the truck.



10. The cost for the company to ship the boxes varies indirectly with the number of boxes being shipped. If 25 boxes are shipped at once, it will cost \$10 per box. If 50 boxes are shipped at once, the cost will be \$5 per box.

a. Write an indirect variation equation that relates the cost per box to the number of boxes being shipped.

$y = \frac{250}{x}$

b. How much would it cost to ship only 10 boxes?

\$25

11. Is an indirect variation function a linear function? Explain.

No, an indirect function $y = \frac{k}{x}$ is not a linear function because the graph of an indirect variation is not a line.

MINI-LESSON: Recognizing Variation Equations

Give each student three index cards. On the first card they should write *Direct Variation* and an equation in the form $y = kx$. On the second card they should write *Indirect Variation* and an equation in the form $y = \frac{k}{x}$. On the third card they should write *Neither Direct nor Indirect* and write an equation that is neither a direct nor indirect variation. On the back of each card, students should create a table of values that corresponds to the equation on the front of the card. Collect and shuffle the cards. Distribute three cards to each student, table side up. Students should use the table to decide if it is a direct variation, an indirect variation or neither. If it is a variation, they should determine the constant of variation. They can turn over the card to check the answer.

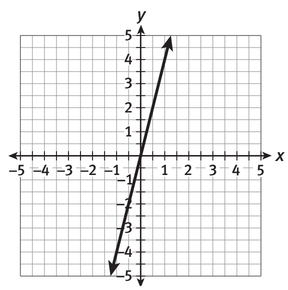
Lesson 10-2
Indirect Variation

My Notes

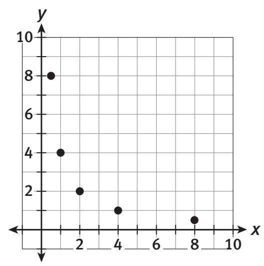
Check Your Understanding

12. Identify the following graphs as direct variation, indirect variation, neither, or both.

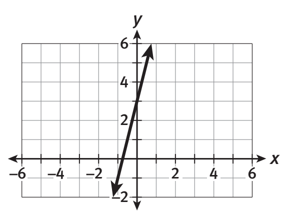
a.



b.



c.



13. Which equations are examples of indirect variation? Justify your answers.

A. $y = 2x$ **B.** $y = \frac{x}{2}$

C. $y = \frac{2}{x}$ **D.** $xy = 2$

14. In the equation $y = \frac{80}{x}$, what is the constant of variation?

LESSON 10-2 PRACTICE

15. Graph each function. Identify whether the function is an indirect variation.

a.

x	-4	-2	-1	1	2	4
y	$-\frac{3}{4}$	$-\frac{3}{2}$	-3	3	$\frac{3}{2}$	$\frac{3}{4}$

b.

x	-4	-2	-1	1	2	4
y	11	5	3	-4	-7	-13

16. Make sense of problems. For Parts (a) and (b) below, y varies indirectly as x.

a. If $y = 6$ when $x = 24$, find y when $x = 16$.

b. If $y = 8$ when $x = 20$, find the value of k.

Check Your Understanding

Students should be able to recognize direct and indirect variations both by their equations and by the shapes of their graphs. As you debrief this lesson, have students explain their reasoning for each answer. Be aware that some students may state that, for example, $y = \frac{x}{2}$ is an indirect variation because it contains a fraction. Help these students realize that the equation could be rewritten as $y = \frac{1}{2}x$ since x is in the numerator. It could be rewritten as $y = \frac{1}{2}x$, which shows a direct variation with constant of variation $\frac{1}{2}$.

Answers

12 a. direct variation

b. indirect variation

c. neither

13. C; The equation is in the form $y = \frac{k}{x}$, where $k = 2$.
D; The equation can be written in the form $y = \frac{k}{x}$, where $k = 2$.

14. 80

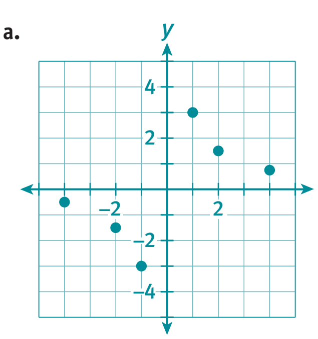
ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

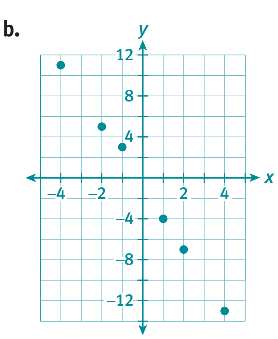
LESSON 10-2 PRACTICE

15. a.



It is an indirect variation.

15. b.



It is not an indirect variation.

16. a. 9

b. 160

ADAPT

If students have difficulty identifying direct and indirect variations, review the characteristics of each. Assign additional practice problems as needed.

Lesson 10-3

PLAN

Materials

- ruler or measuring tape marked with centimeters
- at least 6 each of two sizes of paper cups for each student

Pacing: 1 class period

Chunking the Lesson

#1–2 #3–6 #7–8
#9–10 #11–12 #13
Check Your Understanding
Lesson Practice

TEACH

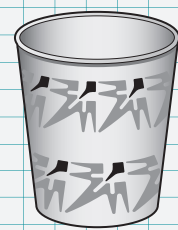
Bell-Ringer Activity

Give each student a centimeter ruler or measuring tape. Ask students to find several measurements accurate to the nearest centimeter. You might have them find the length, width and height of their notebooks or other books, the lengths of pens or pencils, or the length of their thumb or hand span. Assist students who need help measuring accurately.

1–2 Shared Reading, Marking the Text, Use Manipulatives, Discussion Groups Have students work with their groups to collect the data for Item 1. Some students may mistakenly want to measure along the slant height. Differences between heights of stacks with consecutive numbers of cups should be nearly the same, although individual measurements may yield imperfect data. Allow students to discuss the patterns they find in their data with the whole class.

Lesson 10-3
Another Linear Model

My Notes



CONNECT TO GEOMETRY

The carton will be a right rectangular prism. A **rectangular prism** is a closed, three-dimensional figure with three pairs of opposite parallel faces that are congruent rectangles.

Learning Targets:

- Write, graph, and analyze a linear model for a real-world situation.
- Interpret aspects of a model in terms of the real-world situation.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Discussion Groups, Create Representations, Guess and Check, Use Manipulatives

Your design team at the packaging and shipping company has been asked to design a cardboard box to use when packaging paper cups for sale. Your supervisor has given you the following requirements.

- All lateral faces of the container must be rectangular.
- The base of the container must be a square, just large enough to accommodate one cup.
- The height of the container must be given as a function of the number of cups the container will hold.
- All measurements must be in centimeters.

To help discover which features of the cup affect the height of the stack, collect data on two types of cups found around the office.

1. **Use appropriate tools strategically.** Use two different types of cups to complete the tables below.
   Answers will vary depending on cup dimensions.

CUP 1	
Number of Cups	Height of Stack
1	
2	
3	
4	
5	
6	

CUP 2	
Number of Cups	Height of Stack
1	
2	
3	
4	
5	
6	

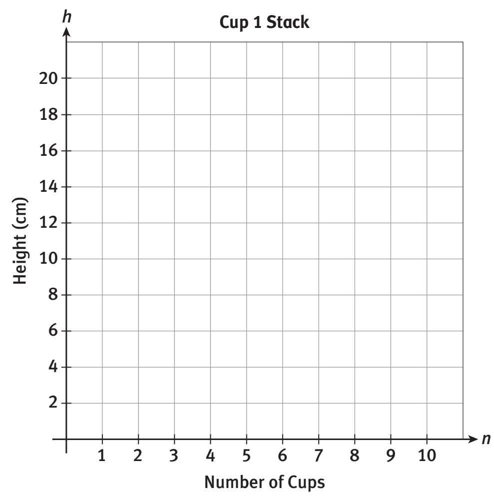
2. **Express regularity in repeated reasoning.** What patterns do you notice that might help you figure out the relationship between the height of the stack and the number of cups in that stack?
   Answers may vary. The height increased by the same amount each time a new cup was added to the stack.

My Notes

Use your data for Cup 1 to complete Items 3–13.

3. Make a graph of the data you collected.

Answers will vary depending on cup dimensions. Graphs should reflect tabular data for Cup 1 and be discrete.



4. Predict, without measuring, the height of a stack of 16 cups. Explain how you arrived at your prediction.

Answers will vary depending on cup dimensions, but should be based on the pattern students observed in Items 1 and 2.

5. Predict, without measuring, the height of a stack of 50 cups. Explain how you arrived at your prediction.

Answers will vary depending on cup dimensions, but should be based on the pattern students observed in Items 1 and 2.

6. Write an equation that gives the height of a stack of cups, h, in terms of n, the number of cups in the stack.

Answers should be equivalent to $h = d \bullet n + (c - d)$ or $h = c + (n - 1)d$ where d is the difference in height between consecutive entries on the table and c is the height of one cup. Values of c and d in students' solutions will depend on cup dimensions.

7. Use your equation from Item 6 to find h when $n = 16$ and when $n = 50$. Do your answers to this question agree with your predictions in Items 4 and 5?

Answers will vary.

3–6 Discussion Groups, Create Representations, Look for a Pattern, Guess and Check, Debriefing

Students may want to connect the points on the graph. Help students realize that the graphs should consist of only discrete collinear points. If students connect the points, encourage self-editing by asking if packing a fraction of a cup makes sense. Students will probably not extend the graph to accommodate 50 cups in Item 7. It is more likely they will use the pattern in the table. Students who are unable to see the additive pattern may have difficulty connecting it to linear functions.

Teacher to Teacher

Some students may extend the grid, draw a line through the points, and extend the line to $x = 16$ to estimate the height of the stack. Others might use the additive pattern in the table. A common error is to add 16 times the differences in the table to 1 cup. Another common error is to determine the height of 8 cups and then double it.

Some students may require significant assistance when writing the equation in Item 6. Students will create different forms of the equation. Allow time during debriefing for a whole-class discussion about the equation forms and their relevance in the situation.

7–8 Create Representations, Discussion Groups, Guess and Check Correct equations should yield values close to the predictions made in the beginning of the lesson. Again, students may try to graph the equation as a solid line. The graph should contain only discrete points because the input values are numbers of cups.

9–10 Activating Prior Knowledge, Think-Pair-Share Students' graphs will be similar. The equation should give ordered pairs very close to, if not exactly the same as, the ordered pairs found in the collected data.

11–12 Think-Pair-Share, Sharing and Responding The length and width of the carton are required to complete the design. Students must realize that these dimensions depend on the diameter of the top of a cup. In Item 12, students should evaluate the equation for $n = 25$ and identify that value as the height of the carton. Emphasize units.

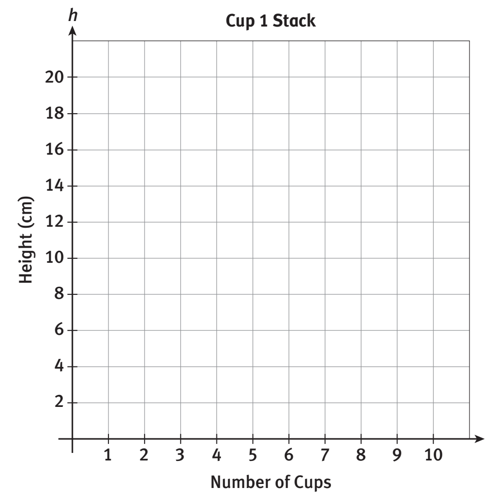
Differentiating Learning

Support the acquisition of academic vocabulary for English Language Learners during this hands-on activity. As you use clear academic language when describing and discussing the task, support it with nonverbal cues. Ask students to repeat or paraphrase what you say. To facilitate continued acquisition of academic language during group work, pair students with strong English skills with those who need more practice with academic vocabulary.

My Notes

8. Sketch the graph of your equation from Item 6.

Answers will vary depending on the values of *c* and *d*. Graphs should be discrete and include values of $n > 0$.



9. How are the graphs you made in Items 3 and 8 the same? How are they different?

Answers may vary. Both graphs are discrete but this new graph includes more points. The graph in Item 3 contains only data that was collected from measuring a finite number of cups. The *y*-values may be slightly different because the graph from Item 3 was based on measuring the height of the stack and not the equation.

10. Do the graphs in Items 3 and 8 represent direct variation, indirect variation, or neither? Explain.

Neither; it is not a direct variation because there is an initial height for the first cup so the graph does not pass through (0, 0). It is not an indirect variation because the graph appears to be linear.

11. Remember that you are designing a container with a square base. What dimension(s), other than the height of the stack, do you need to design your cup container? Use Cup 1 to find this/these dimension(s).

Answers may vary. The diameter of the wider end of the cup. The actual diameter will depend on the dimensions of Cup 1.

12. Find the dimensions of a container that will hold a stack of 25 cups.

Answers will vary depending on the dimensions of Cup 1.

Lesson 10-3
Another Linear Model

My Notes

13. Your team has been asked to communicate its findings to your supervisor. Write a report to her that summarizes your findings about the cup container design. Include the following information in your report.
- The equation your team discovered to find the height of the stack of Cup 1 style cups
- A description of how your team discovered the equation and the minimum number of cups needed to find it
- An explanation of how the numbers in the equation relate to the physical features of the cup
- An equation that could be used to find the height of the stack of Cup 2 style cups

Answers will vary.

MATH TIP

When writing your answer to Item 13, you can use a RAFT.

- Role—team leader
- Audience—your boss
- Format—a letter
- Topic—stacks of cups

Check Your Understanding

14. A group of students performed the cup activity described in this lesson. For their Cup 1, they found the equation $h = 0.25n + 8.5$, where h is the height in inches of a stack of cups and n is the number of cups.

a. What would be the height of 25 cups? Of 50 cups?

b. Graph this equation. Describe your graph.

LESSON 10-3 PRACTICE

15. Reason quantitatively. A group of students performed the cup activity in this lesson using plastic drinking cups. Their data is shown below.

CUP 1	
Number of Cups	Height of Stack
1	14.5 cm
2	16 cm
3	17.5 cm
4	19 cm
5	20.5 cm

CUP 2	
Number of Cups	Height of Stack
1	10.5 cm
2	11.75 cm
3	13 cm
4	14.25 cm
5	15.5 cm

For each cup, write and graph an equation. Describe your graphs.

16. A consultant earns a flat fee of \$75 plus \$50 per hour for a contracted job. The table shows the consultant's earnings for the first four hours she works.

Hours	0	1	2	3	4
Earnings	\$75	\$125	\$175	\$225	\$275

The consultant has a 36-hour contract. How much will she earn?

ACTIVITY 10 Continued

13 Shared Reading, Marking the Text, RAFT, Prewriting, Self Revision/Peer Revision Read this aloud so all students know what is expected. The minimum number of cups needed to determine the equation is 2. Note that students use Cup 2 data for their second equation. Students with a thorough understanding of linear equations and their relationship to this context should be able to replicate the process they followed for Cup 1. Encourage the use of correct mathematical vocabulary as students begin to write.

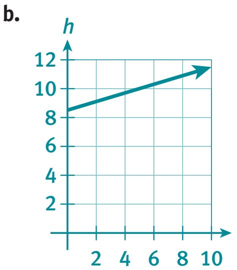
Check Your Understanding

Debrief this lesson by having students explain how to use the given equation, including how to determine a reasonable domain for the situation. Model and encourage correct use of mathematical vocabulary as students describe their graphs.

Answers

14. a. 14.75 inches; 21 inches

b.



The graph is a line.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

ADAPT

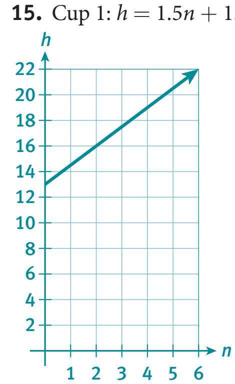
Check students' answers to the Lesson Practice to assess whether they can write and graph an equation that represents a set of linear data. If students have difficulty writing the equations, have them describe each pattern verbally. Then help them translate their words into an equation.

Differentiating Learning

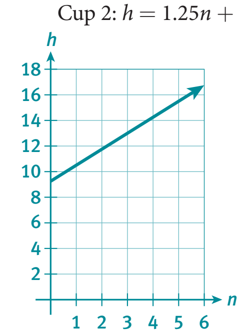
Extend the lesson to make a connection to arithmetic sequences. The height equations could be expressed as $a_n = d(n-1) + a_1$. Using sequences is appropriate here because the domain is the set of counting numbers.

LESSON 10-3 PRACTICE

15. Cup 1: $h = 1.5n + 13$



Cup 2: $h = 1.25n + 9.25$



Both graphs are lines.

16. \$1,875

Lesson 10-4

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–4 #5–8
Check Your Understanding
#12–13
Example A #14–16 #17–18
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Have students consider the concept of an opposite or inverse by having them tell how to undo each of the following:

- climb up 3 stairs
- travel east 4 miles
- put on socks, then put on shoes
- close the door, then lock the door

Be sure students note that in the compound actions, they not only use opposite actions, but also reverse the order of the actions.

1–4 Visualization, Create Representations, Activating Prior Knowledge, Think-Pair-Share, Sharing and Responding These items provide a good opportunity for formative assessment. It is important that students are able to interpret the constants 12.5 and 0.5 as the height of one cup minus the difference and 0.5 as the difference to give a cup height of 13 cm.

Lesson 10-4
Inverse Functions

My Notes

Learning Targets:
- Write the inverse function for a linear function.
- Determine the domain and range of an inverse function.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share, Discussion Groups, Construct an Argument

After reading your report, your supervisor was able to determine the equation for the height of the stack for the specific cup that the company will manufacture. The company will use the function $S(n) = 0.5n + 12.5$.

1. What do S, n, and $S(n)$ represent?
S is the height of a stack, n is the number of cups, and $S(n)$ is the height of a stack that has n cups.

2. What do the numbers 0.5 and the 12.5 in the function S tell you about the physical features of the cup?
12.5 is the height of one cup minus the difference, and 0.5 is the height increase when a new cup is added to the stack.

3. Evaluate $S(1)$ to find the height of a single cup.
$S(1) = 0.5(1) + 12.5 = 13$ cm

4. How tall is a stack of 35 cups? Show your work using function notation.
$S(35) = 0.5(35) + 12.5 = 30$ cm

5. If you add 2 cups to a stack, by how much does the height of the stack increase?
The height increases by 1 cm.

6. If you add 20 cups to a stack, by how much does the height of the stack increase?
The height increases by 10 cm.

7. **Critique the reasoning of others.** A member of one of the teams stated: "If you double the number of cups in a stack, then the height of the stack is also doubled." Is this statement correct? Explain.
Answers may vary. The height of 1 cup is 13 cm and the height of a stack of 2 cups is 13.5 cm. The height of a stack of 3 cups is 14 cm and the height of a stack of 6 cups is 15.5 cm. These two examples show that doubling the number of cups does not double the height of the stack.

Lesson 10-4
Inverse Functions

My Notes

8. If you were to graph the function $S(n) = 0.5n + 12.5$, you would see that the points lie on a line.

a. What is the slope of this line?
0.5

b. Interpret the slope of the line as a rate of change that relates a change in height to a change in the number of cups.
Each additional cup adds 0.5 cm to the height of the stack.

Check Your Understanding

Use this table for Items 9 and 10.

x	1	2	3	4	5
y	1	5	9	13	17

9. Write an equation for y in terms of x.

10. Explain how the numbers in your equation relate to the numbers in the table.

11. Evaluate the function you wrote in Item 9 for each of the following values of x.

a. $x = 8$ **b.** $x = 12$ **c.** $x = 15$ **d.** $x = 0$

12. a. The supervisor wanted to increase the height of a container by 5 cm. How many more cups would fit in the container?
16 cups

b. If the supervisor wanted to increase the height of a container by 6.4 cm, how many more cups would fit in the container?
The container would fit 12 more cups, which would use 6 cm of the additional 6.4 cm.

c. How many cups fit in a container that is 36 cm tall?
47 cups; $0.5n + 12.5 = 36$ when $n = 47$

d. How many cups fit in a container that is 50 cm tall?
75 cups; $0.5n + 12.5 = 50$ when $n = 75$

5–8 Discussion Groups, Look for a Pattern, Construct an Argument, Debriefing These items address slope as a rate of change, $\frac{\Delta\text{height}}{\Delta\text{cups}}$. The implication is that for each cup added, the height increases by 0.5 cm, or that for each two cups added, the height increases 1 cm. The most direct way for students to address Item 7 is by using a counterexample. Some students may use words or pictures. For example, a diagram of cups stacked on top, but not inside of, each other is a powerful argument. Debrief by allowing students to discuss responses to Item 7, using manipulatives as models if necessary.

Check Your Understanding
Debrief students' answers to make sure they know how to write a linear equation from a table of coordinates of points on a line and how to use the equation to find y-values when given x-values. This portion of the lesson prepares students to extend their understanding of functions to include the inverse of functions.

Answers

9. $y = 4x - 3$

10. Answers may vary. Sample answer: The y-values increase by 4 as the x-values increase by 1, so the coefficient of x in the equation is 4. Comparing the x-value to the corresponding y-value, the only expression in terms of x that gives y is $4x - 3$.

11. a. $y = 29$
b. $y = 45$
c. $y = 57$
d. $y = 3$

12–13 Discussion Groups, Create Representations, Work Backward, Debriefing Students may use proportions to solve using the rate of change $\frac{\Delta\text{height}}{\Delta\text{cups}} = \frac{1}{2}$. In 12b, the numerical answer comes to 12.8. The number of cups must be an integer, so the maximum number of cups is 12. Some students may be tempted to round 12.8 to 13. Be sure students understand that 13 cups would exceed the height in question.

12–13 (continued) In Item 13, some students may immediately recognize that solving the equation for n is all that is necessary. Others may find the equation of the line by using ordered pairs of the form (S, n). What students may not realize is that they are finding the inverse of the original function. In the inverse function, the input values are heights S and output values are numbers of cups n. Debrief these items by asking students to discuss what it means to solve an equation for a variable. Have students discuss the advantages and disadvantages of solving the equation for the dependent variable before trying to find the independent variable. Ask students to compare the slope of the new equation to the slope of the original equation.

Developing Math Language

The concept of an *inverse function* is important in algebra. Have students highlight it and then add it to their math notebooks. As you add it to the classroom Word Wall, have students explain it in their own words. You might note that the function $f(x)$ is useful when you know a value of x and want to find the corresponding value of y. The inverse function $f^{-1}(x)$ is useful when you know a value of y and want to find the corresponding value of x. Note that $f^{-1}(x)$ means the inverse of $f(x)$, not $\frac{1}{f(x)}$.

Example A Note Taking Observe that it is not difficult to find the inverse of a function, but that there are several steps that must be completed in order. Use the Try These to have students demonstrate how to complete each step in the context of an actual function. Ask students to explain the process to their shoulder partner before moving on.

13. The function $S(n) = 0.5n + 12.5$ describes the height S in terms of the number of cups n.

a. Solve this equation for n to describe the number of cups n in terms of the height S.

$n = 2S - 25$

b. How many cups fit in a carton that is 85 cm tall? Compare your method of answering this question to your method used in Items 12c and 12d.

$n = 2(85) - 25 = 145$. Comparisons may vary.

c. What is the slope of the line represented by your equation in Part (a)? Interpret it as a rate of change and compare it to the rate of change found in Item 8b.

The slope is $2 = \frac{2}{1} = \frac{\Delta \text{ cups}}{\Delta \text{ height}}$. For each additional 1 cm of height, there are two additional cups.

READING MATH

$f^{-1}(x)$ is read as "f inverse of x". It does **not** mean "f to the negative one power."

An ***inverse function*** is a function that interchanges the independent and dependent variables of another function. In Item 13, you found the inverse function for $S(n)$. In general, the inverse function for $f(x)$ is $f^{-1}(x)$.

Example A

Use the table below to fill in the steps to find the inverse function for $f(x) = 2x + 3$.

Write the function, replacing $f(x)$ with y.	$y = 2x + 3$
Switch x and y.	$x = 2y + 3$
Solve for y in terms of x.	$x - 3 = 2y$ $\frac{x-3}{2} = y$ or $y = \frac{1}{2}x - \frac{3}{2}$
Replace y with $f^{-1}(x)$.	$f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

Try These A

Determine the inverses of each of the following of functions.

a. $f(x) = -4x - 5$ $f^{-1}(x) = -\frac{1}{4}x - \frac{5}{4}$

b. $f(x) = \frac{2}{3}x + 2$ $f^{-1}(x) = \frac{3}{2}x - 3$

c. $f(x) = -\frac{1}{2}x + 4$ $f^{-1}(x) = -2x + 8$

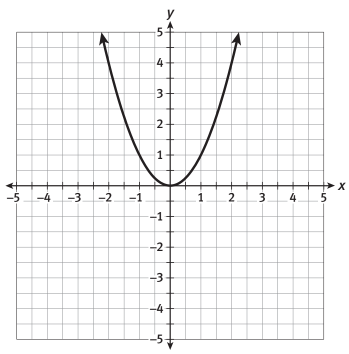
Lesson 10-4
Inverse Functions

Only those functions that are ***one-to-one*** functions have an inverse function. Functions that are not one-to-one must have their domain restricted for an inverse function to exist.

14. Is $S(n) = 0.5n + 12.5$ a one-to-one function? Explain.
Yes; each value of *n* is paired with a different value of *S*(*n*).

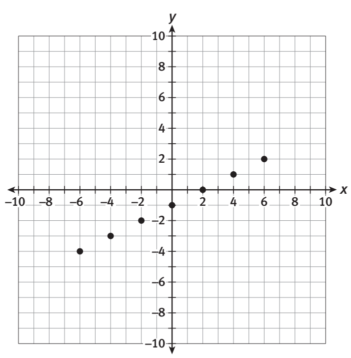
15. Do the following graphs of functions show one-to-one functions? Justify your answers.

a.



No; there are points such as (2, 4) and (−2, 4) where two different *x*-values are paired with the same *y*-value.

b.



Yes; each value of *x* is paired with a different value of *y*.

A visual test for a one-to-one function is the horizontal line test. If you can draw a horizontal line that intersects the graph of a function in more than one place, that function is not one-to-one.

16. Construct viable arguments. Are linear functions one-to-one functions? Justify your response.
Most linear functions are one-to-one; their graphs pass the horizontal line test. However, linear functions whose graphs are horizontal lines do not pass the horizontal line test and are therefore not one-to-one.

My Notes

MATH TERMS

For a function to be **one-to-one** means that no two values of *x* are paired with the same value of *y*.

ACTIVITY 10 Continued

14–16 Shared Reading, Marking the Text, Interactive Word Wall, Discussion Groups, Visualization, Construct an Argument, Debriefing Students should be able to understand the difference between one-to-one functions and other functions both symbolically and graphically. Relate the horizontal line test for a one-to-one function to the vertical line test for a function.

Developing Math Language

Review the definition of a function. Have student volunteers give examples and nonexamples of a function using ordered pairs or mappings. Extend the concept of a function to include a *one-to-one function*. Say that for all functions, each *x* can be paired with only one *y*. If, in addition to that, each *y* is paired with exactly one *x*, it is a one-to-one function. Use mappings to illustrate examples and nonexamples. Then discuss how to tell from a list of ordered pairs or a graph whether or not the function is a one-to-one function.

17–18 Marking the Text, Activating Prior Knowledge , Think-Pair-Share
Discuss the fact that the domain and range of a function are reversed for its inverse function. Ask students to explain in their own words why this makes sense.

Check Your Understanding
As you debrief this lesson, model and encourage the use of mathematical language. Discuss when the function is most useful to solve a problem and when the inverse function is most useful.

Answers

19. \$18.50

20. domain = {all real numbers greater than 0}, range = {all real numbers greater than 3.5}

21. $f^{-1}(x) = \frac{x - 3.5}{2.5}$; domain = {all real numbers greater than 3.5}, range = {all real numbers greater than 0}

22. x represents cost of the cab ride.

23. $f^{-1}(46) = \frac{46 - 3.5}{2.5} = 17$ miles

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 10-4 PRACTICE

24. $f^{-1}(x) = \frac{x + 5}{3}$

25. $f^{-1}(x) = -\frac{1}{2}x - 5$

26. $f^{-1}(x) = \frac{3}{7}x + \frac{1}{14}$

ADAPT

If students have difficulty writing inverse functions, review the steps presented in the lesson while making visual connections to graphs, tables and mappings. Assign additional practice as needed.

My Notes

17. A function is defined by the ordered pairs {(−3, −1), (−1, 0), (1, 1), (3, 2), (5, 3)}. What are the domain and range of the function?
Domain = {−3, −1, 1, 3, 5}; Range = {−1, 0, 1, 2, 3}

Because inputs and outputs are switched when writing the inverse of a function, the domain of a function is the range of its inverse function, and the range of a function is the domain of its inverse function.

18. What are the domain and range of the inverse function for the function in Item 17?
Domain = {−1, 0, 1, 2, 3}; Range = {−3, −1, 1, 3, 5}

Check Your Understanding

The function $f(x) = 2.5x + 3.5$ gives the cost $f(x)$ of a cab ride of x miles.

19. What is the cost of a 6-mile ride?

20. What are the reasonable domain and range of the function?

21. Write the inverse function, $f^{-1}(x)$. What are the domain and range of $f^{-1}(x)$?

22. What does x represent in the inverse function?

23. A cab ride costs \$46. Show how to use the inverse function to find the distance of the cab ride in miles.

LESSON 10-4 PRACTICE

Make use of structure. Find the inverse function, $f^{-1}(x)$, for the functions in Items 24–26.

24. $f(x) = 3x - 5$

25. $f(x) = -2x + 10$

26. $f(x) = \frac{7x}{3} - \frac{1}{6}$

27. The yearly membership fee for the Art Museum is \$75. After paying the membership fee, the cost to enter each exhibit is \$7.50.

a. Write a function for the total cost of a member for one year of attending the art museum.
b. What is the total cost for a member who sees 12 exhibits?
c. What are the domain and range for the function?
d. What is $f^{-1}(x)$? What are the domain and range for $f^{-1}(x)$?
e. What does x represent in $f^{-1}(x)$?
f. How many exhibits can a member see in a year for a total of \$210, including the membership fee?

27. a. $f(x) = 75 + 7.5x$
b. \$165
c. Domain is the natural numbers; range is real numbers greater than or equal to 75.
d. $f^{-1}(x) = \frac{2}{15}x - 10$; Domain is real numbers greater than or equal to 75, and the range is the natural numbers.
e. x represents the total cost for a member for one year.
f. 18 exhibits

ACTIVITY 10 PRACTICE

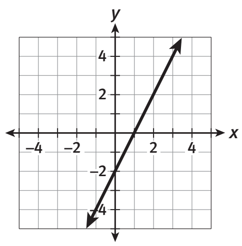
Write your answers on notebook paper.
Show your work.

Lesson 10-1

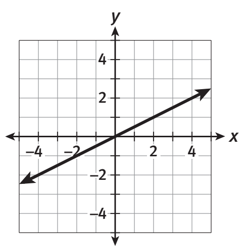
1. The value of y varies directly as x and $y = 125$ when $x = 25$. What is the value of y when $x = 2$?

2. Which is the graph of a direct variation?

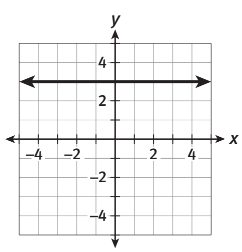
A.



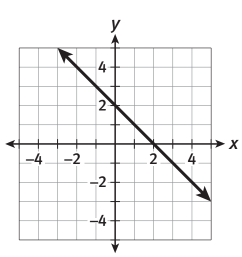
B.



C.



D.



3. Which equation **does not** represent a direct variation?
   A. $y = \frac{x}{3}$
   B. $y = \frac{2}{5}x$
   C. $y = \frac{3}{x}$
   D. $y = \frac{5x}{2}$

4. The value of y varies directly as x and $y = 9$ when $x = 6$. What is the value of y when $x = 15$?

5. The tailor determines that the cost of material varies directly with the amount of material. The cost is \$42 for 14 yards of material. What is the cost for 70 yards of material?

Lesson 10-2

6. The value of y varies indirectly as x and $y = 4$ when $x = 20$. What is the value of y when $x = 40$?
   A. $y = 2$
   B. $y = 8$
   C. $y = 50$
   D. $y = 80$

7. The temperature varies indirectly as the distance from the city. The temperature equals 3°C when the distance from the city is 40 miles. What is the temperature when the distance is 20 miles from the city?

8. The amount of gas left in the gas tank of a car varies indirectly to the number of miles driven. There are 9 gallons of gas left after 24 miles. How much gas is left after the car is driven 120 miles?

ACTIVITY 10 Continued

ACTIVITY PRACTICE

1. 10
2. B
3. C
4. 22.5
5. \$210
6. A
7. 6°
8. 1.8 gallons

9. 3
10. 33
11. Possible answer: Direct; Explanations may vary. If the numbers 0 through 11 are assigned to January through December, the relationship can be modeled by $y = 3x$, a direct variation. If, however, the numbers 1 through 12 are assigned to the months, then the equation is $y = 3x + 3$, which is not a direct or indirect variation.
12. 3; 3 new stores open per month.
13. $h(n) = 3n + 27$
14. 11 chairs; Justifications may vary. 5 feet is equal to 60 inches. A stack of 11 chairs will be $3(11) + 27 =$ 60 inches tall.
15. $f^{-1}(x) = -\frac{1}{8}x + \frac{1}{2}$
16. $f^{-1}(x) = 4x + 12$
17. $f^{-1}(x) = \frac{1}{8}x + \frac{15}{8}$
18. $f^{-1}(x) = x - 1$
19. $f^{-1}(x) = -x + 1$
20. 212°F
21. $\frac{9}{5}$
22. 10°
23. $C = \frac{5}{9}(F - 32)$
24. Answers may vary. Working backwards and using an inverse function are similar in that you are starting with what is usually the output of a problem and finding the input. They differ in that working backwards undoes the problem using the output and undoing the operations. Inverse functions are equations that have already swapped the input and output and are solved as any other equation.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 10-3

The Pete's Pets chain of pet stores is growing. The table below shows the number of stores in business each month. Use the table for Items 9–12.

Month	Stores
January	0
February	3
March	6
April	9
May	12
June	15

9. According to the table, how many new stores open per month?
10. How many stores will be in business by December?
11. Are the Pete's Pets data an example of indirect variation, direct variation, or neither? Explain your reasoning.
12. What is the slope of this function? Interpret the meaning of the slope.

Jeremy collected the following data on stacking chairs. Use the data for Items 13 and 14.

Number of Chairs	Height (in.)
1	30
2	33
3	36
4	39
5	42
6	45

13. Write a linear function that models the data.
14. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack? Justify your answer.

Lesson 10-4

Write the inverse function for each of the following.

15. $f(x) = -8x + 4$
16. $f(x) = \frac{1}{4}x - 3$
17. $f(x) = 8x - 15$
18. $f(x) = x + 1$
19. $f(x) = -x + 1$

The formula to convert degrees Celsius C to degrees Fahrenheit F is $F = \frac{9}{5}C + 32$. Use this formula for Items 20–23.

20. Use the formula to convert 100°C to degrees Fahrenheit.
21. What is the slope?
22. The temperature is 50°F. What is the temperature in degrees Celsius?
23. Solve for C to derive the formula that converts degrees Fahrenheit to degrees Celsius.

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

24. Describe the similarities and differences between finding the inverse of a function and working backward to solve a problem.

Linear Models

Stacking Boxes

Lesson 10-1 Direct Variation

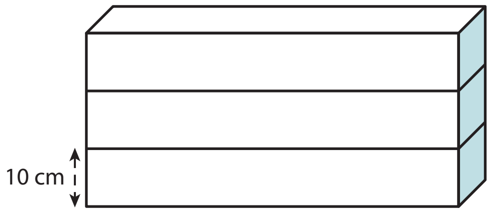
My Notes

Learning Targets:

- Write and graph direct variation.
- Identify the constant of variation.

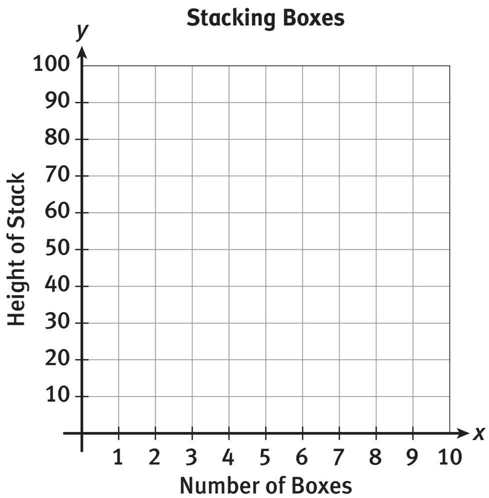
SUGGESTED LEARNING STRATEGIES: Create Representations, Interactive Word Wall, Marking the Text, Sharing and Responding, Discussion Groups

You work for a packaging and shipping company. As part of your job there, you are part of a package design team deciding how to stack boxes for packaging and shipping. Each box is 10 cm high.



1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

Number of Boxes	Height of the Stack (cm)
0	0
1	10
2	20
3	
4	
5	
6	
7	



2. Write a function to represent the data in the table and graph above.

3. What is a reasonable and realistic domain for the function? Explain.

4. What is a reasonable and realistic range for the function? Explain.

WRITING MATH

Either y or $f(x)$ can be used to represent the output of a function.

My Notes

MATH TERMS

A **direct proportion** is a relationship in which the ratio of one quantity to another remains constant.

5. What do $f(x)$, or y, and x represent in your equation from Item 2?

6. Describe any patterns that you notice in the table and graph representing your function.

7. The number of boxes is ***directly proportional*** to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.

When two values are directly proportional, there is a ***direct variation***. In terms of stacking boxes, the height of the stack *varies directly* as the number of boxes.

8. Using variables x and y to represent the two values, you can say that y varies directly as x. Use your answer to Item 6 to explain this statement.

9. Direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient k is the ***constant of variation***.

a. Consider your answer to Item 2. What is the constant of variation in your function?

b. Why do you think the coefficient is called the constant of variation?

c. **Reason quantitatively.** Explain why the value of k cannot be equal to 0.

d. Write an equation for finding the constant of variation by solving the equation $y = kx$ for k.

My Notes

10. a. Interpret the meaning of the point (0, 0) in your table and graph.

b. True or False? Explain your answer. "The graphs of all direct variations are lines that pass through the point (0, 0)."

c. Identify the slope and y-intercept in the graph of the stacking boxes.

d. Describe the relationship between the constant of variation and the slope.

Direct variation can be used to answer questions about stacking and shipping your boxes.

11. The height y of a different stack of boxes varies directly as the number of boxes x. For this type of box, 25 boxes are 500 cm high.

a. Find the value of k. Explain how you found your answer.

b. Write a direct variation equation that relates y, the height of the stack, to x, the number of boxes in the stack.

c. How high is a stack of 20 boxes? Explain how you would use your direct variation equation to find the height of the stack.

My Notes

12. At the packaging and shipping company, you get paid each week. One week you earned \$48 for 8 hours of work. Another week you earned \$30 for 5 hours of work.

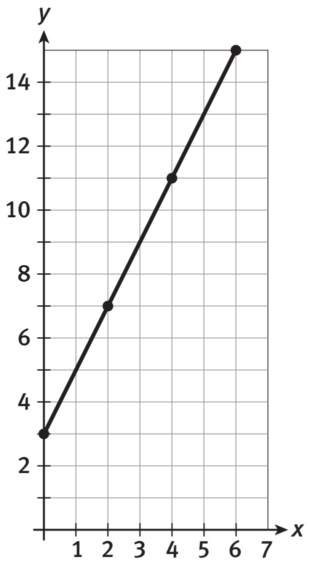
a. Write a direct variation equation that relates your wages to the number of hours you worked each week. Explain the meaning of each variable and identify the constant of variation.

b. How much would you earn if you worked 3.5 hours in one week?

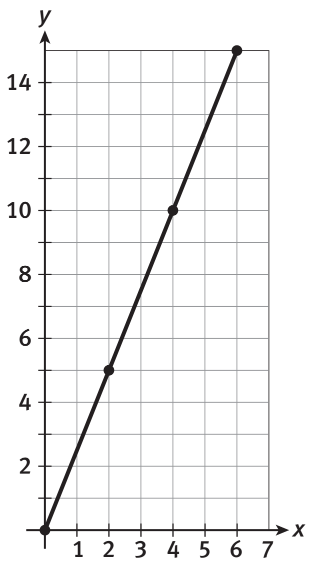
Check Your Understanding

13. Tell whether the tables, graphs, and equations below represent direct variations. Justify your answers.

a.



b.



c.

x	y
2	12
4	24
6	36

d.

x	y
2	8
4	12
6	16

e. $y = 20x$

f. $y = 3x + 2$

My Notes

LESSON 10-1 PRACTICE

14. In the equation $y = 15x$, what is the constant of variation?

15. In the equation $y = 8x$, what is the constant of variation?

16. The value of y varies directly with x and the constant of variation is 7. What is the value of x when $y = 63$?

17. The value of y varies directly with x and the constant of variation is 12. What is the value of y when $x = 5$?

18. **Model with mathematics.** The height of a stack of boxes varies directly with the number of boxes. A stack of 12 boxes is 15 feet high. How tall is a stack of 16 boxes?

19. Jan's pay is in direct variation to the hours she works. Jan earns $54 for 12 hours of work. How much will she earn for 18 hours work?

My Notes

MATH TIP

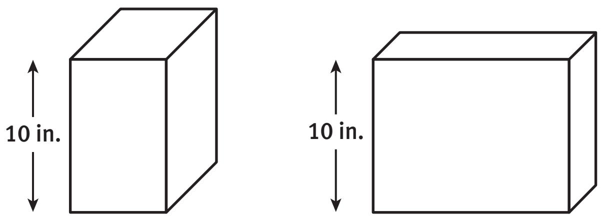
The volume of a rectangular prism is found by multiplying length, width, and height: $V = lwh$.

Learning Targets:

- Write and graph indirect variations.
- Distinguish between direct and indirect variation.

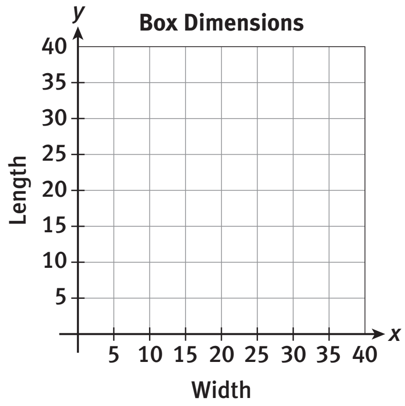
SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Sharing and Responding, Think-Pair-Share, Discussion Groups

When packaging a different product, your team at the packaging and shipping company determines that all boxes for this product will have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.



1. To explore the relationship between length and width, complete the table and make a graph of the points.

Width (x)	Length (y)
1	40
2	20
4	10
5	
8	
10	
20	



2. How are the lengths and widths in Item 1 related? Write an equation that shows this relationship.

3. Use the equation you wrote in Item 2 to write a function to represent the data in the table and graph above.

4. Describe any patterns that you notice in the table and graph representing your function.

My Notes

MATH TIP

Indirect variation is also known as **inverse variation.**

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an ***indirect variation***.

5. Recall that direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient k is the constant of variation.

a. How would you define indirect variation in terms of y, k, and x?

b. Are there any limitations on these variables as there are on k in direct variation? Explain.

c. Write an equation for finding the constant of variation by solving for k in your answer to Part (a).

6. Reason abstractly. Compare and contrast the equations of direct and indirect variation.

7. Compare and contrast the graphs of direct and indirect variation.

8. Use your function in Item 3 to determine the following measurements for your company.

a. Find the length of a box whose width is 80 inches.

b. Find the length of a box whose width is 0.4 inches.

My Notes

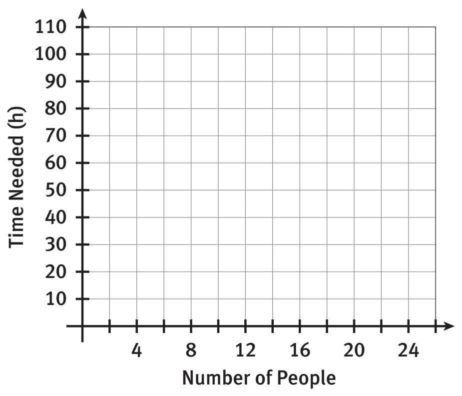
9. The time, y, needed to load the boxes on a truck for shipping varies indirectly as the number of people, x, working. If 10 people work, the job is completed in 20 hours.

a. Explain how to find the constant of variation. Then find it.

b. Write an indirect variation equation that relates the time to load the boxes to the number of people working.

c. How long does it take 8 people to finish loading the boxes? Use your equation to answer this question.

d. On the grid below, make a graph to show the time needed for 2, 4, 5, 8, 10, and 25 people to load the boxes on the truck.



10. The cost for the company to ship the boxes varies indirectly with the number of boxes being shipped. If 25 boxes are shipped at once, it will cost $10 per box. If 50 boxes are shipped at once, the cost will be $5 per box.

a. Write an indirect variation equation that relates the cost per box to the number of boxes being shipped.

b. How much would it cost to ship only 10 boxes?

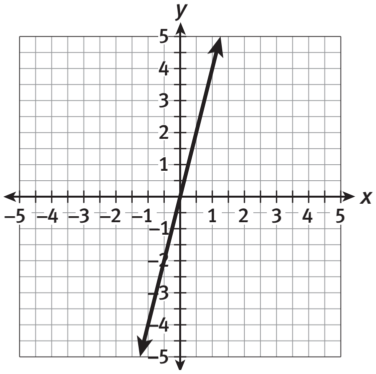
11. Is an indirect variation function a linear function? Explain.

My Notes

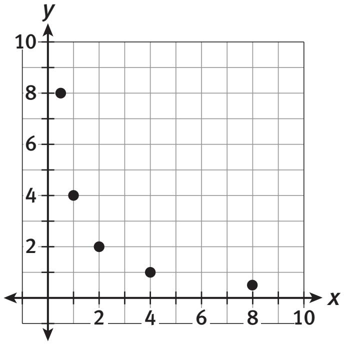
Check Your Understanding

12. Identify the following graphs as direct variation, indirect variation, neither, or both.

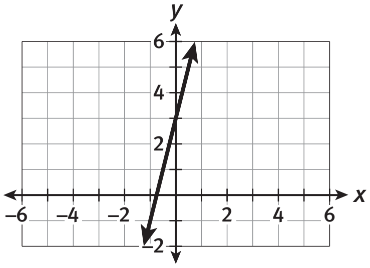
a.



b.



c.



13. Which equations are examples of indirect variation? Justify your answers.

A. $y = 2x$

B. $y = \frac{x}{2}$

C. $y = \frac{2}{x}$

D. $xy = 2$

14. In the equation $y = \frac{80}{x}$, what is the constant of variation?

LESSON 10-2 PRACTICE

15. Graph each function. Identify whether the function is an indirect variation.

a.

x	-4	-2	-1	1	2	4
y	$-\frac{3}{4}$	$-\frac{3}{2}$	-3	3	$\frac{3}{2}$	$\frac{3}{4}$

b.

x	-4	-2	-1	1	2	4
y	11	5	3	-4	-7	-13

16. Make sense of problems. For Parts (a) and (b) below, y varies indirectly as x.

a. If $y = 6$ when $x = 24$, find y when $x = 16$.

b. If $y = 8$ when $x = 20$, find the value of k.

My Notes

CONNECT TO GEOMETRY

The carton will be a right rectangular prism. A **rectangular prism** is a closed, three-dimensional figure with three pairs of opposite parallel faces that are congruent rectangles.

Learning Targets:

- Write, graph, and analyze a linear model for a real-world situation.
- Interpret aspects of a model in terms of the real-world situation.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Discussion Groups, Create Representations, Guess and Check, Use Manipulatives

Your design team at the packaging and shipping company has been asked to design a cardboard box to use when packaging paper cups for sale. Your supervisor has given you the following requirements.

- All lateral faces of the container must be rectangular.
- The base of the container must be a square, just large enough to accommodate one cup.
- The height of the container must be given as a function of the number of cups the container will hold.
- All measurements must be in centimeters.

To help discover which features of the cup affect the height of the stack, collect data on two types of cups found around the office.

1. **Use appropriate tools strategically.** Use two different types of cups to complete the tables below.

CUP 1	
Number of Cups	Height of Stack
1	
2	
3	
4	
5	
6	

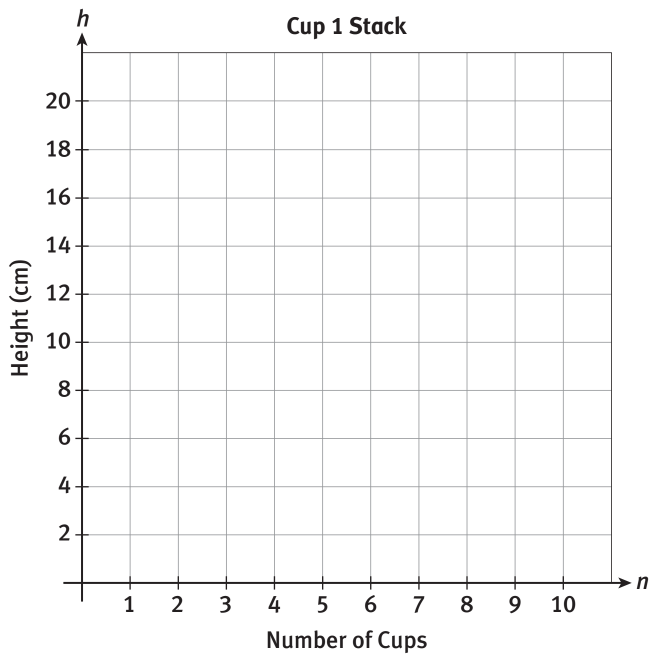
CUP 2	
Number of Cups	Height of Stack
1	
2	
3	
4	
5	
6	

2. **Express regularity in repeated reasoning.** What patterns do you notice that might help you figure out the relationship between the height of the stack and the number of cups in that stack?

My Notes

Use your data for Cup 1 to complete Items 3–13.

3. Make a graph of the data you collected.



4. Predict, without measuring, the height of a stack of 16 cups. Explain how you arrived at your prediction.

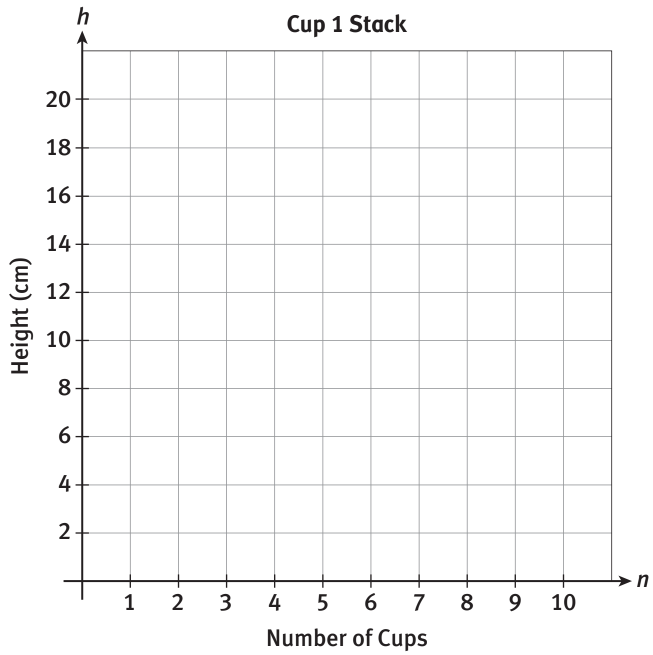
5. Predict, without measuring, the height of a stack of 50 cups. Explain how you arrived at your prediction.

6. Write an equation that gives the height of a stack of cups, h, in terms of n, the number of cups in the stack.

7. Use your equation from Item 6 to find h when $n = 16$ and when $n = 50$. Do your answers to this question agree with your predictions in Items 4 and 5?

My Notes

8. Sketch the graph of your equation from Item 6.



9. How are the graphs you made in Items 3 and 8 the same? How are they different?

10. Do the graphs in Items 3 and 8 represent direct variation, indirect variation, or neither? Explain.

11. Remember that you are designing a container with a square base. What dimension(s), other than the height of the stack, do you need to design your cup container? Use Cup 1 to find this/these dimension(s).

12. Find the dimensions of a container that will hold a stack of 25 cups.

My Notes

13. Your team has been asked to communicate its findings to your supervisor. Write a report to her that summarizes your findings about the cup container design. Include the following information in your report.
- The equation your team discovered to find the height of the stack of Cup 1 style cups
- A description of how your team discovered the equation and the minimum number of cups needed to find it
- An explanation of how the numbers in the equation relate to the physical features of the cup
- An equation that could be used to find the height of the stack of Cup 2 style cups

MATH TIP

When writing your answer to Item 13, you can use a RAFT.

- Role—team leader
- Audience—your boss
- Format—a letter
- Topic—stacks of cups

Check Your Understanding

14. A group of students performed the cup activity described in this lesson. For their Cup 1, they found the equation $h = 0.25n + 8.5$, where h is the height in inches of a stack of cups and n is the number of cups.

a. What would be the height of 25 cups? Of 50 cups?

b. Graph this equation. Describe your graph.

LESSON 10-3 PRACTICE

15. Reason quantitatively. A group of students performed the cup activity in this lesson using plastic drinking cups. Their data is shown below.

CUP 1	
Number of Cups	Height of Stack
1	14.5 cm
2	16 cm
3	17.5 cm
4	19 cm
5	20.5 cm

CUP 2	
Number of Cups	Height of Stack
1	10.5 cm
2	11.75 cm
3	13 cm
4	14.25 cm
5	15.5 cm

For each cup, write and graph an equation. Describe your graphs.

16. A consultant earns a flat fee of \$75 plus \$50 per hour for a contracted job. The table shows the consultant's earnings for the first four hours she works.

Hours	0	1	2	3	4
Earnings	\$75	\$125	\$175	\$225	\$275

The consultant has a 36-hour contract. How much will she earn?

My Notes

Learning Targets:

- Write the inverse function for a linear function.
- Determine the domain and range of an inverse function.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share, Discussion Groups, Construct an Argument

After reading your report, your supervisor was able to determine the equation for the height of the stack for the specific cup that the company will manufacture. The company will use the function $S(n) = 0.5n + 12.5$.

1. What do S, n, and $S(n)$ represent?

2. What do the numbers 0.5 and the 12.5 in the function S tell you about the physical features of the cup?

3. Evaluate $S(1)$ to find the height of a single cup.

4. How tall is a stack of 35 cups? Show your work using function notation.

5. If you add 2 cups to a stack, by how much does the height of the stack increase?

6. If you add 20 cups to a stack, by how much does the height of the stack increase?

7. **Critique the reasoning of others.** A member of one of the teams stated: "If you double the number of cups in a stack, then the height of the stack is also doubled." Is this statement correct? Explain.

My Notes

8. If you were to graph the function $S(n) = 0.5n + 12.5$, you would see that the points lie on a line.

a. What is the slope of this line?

b. Interpret the slope of the line as a rate of change that relates a change in height to a change in the number of cups.

Check Your Understanding

Use this table for Items 9 and 10.

x	1	2	3	4	5
y	1	5	9	13	17

9. Write an equation for y in terms of x.

10. Explain how the numbers in your equation relate to the numbers in the table.

11. Evaluate the function you wrote in Item 9 for each of the following values of x.

a. $x = 8$ **b.** $x = 12$ **c.** $x = 15$ **d.** $x = 0$

12. a. The supervisor wanted to increase the height of a container by 5 cm. How many more cups would fit in the container?

b. If the supervisor wanted to increase the height of a container by 6.4 cm, how many more cups would fit in the container?

c. How many cups fit in a container that is 36 cm tall?

d. How many cups fit in a container that is 50 cm tall?

My Notes

13. The function $S(n) = 0.5n + 12.5$ describes the height S in terms of the number of cups n.

a. Solve this equation for n to describe the number of cups n in terms of the height S.

b. How many cups fit in a carton that is 85 cm tall? Compare your method of answering this question to your method used in Items 12c and 12d.

c. What is the slope of the line represented by your equation in Part (a)? Interpret it as a rate of change and compare it to the rate of change found in Item 8b.

READING MATH

$f^{-1}(x)$ is read as "f inverse of x". It does **not** mean "f to the negative one power."

An ***inverse function*** is a function that interchanges the independent and dependent variables of another function. In Item 13, you found the inverse function for $S(n)$. In general, the inverse function for $f(x)$ is $f^{-1}(x)$.

Example A

Use the table below to fill in the steps to find the inverse function for $f(x) = 2x + 3$.

Write the function, replacing $f(x)$ with y.	
Switch x and y.	
Solve for y in terms of x.	
Replace y with $f^{-1}(x)$.	

Try These A

Determine the inverses of each of the following of functions.

a. $f(x) = -4x - 5$ **b.** $f(x) = \frac{2}{3}x + 2$ **c.** $f(x) = -\frac{1}{2}x + 4$

My Notes

MATH TERMS

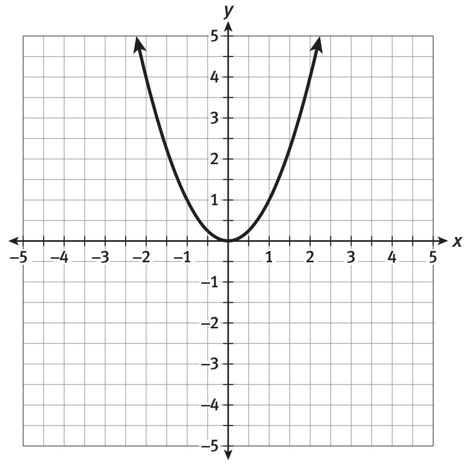
For a function to be **one-to-one** means that no two values of *x* are paired with the same value of *y*.

Only those functions that are ***one-to-one*** functions have an inverse function. Functions that are not one-to-one must have their domain restricted for an inverse function to exist.

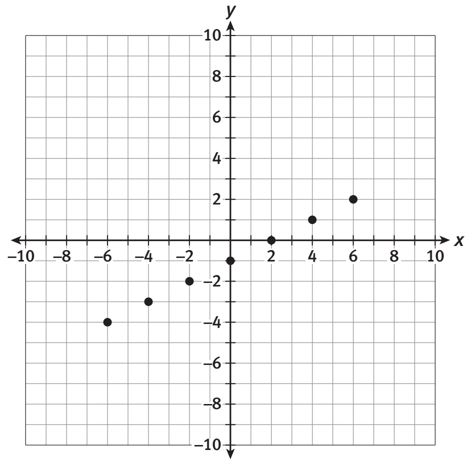
14. Is $S(n) = 0.5n + 12.5$ a one-to-one function? Explain.

15. Do the following graphs of functions show one-to-one functions? Justify your answers.

a.



b.



A visual test for a one-to-one function is the horizontal line test. If you can draw a horizontal line that intersects the graph of a function in more than one place, that function is not one-to-one.

16. Construct viable arguments. Are linear functions one-to-one functions? Justify your response.

My Notes

17. A function is defined by the ordered pairs $\{(-3, -1), (-1, 0), (1, 1), (3, 2), (5, 3)\}$. What are the domain and range of the function?

Because inputs and outputs are switched when writing the inverse of a function, the domain of a function is the range of its inverse function, and the range of a function is the domain of its inverse function.

18. What are the domain and range of the inverse function for the function in Item 17?

Check Your Understanding

The function $f(x) = 2.5x + 3.5$ gives the cost $f(x)$ of a cab ride of x miles.

19. What is the cost of a 6-mile ride?

20. What are the reasonable domain and range of the function?

21. Write the inverse function, $f^{-1}(x)$. What are the domain and range of $f^{-1}(x)$?

22. What does x represent in the inverse function?

23. A cab ride costs \$46. Show how to use the inverse function to find the distance of the cab ride in miles.

LESSON 10-4 PRACTICE

Make use of structure. Find the inverse function, $f^{-1}(x)$, for the functions in Items 24–26.

24. $f(x) = 3x - 5$

25. $f(x) = -2x + 10$

26. $f(x) = \frac{7x}{3} - \frac{1}{6}$

27. The yearly membership fee for the Art Museum is \$75. After paying the membership fee, the cost to enter each exhibit is \$7.50.

a. Write a function for the total cost of a member for one year of attending the art museum.

b. What is the total cost for a member who sees 12 exhibits?

c. What are the domain and range for the function?

d. What is $f^{-1}(x)$? What are the domain and range for $f^{-1}(x)$?

e. What does x represent in $f^{-1}(x)$?

f. How many exhibits can a member see in a year for a total of \$210, including the membership fee?

ACTIVITY 10 PRACTICE

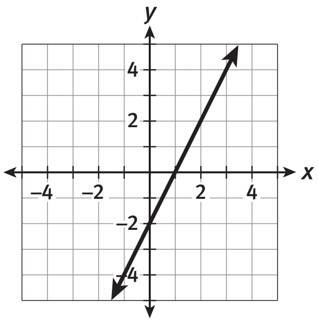
Write your answers on notebook paper.
Show your work.

Lesson 10-1

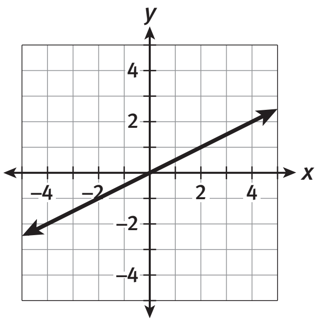
1. The value of y varies directly as x and $y = 125$ when $x = 25$. What is the value of y when $x = 2$?

2. Which is the graph of a direct variation?

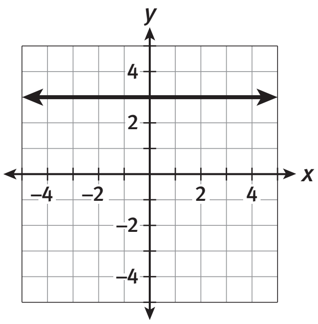
A.



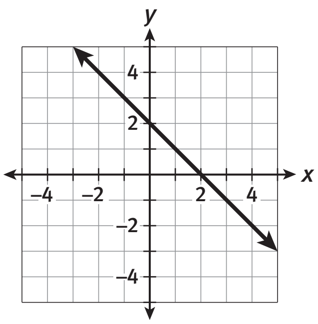
B.



C.



D.



3. Which equation **does not** represent a direct variation?

   A. $y = \frac{x}{3}$

   B. $y = \frac{2}{5}x$

   C. $y = \frac{3}{x}$

   D. $y = \frac{5x}{2}$

4. The value of y varies directly as x and $y = 9$ when $x = 6$. What is the value of y when $x = 15$?

5. The tailor determines that the cost of material varies directly with the amount of material. The cost is \$42 for 14 yards of material. What is the cost for 70 yards of material?

Lesson 10-2

6. The value of y varies indirectly as x and $y = 4$ when $x = 20$. What is the value of y when $x = 40$?

   A. $y = 2$

   B. $y = 8$

   C. $y = 50$

   D. $y = 80$

7. The temperature varies indirectly as the distance from the city. The temperature equals 3°C when the distance from the city is 40 miles. What is the temperature when the distance is 20 miles from the city?

8. The amount of gas left in the gas tank of a car varies indirectly to the number of miles driven. There are 9 gallons of gas left after 24 miles. How much gas is left after the car is driven 120 miles?

Lesson 10-3

The Pete's Pets chain of pet stores is growing. The table below shows the number of stores in business each month. Use the table for Items 9–12.

Month	Stores
January	0
February	3
March	6
April	9
May	12
June	15

9. According to the table, how many new stores open per month?

10. How many stores will be in business by December?

11. Are the Pete's Pets data an example of indirect variation, direct variation, or neither? Explain your reasoning.

12. What is the slope of this function? Interpret the meaning of the slope.

Jeremy collected the following data on stacking chairs. Use the data for Items 13 and 14.

Number of Chairs	Height (in.)
1	30
2	33
3	36
4	39
5	42
6	45

13. Write a linear function that models the data.

14. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack? Justify your answer.

Lesson 10-4

Write the inverse function for each of the following.

15. $f(x) = -8x + 4$

16. $f(x) = \frac{1}{4}x - 3$

17. $f(x) = 8x - 15$

18. $f(x) = x + 1$

19. $f(x) = -x + 1$

The formula to convert degrees Celsius C to degrees Fahrenheit F is $F = \frac{9}{5}C + 32$. Use this formula for Items 20–23.

20. Use the formula to convert 100°C to degrees Fahrenheit.

21. What is the slope?

22. The temperature is 50°F. What is the temperature in degrees Celsius?

23. Solve for C to derive the formula that converts degrees Fahrenheit to degrees Celsius.

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

24. Describe the similarities and differences between finding the inverse of a function and working backward to solve a problem.