Writing Quadratic Equations

What Goes Up Must Come Down
Lesson 10-1 Parabolas and Quadratic Equations
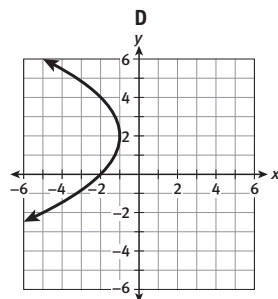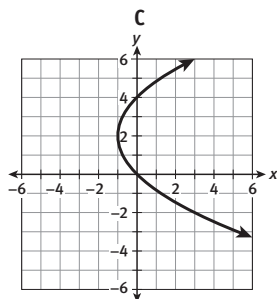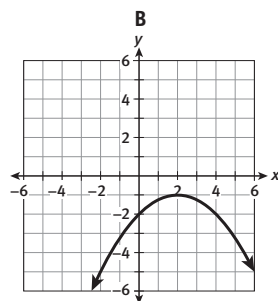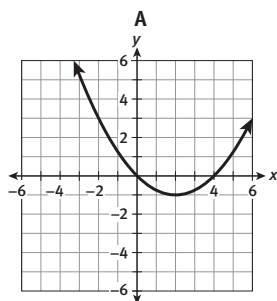
Learning Targets:
- Derive a general equation for a parabola based on the definition of a parabola.
- Write the equation of a parabola given a graph and key features.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Discussion Groups, Interactive Word Wall, Create Representations, Close Reading

Take a look at the graphs shown below.

A

B

C

D

1. **Make use of structure.** Match each equation with one of the graphs above.

$x = \frac{1}{4}(y - 2)^2 - 1$ $y = \frac{1}{4}(x - 2)^2 - 1$

C   **A**

$y = -\frac{1}{4}(x - 2)^2 - 1$ $x = -\frac{1}{4}(y - 2)^2 - 1$

B   **D**

My Notes

Activity Standards Focus

In Activity 10, students write equations of parabolas given a graph or key features of the parabola. They determine a quadratic function given three points on a plane that the function passes through. They also find a quadratic model for a given set of data values and use the model to make predictions about the data. Throughout this activity, emphasize the definition of a parabola and how the equation of a parabola relates to a quadratic function.

Lesson 10-1

PLAN

Pacing: 1 class period
Chunking the Lesson

#1–3   #4–6

Check Your Understanding

#10–11   #12–13   #14–17

Check Your Understanding

#21

Check Your Understanding
Lesson Practice

TEACH

📷 **Bell-Ringer Activity**

Have students make a table of values for the equations $y = 2x$, $y = 2x^2$, and $y = 2x^3$ using domain values of $-3, -2, -1, 0, 1, 2, 3$. Then have them graph the equations.

1–3 Think-Pair-Share, Critique Reasoning Students will likely use several different methods to match the graphs to the equations. After students have shared their methods, ask students to determine the most efficient method.

Common Core State Standards for Activity 10

HSA-CED.A.1   Create equations and inequalities in one variable and use them to solve problems.

HSA-CED.A.2   Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4–6 Activating Prior Knowledge, Graphic Organizer When comparing the graphs of A and B to the graphs of C and D, students may note that A and B are functions while C and D are not. While this is true, it is important that students note that the orientations of the axes of symmetry are different. The notion of functions will be covered in Item 8.

TEACHER to TEACHER

Parabolas are defined both geometrically and algebraically. Geometrically, a parabola is a conic section and meets the geometric criteria set forth in this activity. Algebraically, a parabola is the graph of any quadratic equation.

Technology Tip

Point out that splitting an equation that includes the $\pm$ symbol results in two separate equations that are reflections of each other. Have students enter each equation separately. For additional technology resources, visit SpringBoard Digital.

My Notes

TECHNOLOGY TIP

If an equation includes the $\pm$ symbol, you will need to enter it in a graphing calculator as two separate equations. For example, enter the equation $y = 2 \pm \sqrt{x}$ as $y = 2 + \sqrt{x}$ and $y = 2 - \sqrt{x}$.

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2. Explain how you matched each equation with one of the graphs.
Sample answer: A is the only graph that includes (4, 0), B is the only graph that includes (0, −2), C is the only graph that includes (0, 4), and D is the only graph that includes (−2, 0). Use substitution to determine which of these ordered pairs is a solution of each equation.

3. **Use appropriate tools strategically.** Use a graphing calculator to confirm your answers to Item 1. Which equations must be rewritten to enter them in the calculator? Rewrite any equations from Item 1 as necessary so that you can use them with your calculator.
Rewrite $x = \frac{1}{4}(y-2)^2 - 1$ as $y = 2 \pm 2\sqrt{x+1}$;
rewrite $x = -\frac{1}{4}(y-2)^2 - 1$ as $y = 2 \pm 2\sqrt{-x-1}$;
check students' calculator graphs.

4. a. How do graphs A and B differ from graphs C and D?
Sample answer: A and B are parabolas that open up or down. They are symmetric about a vertical line. C and D are parabolas that open right or left. They are symmetric about a horizontal line.

b. How do the equations of graphs A and B differ from the equations of graphs C and D?
Sample answer: The equations for A and B are solved for y, and the expression equal to y is a quadratic expression in terms of x. The equations for C and D are solved for x, and the expression equal to x is a quadratic expression in terms of y.

Lesson 10-1
Parabolas and Quadratic Equations

ACTIVITY 10
continued

My Notes

5. Work with your group. Consider graphs A and B and their equations.
  a. Describe the relationship between the graphs.
  Sample answer: The graphs are reflections of each other across the line $y = -1$.

  b. What part of the equation determines whether the graph opens up or down? How do you know?
  The equations are identical except for the sign of $\frac{1}{4}$, so the sign of this number determines whether the graph opens up or down. If the sign is positive, the graph opens up; if it is negative, the graph opens down.

  c. **Attend to precision.** What are the coordinates of the lowest point on graph A? What are the coordinates of the highest point on graph B? How do the coordinates of these points relate to the equations of the graphs?
  A: $(2, -1)$; B: $(2, -1)$; The x-coordinate is the number subtracted from x inside the parentheses. The y-coordinate is the number added outside the parentheses.

6. Continue to work with your group. Consider graphs C and D and their equations.
  a. Describe the relationship between the graphs.
  Sample answer: The graphs are reflections of each other across the line $x = -1$.

  b. What part of the equation determines whether the graph opens to the right or left? How do you know?
  The equations are identical except for the sign of $\frac{1}{4}$, so the sign of this number determines whether the graph opens to the right or left. If the sign is positive, the graph opens to the right; if it is negative, the graph opens to the left.

  c. What are the coordinates of the leftmost point on graph C? What are the coordinates of the rightmost point on graph D? How do the coordinates of these points relate to the equations of the graphs?
  C: $(-1, 2)$; D: $(-1, 2)$; The x-coordinate is the number added outside the parentheses. The y-coordinate is the number subtracted from y inside the parentheses.

DISCUSSION GROUP TIP

As you share ideas for Items 5 and 6 in your group, ask your group members or your teacher for clarification of any language, terms, or concepts that you do not understand.

MATH TIP

A graph is said to open upward when both ends of the graph point up. A graph is said to open downward when both ends of the graph point down.

The vertex of a graph that opens upward is the *minimum* of the graph, and is its lowest point. The vertex of a graph that opens downward is the *maximum* of the graph, and is its highest point.

ACTIVITY 10 Continued

4–6 (continued) Help students create a graphic organizer to summarize the connections between the equation and the graph of a parabola. There should be three levels of organization: What is the orientation of the parabola? Which specific direction does the parabola open? What is the vertex of the parabola?

Differentiating Instruction

Challenge students to determine why a parabola with a negative x^2 coefficient opens down and one with a positive x^2 coefficient opens up. Have students create a table of values in which the domain values tend toward infinity and negative infinity for the equations $y = -2x^2$ and $y = 2x^2$. As students create these tables, they should note that squaring always results in a positive value and the lead coefficient will determine the sign of the y-value. They should also note that as domain values tend toward infinity and negative infinity, the y-values will either increase or decrease without bound.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to graphing quadratic equations. Ask students whether the graph in Item 9 represents a function. [*yes*]

Answers

7. B

8. The equations $y = \frac{1}{4}(x-2)^2 - 1$ and $y = -\frac{1}{4}(x-2)^2 - 1$ represent functions because the graphs of these equations show that there is only 1 value of y for each value of x. The equations $x = \frac{1}{4}(y-2)^2 - 1$ and $x = -\frac{1}{4}(y-2)^2 - 1$ are not functions because the graphs of these equations show that there are values of x for which there is more than one value of y.

9. To the left; Sample explanation: The equation is solved for x, and the expression equal to x is a quadratic expression in terms of y, which indicates that the graph opens to the right or to the left. The sign of the number multiplied by the squared quantity is negative, so the graph opens to the left.

Developing Math Language

Have students make four drawings in their notes.

1. a focus and directrix that would result in a parabola that opens up

2. a focus and directrix that would result in a parabola that opens down

3. a focus and directrix that would result in a parabola that opens left

4. a focus and directrix that would result in a parabola that opens right

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My Notes

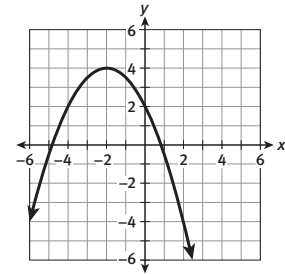## Check Your Understanding

7. Which equation does the graph at right represent? Explain your answer.

A. $y = -\frac{1}{2}(x+2)^2 - 4$

B. $y = -\frac{1}{2}(x+2)^2 + 4$

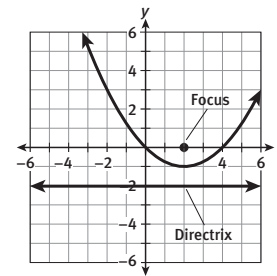**C.** $y = -\frac{1}{2}(x-2)^2 + 4$

8. Construct viable arguments. Which of the equations in Item 1 represent functions? Explain your reasoning.

9. Consider the equation $x = -2(y+4)^2 - 1$. Without graphing the equation, tell which direction its graph opens. Explain your reasoning.

MATH TERMS

A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line.

The fixed point is called the **focus**.

The fixed line is called the **directrix**.

MATH TIP

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

MATH TIP

The distance between a point and a horizontal line is the length of the vertical segment with one endpoint at the point and one endpoint on the line.

The graphs shown at the beginning of this lesson are all parabolas. A *parabola* can be defined as the set of points that are the same distance from a point called the *focus* and a line called the *directrix*.

10. The focus of graph A, shown below, is (2, 0), and the directrix is the horizontal line $y = -2$.

a. The point $(-2, 3)$ is on the parabola. Find the distance between this point and the focus.

distance to focus: $\sqrt{(2 - (-2))^2 + (0 - 3)^2} = 5$

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b. Find the distance between the point $(-2, 3)$ and the directrix.

distance to directrix: $\sqrt{(-2-(-2))^2+(-2-3)^2} = 5; x = 2$

c. Reason quantitatively. Compare your answers in parts a and b. What do you notice?

The point $(-2, 3)$ on the parabola is the same distance from the focus as from the directrix.

11. The focus of graph D, shown below, is $(-2, 2)$, and the directrix is the vertical line $x = 0$.

a. The point $(-2, 4)$ is on the parabola. Show that this point is the same distance from the focus as from the directrix.

distance to focus: $\sqrt{(-2-(-2))^2+(2-4)^2} = 2;$

distance to directrix: $\sqrt{(0-(-2))^2+(4-4)^2} = 2$

b. The point $(-5, -2)$ is also on the parabola. Show that this point is the same distance from the focus as from the directrix.

distance to focus: $\sqrt{(-2-(-5))^2+(2-(-2))^2} = 5;$

distance to directrix: $\sqrt{(0-(-5))^2+(-2-(-2))^2} = 5$

My Notes

MATH TIP

The distance between a point and a vertical line is the length of the horizontal segment with one endpoint at the point and one endpoint on the line.

10–11 Create Representations, Identify a Subtask For students who struggle with either remembering or using the distance formula correctly, remind them that it is possible to determine the distance between two points that fall on a horizontal or vertical line by counting. Also, you can determine the distance between two points that do not fall on a vertical or horizontal line by constructing a right triangle and using the Pythagorean Theorem to determine the hypotenuse of the triangle.

Technology Tip

Use a dynamic mathematics software program such as GeoGebra to create a parabola by selecting a point and a directrix and using the parabola tool. You can then construct points on the parabola and measure the distances between the point and the focus and the point and the directrix.

For additional technology resources, visit SpringBoard Digital.

12–13 Create Representations, Summarizing Students commonly reverse the equations of horizontal and vertical lines. Be sure to emphasize the relationship between the equation of the axis of symmetry and the coordinates of the vertex.

Differentiating Instruction

For students who need a challenge, present them with the following task: Sketch a line through the focus of a parabola that is perpendicular to the axis of symmetry. This line will intersect the parabola in two points, P_1 and P_2. Explain why the distance between P_1 and P_2 is double the distance between the focus and the directrix of the parabola.


MATH TERMS

The **axis of symmetry** is a line that divides the parabola into two congruent halves. The axis of symmetry passes through the focus and is perpendicular to the directrix.

The **vertex** is the point on the parabola that lies on the axis of symmetry. The vertex is the midpoint of the segment connecting the focus and the directrix.

The focus of the parabola shown below is $(-2, -1)$, and the directrix is the line $y = -5$.
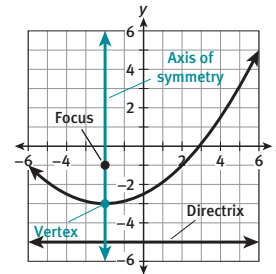
12. a. Draw and label the ***axis of symmetry*** on the graph above. What is the equation of the axis of symmetry?

$x = -2$

b. Explain how you identified the axis of symmetry of the parabola.
Sample answer: The directrix is horizontal, so I drew a vertical line through the focus.

13. a. Draw and label the ***vertex*** on the graph above. What are the coordinates of the vertex?

$(-2, -3)$

b. Explain how you identified the vertex of the parabola.
Sample answer: I drew a point where the axis of symmetry intersects the parabola.

c. What is another way you could have identified the vertex?
Sample answer: I could have drawn a vertical segment from the focus to the directrix. Then I could have drawn a point at the midpoint of this segment.

Lesson 10-1
Parabolas and Quadratic Equations

You can use what you have learned about parabolas to derive a general equation for a parabola whose vertex is located at the origin. Start with a parabola that has a vertical axis of symmetry, a focus of $(0, p)$, and a directrix of $y = -p$. Let $P(x, y)$ represent any point on the parabola.

My Notes
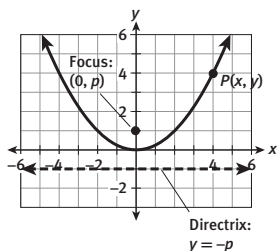
14. Write, but do not simplify, an expression for the distance from point P to the focus.

$\sqrt{(x - 0)^2 + (y - p)^2}$ **or equivalent**

15. Write, but do not simplify, an expression for the distance from point P to the directrix.

$\sqrt{(x - x)^2 + (y - (-p))^2}$ **or equivalent**

16. **Make use of structure.** Based on the definition of a parabola, the distance from point P to the focus is the same as the distance from point P to the directrix. Set your expressions from Items 14 and 15 equal to each other, and then solve for y.

$$\sqrt{(x - 0)^2 + (y - p)^2} = \sqrt{(x - x)^2 + (y - (-p))^2}$$
$$(x - 0)^2 + (y - p)^2 = (x - x)^2 + (y - (-p))^2$$
$$x^2 + (y - p)^2 = (y + p)^2$$
$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$
$$x^2 - 2py = 2py$$
$$x^2 = 4py$$
$$\frac{1}{4p} x^2 = y$$

MATH TIP

In Item 16, start by squaring each side of the equation to eliminate the square root symbols. Next, simplify each side and expand the squared terms.

14–17 Think-Pair-Share, Look for a Pattern Have volunteers share their answers to these items. Some students may choose to simplify the expressions under the radicals prior to squaring both sides. Watch for the common student error of improperly expanding a squared binomial.

Differentiating Instruction

Have students determine how the value of p will affect the shape of the parabola $y = \frac{1}{4p} x^2$. In particular, have students answer these questions: What happens to the shape of the parabola as the value of p increases toward infinity? What happens to the shape of the parabola as the value of p gets closer and closer to zero?

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to the general equation of a parabola.

Answers

18. $x = \dfrac{1}{4p} y^2$. Sample derivation:

distance from P to focus = distance from P to directrix

$\sqrt{(x-p)^2 + (y-0)^2}$

$\quad = \sqrt{(x-(-p))^2 + (y-y)^2}$

$(x-p)^2 + (y-0)^2$

$\quad = (x-(-p))^2 + (y-y)^2$

$(x-p)^2 + y^2 = (x+p)^2$

$x^2 - 2px + p^2 + y^2 = x^2 + 2px + p^2$

$-2px + y^2 = 2px$

$y^2 = 4px$

$\dfrac{1}{4p} y^2 = x$

19. $y = -\dfrac{1}{12} x^2$; The vertex and the focus of the parabola are on the y-axis, so the y-axis is the axis of symmetry. The parabola has its vertex at the origin and a vertical axis of symmetry, so its equation has the form $y = \dfrac{1}{4p} x^2$, where p is the y-coordinate of the focus. The focus is $(0, -3)$, and the equation of the parabola is $y = \dfrac{1}{4(-3)} x^2 = -\dfrac{1}{12} x^2$.

20. **a.** $y = 4$; The directrix is vertical, so the axis of symmetry is a horizontal line through the focus. The focus has a y-coordinate of 4, so the axis of symmetry is the line $y = 4$.

b. $(1, 4)$; The vertex is the midpoint of the segment that connects the focus and the directrix. The endpoints of this segment have coordinates $(3, 4)$ and $(-1, 4)$, so the vertex has coordinates $(1, 4)$.

c. To the right; The axis of symmetry is horizontal and the focus is to the right of the directrix, so the parabola opens to the right.
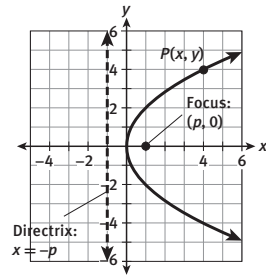

My Notes

17. What is the general equation for a parabola with its vertex at the origin, a focus of $(0, p)$, and a directrix of $y = -p$?

$y = \dfrac{1}{4p} x^2$ **or equivalent**

Check Your Understanding

18. See the diagram at right. Derive the general equation of a parabola with its vertex at the origin, a horizontal axis of symmetry, a focus of $(p, 0)$, and a directrix of $x = -p$. Solve the equation for x.

19. **Model with mathematics.** The vertex of a parabola is at the origin and its focus is $(0, -3)$. What is the equation of the parabola? Explain your reasoning.

20. A parabola has a focus of $(3, 4)$ and a directrix of $x = -1$. Answer each question about the parabola, and explain your reasoning.

a. What is the axis of symmetry?

b. What is the vertex?

c. In which direction does the parabola open?

MATH TIP

A parabola always opens toward the focus and away from the directrix.

You can also write general equations for parabolas that do not have their vertex at the origin. You will derive these equations later in this activity.

	Vertical Axis of Symmetry	Horizontal Axis of Symmetry
Vertex	(h, k)	(h, k)
Focus	$(h, k + p)$	$(h + p, k)$
Directrix	horizontal line $y = k - p$	vertical line $x = h - p$
Equation	$y = \dfrac{1}{4p}(x - h)^2 + k$	$x = \dfrac{1}{4p}(y - k)^2 + h$

Lesson 10-1
Parabolas and Quadratic Equations

My Notes

21. **Reason quantitatively.** Use the given information to write the equation of each parabola.

a. axis of symmetry: $y = 0$; vertex: $(0, 0)$; directrix: $x = \frac{1}{2}$

$$x = -\frac{1}{2}y^2$$

b. vertex: $(3, 4)$; focus: $(3, 6)$

$$y = \frac{1}{8}(x - 3)^2 + 4$$

c. vertex: $(-2, 1)$; directrix: $y = 4$

$$y = -\frac{1}{12}(x + 2)^2 + 1$$

d. focus: $(-4, 0)$; directrix: $x = 4$

$$x = -\frac{1}{16}y^2$$

e. opens up; focus: $(5, 7)$; directrix: $y = 3$

$$y = \frac{1}{8}(x - 5)^2 + 5$$

> **MATH TIP**
>
> You may find it helpful to make a quick sketch of the information you are given.

21 Create Representations, Identify a Subtask, Debriefing Prior to writing the equation of each parabola, ask students to supply the missing information for each parabola. For example, students would determine the vertex and the axis of symmetry for Part d.

> **TEACHER to TEACHER**
>
> Parabolas have many applications in the real world. One application is the use of parabolic reflectors in reflecting telescopes. These parabolic reflectors range in diameter from 3 inches in home telescopes to 200 inches in research telescopes. A parabolic reflector is a paraboloid which is formed by rotating a parabola about its axis of symmetry. Help students to visualize this rotation and the formation of the paraboloid.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to equations of parabolas.
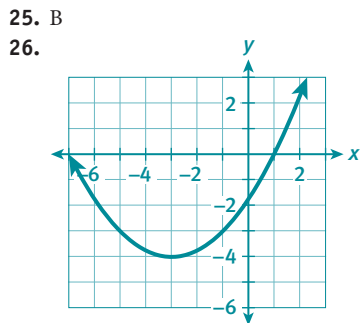
Answers

23. No. Sample explanation: To write the equation of a parabola, you need to know the value of p. To determine the value of p given the vertex, you would also need to know either the focus or the directrix of the parabola.

24. vertex: $(1, 2)$; axis of symmetry: $y = 2$; focus: $(3, 2)$; directrix: $x = -1$

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 10-1 PRACTICE

25. B

26.

27. axis of symmetry: $y = 2$; vertex: $(-3, 2)$; opens to the right

28. $y = -\dfrac{1}{2}x^2$

29. $x = \dfrac{1}{16}y^2$

30. $x = -\dfrac{1}{20}(y - 5)^2$

31. $y = \dfrac{1}{12}(x - 3)^2 - 4$

32. $x = \dfrac{1}{4}(y - 4)^2 - 2$

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand the geometric definition of a parabola, its component parts, and the general form of the equation of a parabola. Students should be able to match graphs to their equations and vice versa. Encourage students who require extra practice to create their own problems using Lesson Practice Items 28–32 as a template. Students can check their own work by graphing the equation they write on a graphing calculator.

My Notes

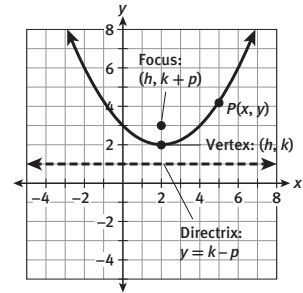### Check Your Understanding

22. See the diagram at right. Derive the general equation of a parabola with its vertex at (h, k), a vertical axis of symmetry, a focus of $(h, k + p)$, and a directrix of $y = k - p$. Solve the equation for y.
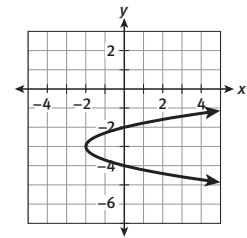
23. **Construct viable arguments.** Can you determine the equation of a parabola if you know only its axis of symmetry and its vertex? Explain.

24. The equation of a parabola is $x = \dfrac{1}{8}(y - 2)^2 + 1$. Identify the vertex, axis of symmetry, focus, and directrix of the parabola.

LESSON 10-1 PRACTICE

25. Which equation does the graph at right represent?
    A. $x = -2(y + 3)^2 - 2$
    B. $x = 2(y + 3)^2 - 2$
    C. $y = -2(x + 3)^2 - 2$
    D. $y = 2(x + 3)^2 - 2$

26. Graph the parabola given by the equation $y = \dfrac{1}{4}(x + 3)^2 - 4$.

27. **Make sense of problems.** The focus of a parabola is $(0, 2)$, and its directrix is the vertical line $x = -6$. Identify the axis of symmetry, the vertex, and the direction the parabola opens.

Use the given information to write the equation of each parabola.

28. vertex: $(0, 0)$; focus: $\left(0, -\dfrac{1}{2}\right)$

29. focus: $(4, 0)$; directrix: $x = -4$

30. opens to the left; vertex: $(0, 5)$; focus: $(-5, 5)$

31. axis of symmetry: $x = 3$; focus: $(3, -1)$; directrix: $y = -7$

32. vertex: $(-2, 4)$; directrix: $x = -3$

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22. Sample derivation:

$$\text{distance from } P \text{ to focus} = \text{distance from } P \text{ to directrix}$$

$$\sqrt{(x - h)^2 + (y - (k + p))^2} = \sqrt{(x - x)^2 + (y - (k - p))^2}$$

$$(x - h)^2 + (y - (k + p))^2 = (x - x)^2 + (y - (k - p))^2$$

$$(x - h)^2 + y^2 - 2(k + p)y + (k + p)^2 = y^2 - 2(k - p)y + (k - p)^2$$

$$(x - h)^2 + y^2 - 2ky - 2py + k^2 + 2pk + p^2 = y^2 - 2ky + 2py + k^2 - 2pk + p^2$$

$$(x - h)^2 - 2py + 2pk = 2py - 2pk$$

$$(x - h)^2 + 4pk = 4py$$

$$\frac{1}{4p}(x - h)^2 + k = y$$

Lesson 10-2
Writing a Quadratic Function Given Three Points

Learning Targets:
- Explain why three points are needed to determine a parabola.
- Determine the quadratic function that passes through three given points on a plane.

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Questioning the Text, Create Representations, Identify a Subtask

Recall that if you are given any two points on the coordinate plane, you can write the equation of the line that passes through those points. The two points are said to determine the line because there is only one line that can be drawn through them.

Do two points on the coordinate plane determine a parabola? To answer this question, work through the following items.

1. Follow these steps to write the equation of a quadratic function whose graph passes through the points $(2, 0)$ and $(5, 0)$.

   a. Write a quadratic equation in standard form with the solutions $x = 2$ and $x = 5$.
   $x^2 - 7x + 10 = 0$ or a nonzero multiple of this equation

   b. Replace 0 in your equation from part a with y to write the corresponding quadratic function.
   Answers may vary depending on the equation in part a. Sample answer: $y = x^2 - 7x + 10$

   c. Use substitution to check that the points $(2, 0)$ and $(5, 0)$ lie on the function's graph.
   0 ? $(2)^2 - 7(2) + 10$ **0 ? $(5)^2 - 7(5) + 10$**
   0 ? $4 - 14 + 10$ **0 ? $25 - 35 + 10$**
   $0 = 0$ ✓   **$0 = 0$ ✓**

2. a. **Use appropriate tools strategically.** Graph your quadratic function from Item 1 on a graphing calculator.
   Check students' work.

   b. On the same screen, graph the quadratic functions $y = 2x^2 - 14x + 20$ and $y = -x^2 + 7x - 10$.
   Check students' work.

My Notes

> **MATH TIP**
>
> To review writing a quadratic equation when given its solutions, see Lesson 7-3.

Lesson 10-2

PLAN

Pacing: 1 class period
Chunking the Lesson
#1–3   #4–6   #7–10
Check Your Understanding
Lesson Practice

TEACH

📷 Bell-Ringer Activity
Have students write the equation in slope-intercept form of a line that passes through the points $(3, 2)$ and $(-5, 6)$, using the following procedures:

1. Substitute the point $(3, 2)$ into the equation $y = mx + b$.
2. Substitute the point $(-5, 6)$ into the equation $y = mx + b$.
3. Solve the system of equations using substitution or Gaussian elimination.

1–3 Interactive Word Wall, Create Representations Encourage the use of proper math vocabulary to describe the similarities and differences of the three parabolas. Students could use the terms *vertex*, *axis of symmetry*, *maximum*, *minimum*, *y-intercept*, and *x-intercepts* in their descriptions.

> **Universal Access**
>
> Students may question where the equations $y = 2x^2 - 14x + 10$ and $y = -x^2 + 7x - 10$ in Item 2b came from. Return to Item 1 and lead students through writing binomial factors other than $(x - 2)$ and $(x - 5)$ that lead to a quadratic equation with solution set $\{2, 5\}$.

4–6 Predict and Confirm, Create Representations, Visualization Ask students to plot the three points on a coordinate plane and make predictions about the vertex of the parabola. Have students share answers and note that it is impossible to accurately predict the vertex. This will motivate the algebraic solution process.

Universal Access

Using algebraic methods to solve a system of equations in three variables can be time-consuming and frustrating to students. Consider allowing students to use matrix equations and their graphing calculators to determine the values of a, b, and c. Doing this will keep the lesson focused on finding the equation of the parabola.

ACTIVITY 10
continued

My Notes

MATH TIP

Three or more points are *collinear* if they lie on the same straight line.

c. Describe the graphs. Do all three parabolas pass through the points $(2, 0)$ and $(5, 0)$?

Answers may vary, but students should note that all three parabolas pass through the points (2, 0) and (5, 0).
Sample answer: Two of the parabolas open upward, and one opens downward. One parabola is narrower than the others. However, all of the parabolas have the same *x*-intercepts: 2 and 5.

3. Reason abstractly. Do two points on the coordinate plane determine a parabola? Explain.

No. Sample explanation: My graph of the three parabolas shows that more than one parabola can be drawn through the same pair of points, (2, 0) and (5, 0). So, two points are not enough to determine a parabola.

Three points in the coordinate plane that are not on the same line determine a parabola given by a quadratic function. If you are given three noncollinear points on the coordinate plane, you can write the equation of the quadratic function whose graph passes through them.

Consider the quadratic function whose graph passes through the points $(1, 2)$, $(3, 0)$, and $(5, 6)$.

4. Write an equation by substituting the coordinates of the point $(1, 2)$ into the standard form of a quadratic function, $y = ax^2 + bx + c$.

$2 = a + b + c$ or equivalent

5. Write a second equation by substituting the coordinates of the point $(3, 0)$ into the standard form of a quadratic function.

$0 = 9a + 3b + c$ or equivalent

6. Write a third equation by substituting the coordinates of the point $(5, 6)$ into the standard form of a quadratic function.

$6 = 25a + 5b + c$ or equivalent

Lesson 10-2
Writing a Quadratic Function Given Three Points

My Notes

7. Use your equations from Items 4–6 to write a system of three equations in the three variables a, b, and c.

$$\begin{cases} a + b + c = 2 \\ 9a + 3b + c = 0 \quad \text{or equivalent} \\ 25a + 5b + c = 6 \end{cases}$$

8. Use substitution or Gaussian elimination to solve your system of equations for a, b, and c.

   $a = 1$, $b = -5$, $c = 6$

MATH TIP

To review solving a system of three equations in three variables, see Lesson 3-2.

9. Now substitute the values of a, b, and c into the standard form of a quadratic function.

   $y = x^2 - 5x + 6$

10. **Model with mathematics.** Graph the quadratic function to confirm that it passes through the points (1, 2), (3, 0), and (5, 6).
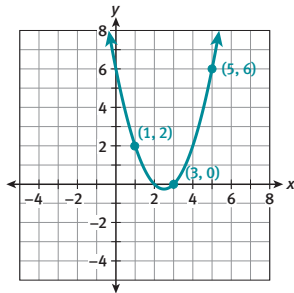
7–10 Debriefing, Identify a Subtask, Summarizing Ask students to provide a summary of the method used to determine the equation of a parabola in standard form that passes through three given points.

TEACHER to TEACHER

If an additional example is needed, students can create examples for a partner by working backward. Have students graph a parabola in standard form on a calculator and use the table or trace function to identify three integral points that lie on the parabola. Students can exchange points and then check each other's work.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to writing equations of quadratic functions.

Answers

11. Substitute the coordinates of each point into the standard form of a quadratic function, $y = ax^2 + bx + c$. Write the 3 resulting equations as a system of equations. Then solve the system for the values of a, b, and c. Finally, use the values of a, b, and c to write the equation of the quadratic function in standard form.

12. **a.** You find that $a = 0$, $b = -1$, and $c = 4$, which results in the function $f(x) = -x + 4$. This function is linear, not quadratic.
    b. The 3 points are on the same line, which means that you cannot write the equation of a quadratic function whose graph passes through the points.

13. **a.** $(-6, 0)$. Sample explanation: For a quadratic function, the axis of symmetry is a vertical line that passes through the vertex, so the axis of symmetry is $(x = -2)$. The point $(2, 0)$ is 4 units to the right of the axis of symmetry, so there will be another point on the graph of the function 4 units to the left of the axis of symmetry with the same y-coordinate. This point has coordinates $(-6, 0)$.
    b. $y = x^2 + 4x - 12$

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to write the equation of a parabola in standard form given three points that lie on the parabola. For additional practice, students can make up their own problems. Have them select three points and write the equation of the parabola using the method learned in this activity. Show students how they can check their work using quadratic regression on their graphing calculators.

My Notes

Lesson 10-2
Writing a Quadratic Function Given Three Points

Check Your Understanding

11. Describe how to write the equation of a quadratic function whose graph passes through three given points.

12. **a.** What happens when you try to write the equation of the quadratic function that passes through the points $(0, 4)$, $(2, 2)$, and $(4, 0)$?
    b. What does this result indicate about the three points?

13. **a. Reason quantitatively.** The graph of a quadratic function passes through the point $(2, 0)$. The vertex of the graph is $(-2, -16)$. Use symmetry to identify another point on the function's graph. Explain how you determined your answer.
    b. Write the equation of the quadratic function.

LESSON 10-2 PRACTICE

Write the equation of the quadratic function whose graph passes through each set of points.

14. $(-3, 2), (-1, 0), (1, 6)$
15. $(-2, -5), (0, -3), (1, 4)$
16. $(-1, -5), (1, -9), (4, 0)$
17. $(-3, 7), (0, 4), (1, 15)$
18. $(1, 0), (2, -7), (5, -16)$
19. $(-2, -11), (-1, -12), (1, 16)$

20. The table below shows the first few terms of a sequence. This sequence can be described by a quadratic function, where $f(n)$ represents the nth term of the sequence. Write the quadratic function that describes the sequence.

MATH TIP

A sequence is an ordered list of numbers or other items. Each number or item in a sequence is called a term.

Term Number, n	1	2	3	4	5
Term of Sequence, $f(n)$	2	6	12	20	30

21. A quadratic function $A(s)$ gives the area in square units of a regular hexagon with a side length of s units.
    a. Use the data in the table below to write the equation of the quadratic function.

CONNECT TO GEOMETRY

A regular hexagon is a six-sided polygon with all sides having the same length and all angles having the same measure.

Side Length, s	2	4	6
Area, $A(s)$	$6\sqrt{3}$	$24\sqrt{3}$	$54\sqrt{3}$

    b. Attend to precision. To the nearest square centimeter, what is the area of a regular hexagon with a side length of 8 cm?

LESSON 10-2 PRACTICE

14. $y = x^2 + 3x + 2$
15. $y = 2x^2 + 5x - 3$
16. $y = x^2 - 2x - 8$
17. $y = 3x^2 + 8x + 4$
18. $y = x^2 - 10x + 9$
19. $y = 5x^2 + 14x - 3$
20. $f(n) = n^2 + n$
21. **a.** $A(s) = \dfrac{3\sqrt{3}}{2} s^2$
    b. 166 cm^2

Lesson 10-3
Quadratic Regression

Learning Targets:
- Find a quadratic model for a given table of data.
- Use a quadratic model to make predictions.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Discussion Groups, Create Representations, Interactive Word Wall, Quickwrite, Close Reading, Predict and Confirm, Look for a Pattern, Group Presentation

A model rocketry club placed an altimeter on one of its rockets. An altimeter measures the altitude, or height, of an object above the ground. The table shows the data the club members collected from the altimeter before it stopped transmitting a little over 9 seconds after launch.
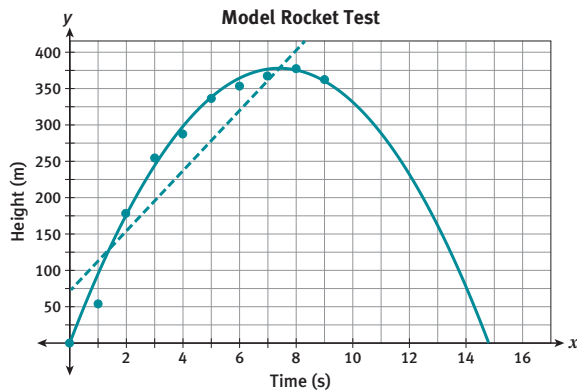
Model Rocket Test

Time Since Launch (s)	0	1	2	3	4	5	6	7	8	9
Height (m)	0	54	179	255	288	337	354	368	378	363

1. Predict the height of the rocket 12 seconds after launch. Explain how you made your prediction.
   Predictions and explanations will vary.

2. **Model with mathematics.** Make a scatter plot of the data on the coordinate grid below.

Model Rocket Test

My Notes

CONNECT TO PHYSICS

A model rocket is not powerful enough to escape Earth's gravity. The maximum height that a model rocket will reach depends in part on the weight and shape of the rocket, the amount of force generated by the rocket motor, and the amount of fuel the motor contains.

MINI-LESSON: Second Differences

If students need additional help with how to use second differences to determine if a set of data is a good candidate for a quadratic model, a mini-lesson is available to provide practice.

See the Teacher Resources at SpringBoard Digital for a student page for this mini-lesson.

Lesson 10-3

PLAN

Pacing: 1 class period
Chunking the Lesson
#1–2   #3–6
#7–9   #10–11
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity
Have students determine whether the function $y = x^2 - 4x + 3$ is a good model for the data points $(-5, 50)$, $(-1, 8)$, $(0, 2)$, $(4, 2)$, and $(5, 7)$. Make sure that students justify their answers.

1–2 Construct an Argument, Visualization, Think Aloud Some students will struggle to understand the physics behind the problem. Help students to realize that while the rocket is launched with an initial velocity, gravity slows the rocket down as it ascends until it finally reaches a point of zero velocity. Once the rocket begins falling back to earth, gravity serves to increase its velocity as it descends.

3–6 Create Representations, Note Taking, Discussion Groups Have students work in groups to perform the quadratic regression. Some students may need to record the keystrokes in their notes for future reference.

TEACHER to TEACHER

How well a regression model fits a set of data can be mathematically represented by the correlation coefficient, r^2, which measures the percent of variability in the y-values that has been explained by the regression equation. This statistic ranges from 0 to 1, with a value of 1 indicating that 100% of the variability of y has been explained by the regression equation. In other words, a value of 1 would occur if the graph of the model passed through each point of the data set.

Developing Math Language

Emphasize that regression, no matter what model, is an attempt to find a sufficiently good functional relationship between an independent variable and a dependent variable. In this case, the independent variable is time and the dependent variable is height.

My Notes

3. Enter the rocket data into a graphing calculator. Enter the time data as List 1 (L1) and the height data as List 2 (L2). Then use the calculator to perform a linear regression on the data. Write the equation of the linear model that results from the regression. Round coefficients and constants to the nearest tenth.

$y = 41.4x + 71.4$

4. Use a dashed line to graph the linear model from Item 3 on the coordinate grid showing the rocket data.
See graph below Item 2.

5. a. **Attend to precision.** To the nearest meter, what height does the linear model predict for the rocket 12 seconds after it is launched?
568 m

b. How does this prediction compare with the prediction you made in Item 1?
Answers will vary.

6. **Construct viable arguments.** Do you think the linear model is a good model for the rocket data? Justify your answer.

Sample answer: No. The linear model indicates that the rocket was already about 71 m off the ground at the time it was launched, when its actual height at this time was 0 m. Also, the linear model indicates that the rocket's height would continue to increase with time without the rocket ever landing. The actual data show that the rocket's height is starting to decrease after 8 seconds.

MATH TIP

A calculator may be able to generate a linear model for a data set, but that does not necessarily mean that the model is a good fit or makes sense in a particular situation.

MATH TERMS

Quadratic regression is the process of determining the equation of a quadratic function that best fits the given data.

A linear regression is the process of finding a linear function that best fits a set of data. A *quadratic regression* is the process of finding a quadratic function that best fits a set of data. The steps for performing a quadratic regression on a graphing calculator are similar to those for performing a linear regression.

Lesson 10-3
Quadratic Regression

My Notes

7. Use these steps to perform a quadratic regression for the rocket data.
   - Check that the data set is still entered as List 1 and List 2.
   - Press [STAT] to select the Statistics menu. Then move the cursor to highlight the Calculate (CALC) submenu.
   - Select 5:QuadReg to perform a quadratic regression on the data in Lists 1 and 2. Press [ENTER].
   - The calculator displays the values of *a*, *b*, and *c* for the standard form of the quadratic function that best fits the data.

   Write the equation of the quadratic model that results from the regression. Round coefficients and constants to the nearest tenth.

   $y = -6.9x^2 + 103.8x - 11.8$

8. Graph the quadratic model from Item 7 on the coordinate grid showing the rocket data.

   See graph below Item 2.

9. **Construct viable arguments.** Contrast the graph of the linear model with the graph of the quadratic model. Which model is a better fit for the data?

   Sample answer: The quadratic model is a better fit for the data because the data points are closer to the parabola overall than to the line. Unlike the linear model, the quadratic model shows that the rocket will eventually return to ground level.

10. **a.** To the nearest meter, what height does the quadratic model predict for the rocket 12 seconds after it is launched?

    Predictions should be close to 240 m.

    b. How does this prediction compare with the prediction you made in Item 1?

    Answers will vary.

11. **Reason quantitatively.** Use the quadratic model to predict when the rocket will hit the ground. Explain how you determined your answer.

    Answers may vary but should be close to 15 s.
    Sample explanation: I set the height *y* of the quadratic model equal to 0, and used the Quadratic Formula to solve for the time *x*. The solutions show that the rocket will hit the ground after about 14.9 s.

TECHNOLOGY TIP

You can graph the equation from a quadratic regression by using these steps: After selecting **5:QuadReg** as described at the left, do not press [ENTER]. Instead, press [VARS] to select the VARS menu. Then move the cursor to highlight the Y-VARS submenu. Select **1:Function**. Then select **1:Y1**. Press [ENTER]. The equation from the quadratic regression is now assigned to Y1. You can press [GRAPH] to view the graph of the equation.

7–9 Predict and Confirm Point out to students that while the quadratic model generated by the calculator is mathematically the best fit for the data, the equation is still imperfect. For example, the model suggests that the rocket is at −11.8 feet at 0 seconds, which clearly is not true. However, the quadratic model is a much better fit than the linear model and will result in better estimates for when the model returns to the ground.

TEACHER to TEACHER

Have students display a scatter plot of the data in Lists 1 and 2 on their calculators. Students will need to access the statplot menu and ensure that Plot 1 is turned on and that the Xlist is L_1 and the Ylist is L_2. Also have students graph the regression equation along with the scatter plot.

10–11 Discussion Groups, Debriefing Have students use the CALC function on a graphing calculator to determine the vertex of the quadratic regression equation. Ask students to interpret the meaning of the *x*- and *y*-coordinates of the vertex and how it relates to the time that the rocket returns to the ground. For additional technology resources, visit SpringBoard Digital.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to quadratic regression.

Answers

12. An underestimate; The parachute slows the rocket down, which means that it will take the rocket longer to reach the ground than the model predicts. The prediction from the quadratic model is an underestimate of the time at which the rocket will reach the ground.

13. a. Yes. Three noncollinear points determine a parabola, so you can perform a quadratic regression if you have at least 3 data points.

b. The model would fit the data set exactly because there is only 1 parabola representing a quadratic function that can be drawn through any set of 3 noncollinear points.

c. If the 3 points lie on the same line, the quadratic regression would show that the coefficient of the x^2-term is 0. In other words, the quadratic regression would result in a linear model. The linear model would fit the data exactly, because the 3 points lie on the same line.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to use their graphing calculator to complete quadratic regression. Also make sure that students understand the concept of regression such that they can justify an appropriate regression model given a set of data. Many students will require additional practice. Create additional data sets for students by writing a quadratic function in standard form and using it to provide approximate points that fall on the curve.

My Notes

Check Your Understanding

12. Make sense of problems. Most model rockets have a parachute or a similar device that releases shortly after the rocket reaches its maximum height. The parachute helps to slow the rocket so that it does not hit the ground with as much force. Based on this information, do you think your prediction from Item 11 is an underestimate or an overestimate if the rocket has a parachute? Explain.

13. a. Could you use a graphing calculator to perform a quadratic regression on three data points? Explain.

b. How closely would the quadratic model fit the data set in this situation? Explain.

c. How would your answers to parts a and b change if you knew that the three points lie on the same line?

LESSON 10-3 PRACTICE

Tell whether a linear model or a quadratic model is a better fit for each data set. Justify your answer, and give the equation of the better model.

14.

x	10	12	14	16	18	20	22	24
y	19	15	13	11	9	9	10	11

15.

x	2	4	6	8	10	12	14	16
y	10	22	26	35	45	50	64	66

The tables show time and height data for two other model rockets.

Rocket A

Time (s)	0	1	2	3	4	5	6	7
Height (m)	0	54	179	255	288	337	354	368

Rocket B

Time (s)	0	1	2	3	4	5	6	7
Height (m)	0	37	92	136	186	210	221	229

16. Use appropriate tools strategically. Use a graphing calculator to perform a quadratic regression for each data set. Write the equations of the quadratic models. Round coefficients and constants to the nearest tenth.

17. Use your models to predict which rocket had a greater maximum height. Explain how you made your prediction.

18. Use your models to predict which rocket hit the ground first and how much sooner. Explain how you made your prediction.

LESSON 10-3 PRACTICE

14. Sample justification: A quadratic model is a better fit. A graph of both models shows that the data points are closer to the quadratic model. Also, the values of y first decrease and then begin to increase as x increases, which indicates the shape of a quadratic, not a linear, model. Quadratic model: $y = 0.1x^2 - 4.1x + 49.3$

15. Sample justification: A linear model is a better fit. The values of y increase as x increases without ever decreasing, which indicates the shape of a linear, not a quadratic, model. Linear model: $y = 4.1x + 3.1$

16. Rocket A: $y = -7.6x^2 + 107.9x - 14.9$; Rocket B: $y = -3.9x^2 + 62.1x - 10.4$

17. Rocket A. Sample explanation: Graph both quadratic models on the same coordinate grid. The graphs show that Rocket A reaches a greater height than Rocket B.

18. Predictions may vary but should indicate that Rocket A will hit the ground about 1.7 seconds sooner than Rocket B. Sample explanation: I set the height y of each quadratic model equal to 0 and used the Quadratic Formula to solve for the time x. The solutions show that Rocket A will hit the ground after about 14.1 seconds and Rocket B will hit the ground after about 15.8 seconds, or about 1.7 seconds later.
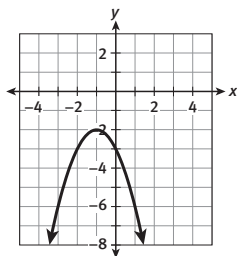
ACTIVITY 10 PRACTICE
Write your answers on notebook paper.
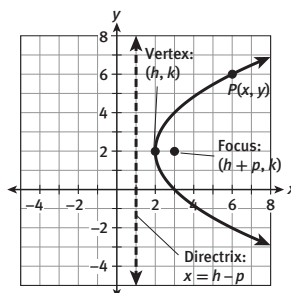**Show your work.**

Lesson 10-1

Use the parabola shown in the graph for Items 1 and 2.

1. What is the equation of the parabola?
   A. $y = -(x - 1)^2 - 2$ **B.** $y = -(x + 1)^2 - 2$
   C. $y = (x - 1)^2 - 2$ **D.** $y = (x + 1)^2 + 2$

2. The focus of the parabola is $\left(-1, -\frac{9}{4}\right)$, and the
   directrix is the line $y = -\frac{7}{4}$. Show that the
   point $(-2, -3)$ on the parabola is the same
   distance from the focus as from the directrix.

3. Graph the parabola given by the equation
   $x = \frac{1}{2}(y - 3)^2 + 3$.

4. Identify the following features of the parabola
   given by the equation $y = \frac{1}{8}(x - 4)^2 + 3$.

   a. vertex   **b.** focus
   c. directrix   **d.** axis of symmetry
   e. direction of opening

5. Describe the relationships among the vertex,
   focus, directrix, and axis of symmetry of a
   parabola.

6. The focus of a parabola is $(3, -2)$, and its
   directrix is the line $x = -5$. What are the vertex
   and the axis of symmetry of the parabola?

For Items 7–11, use the given information to write the
equation of each parabola.

7. vertex: $(0, 0)$; focus: $(0, 5)$

8. vertex: $(0, 0)$; directrix: $x = -3$

9. vertex: $(2, 2)$; axis of symmetry: $y = 2$;
   focus: $(1, 2)$

10. opens downward; vertex: $(-1, -2)$; directrix:
    $y = -1$

11. focus: $(-1, 3)$; directrix: $x = -5$

12. Use the diagram below to help you derive the
    general equation of a parabola with its vertex at
    (h, k), a horizontal axis of symmetry, a focus of
    $(h + p, k)$, and a directrix of $x = h - p$. Solve the
    equation for x.

Lesson 10-2

Write the equation of the quadratic function whose
graph passes through each set of points.

13. $(-3, 0), (-2, -3), (2, 5)$

14. $(-2, -6), (1, 0), (2, 10)$

15. $(-5, -3), (-4, 0), (0, -8)$

16. $(-3, 10), (-2, 0), (0, -2)$

17. $(1, 0), (4, 6), (7, -6)$

18. $(-2, -9), (-1, 0), (1, -12)$
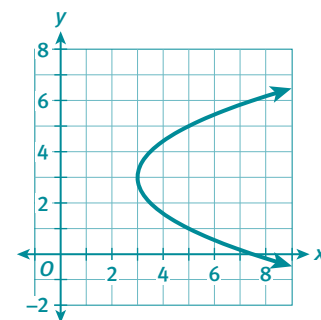

ACTIVITY PRACTICE

1. B
2. distance to focus:

   $$\sqrt{(-1 - (-2))^2 + \left(-\frac{9}{4} - (-3)\right)^2} = \frac{5}{4}$$

   distance to directrix:

   $$\sqrt{(-2 - (-2))^2 + \left(-\frac{7}{4} - (-3)\right)^2} = \frac{5}{4}$$

3.

4. **a.** $(4, 3)$
   b. $(4, 5)$
   c. $y = 1$
   d. $x = 4$
   e. upward

5. Sample answer: The axis of
   symmetry is perpendicular to the
   directrix. The focus and the vertex
   lie on the axis of symmetry. The
   vertex is the midpoint of the
   segment that lies on the axis of
   symmetry and has its endpoints at
   the focus and on the directrix.

6. Vertex: $(-1, -2)$; axis of symmetry:
   $y = -2$

7. $y = \frac{1}{20} x^2$

8. $x = \frac{1}{12} y^2$

9. $x = -\frac{1}{4}(y - 2)^2 + 2$

10. $y = -\frac{1}{4}(x + 1)^2 - 2$

11. $x = \frac{1}{8}(y - 3)^2 - 3$

13. $y = x^2 + 2x - 3$
14. $y = 2x^2 + 4x - 6$
15. $y = -x^2 - 6x - 8$
16. $y = 3x^2 + 5x - 2$
17. $y = -x^2 + 7x - 6$
18. $y = -5x^2 - 6x - 1$

---

12. Sample derivation:

distance from P to focus = distance from P to directrix

$$\sqrt{(x - (h + p))^2 + (y - k)^2} = \sqrt{(x - (h - p))^2 + (y - y)^2}$$

$$(x - (h + p))^2 + (y - k)^2 = (x - (h - p))^2 + (y - y)^2$$

$$x^2 - 2(h + p)x + (h + p)^2 + (y - k)^2 = x^2 - 2(h - p)x + (h - p)^2$$

$$x^2 - 2hx - 2px + h^2 + 2hp + p^2 + (y - k)^2 = x^2 - 2hx + 2px + h^2 - 2hp + p^2$$

$$-2px + 2hp + (y - k)^2 = 2px - 2hp$$

$$(y - k)^2 + 4hp = 4px$$

$$\frac{1}{4p}(y - k)^2 + h = x$$

19. Answers may vary, but equations should be a nonzero multiple of $y = x^2 + 2x - 48$. Sample answer: The parabolas given by the equations $y = x^2 + 2x - 48$ and $y = -x^2 - 2x + 48$ both pass through the points $(-8, 0)$ and $(6, 0)$.

20. a. $(-1, 5)$. Sample explanation: For a quadratic function, the axis of symmetry is a vertical line that passes through the vertex, so the axis of symmetry is $x = 3$. The point $(7, 5)$ is 4 units to the right of the axis of symmetry, so there will be another point on the graph of the function 4 units to the left of the axis of symmetry with the same y-coordinate. This point has coordinates $(-1, 5)$.

   b. $f(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4}$

21. Sample justification: A linear model is a better fit. The values of y increase as x increases without ever decreasing, which indicates the shape of a linear, not a quadratic, model. Linear model: $y = 4.9x + 18.2$

22. Sample justification: A quadratic model is a better fit. A graph of both models shows that the data points are closer to the quadratic model. Also, the values of y first decrease and then begin to increase as x increases, which indicates the shape of a quadratic, not a linear, model. Quadratic model: $y = 0.3x^2 - 5.0x + 23.8$

23. car: $y = 0.047x^2 + 2.207x + 0.214$; truck: $y = 0.064x^2 + 2.210x - 0.500$

24. Predictions should be close to 42 feet.

25. No. Sample explanation: Based on the quadratic model, the stopping distance for the truck at 60 mi/h is about 363 feet. This distance is greater than the distance between the truck and the intersection, so the driver will not be able to stop in time.

26. a. $y = -2.2x^2 + 454.9x - 12,637.0$

   b. Yes. Sample explanation: A graph of the quadratic model and the data from the table shows that the graph of the model is close to the data points. Also, the monthly revenue increases and then decreases as the selling price increases, which indicates a quadratic model could be a good fit for the data.

   c. Answers should be close to $103.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

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19. Demonstrate that the points $(-8, 0)$ and $(6, 0)$ do not determine a unique parabola by writing the equations of two different parabolas that pass through these two points.

20. a. The graph of a quadratic function passes through the point $(7, 5)$. The vertex of the graph is $(3, 1)$. Use symmetry to identify another point on the function's graph. Explain your answer.

   b. Write the equation of the quadratic function.

Lesson 10-3

Tell whether a linear model or a quadratic model is a better fit for each data set. Justify your answer and give the equation of the better model.

21.

x	0	2	4	6	8	10	12	14
y	17	29	40	45	59	63	76	88

22.

x	2	4	6	8	10	12	14	16
y	15	9	5	2	6	7	16	22

The stopping distance of a vehicle is the distance the vehicle travels between the time the driver recognizes the need to stop and the time the vehicle comes to a stop. The table below shows how the speed of two vehicles affects their stopping distances.

Speed (mi/h)	Stopping distance (ft)	
	Car	Truck
10	27	28
15	44	47
20	63	69
25	85	95
30	109	123
35	135	155
40	164	190

23. Use a graphing calculator to perform a quadratic regression on the data for each vehicle. Write the equations of the quadratic models. Round coefficients and constants to the nearest thousandth.

24. Use your models to predict how much farther it would take the truck to stop from a speed of 50 mi/h than it would the car.

25. Suppose the truck is 300 ft from an intersection when the light at the intersection turns yellow. If the truck's speed is 60 mi/h when the driver sees the light change, will the driver be able to stop without entering the intersection? Explain how you know.

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

26. A shoe company tests different prices of a new type of athletic shoe at different stores. The table shows the relationship between the selling price and the monthly revenue per store the company made from selling the shoes.

Selling Price ($)	Monthly Revenue per Store ($)
80	9680
90	10,520
100	11,010
110	10,660
120	10,400
130	9380

   a. Use a graphing calculator to determine the equation of a quadratic model that can be used to predict y, the monthly revenue per store in dollars when the selling price is x dollars. Round values to the nearest tenth.

   b. Is a quadratic model a good model for the data set? Explain.

   c. Use your model to determine the price at which the company should sell the shoes to generate the greatest revenue.

Writing Quadratic Equations

What Goes Up Must Come Down
Lesson 10-1 Parabolas and Quadratic Equations

Learning Targets:

- Derive a general equation for a parabola based on the definition of a parabola.
- Write the equation of a parabola given a graph and key features.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Discussion Groups, Interactive Word Wall, Create Representations, Close Reading

Take a look at the graphs shown below.

A

B

C

D

1. **Make use of structure.** Match each equation with one of the graphs above.

$$x = \frac{1}{4}(y-2)^2 - 1 \qquad\qquad y = \frac{1}{4}(x-2)^2 - 1$$

$$y = -\frac{1}{4}(x-2)^2 - 1 \qquad\qquad x = -\frac{1}{4}(y-2)^2 - 1$$

My Notes

TECHNOLOGY TIP

If an equation includes the $\pm$ symbol, you will need to enter it in a graphing calculator as two separate equations. For example, enter the equation $y = 2 \pm \sqrt{x}$ as $y = 2 + \sqrt{x}$ and $y = 2 - \sqrt{x}$.

2. Explain how you matched each equation with one of the graphs.

3. Use appropriate tools strategically. Use a graphing calculator to confirm your answers to Item 1. Which equations must be rewritten to enter them in the calculator? Rewrite any equations from Item 1 as necessary so that you can use them with your calculator.

4. a. How do graphs A and B differ from graphs C and D?

b. How do the equations of graphs A and B differ from the equations of graphs C and D?

5. Work with your group. Consider graphs A and B and their equations.
   a. Describe the relationship between the graphs.

   b. What part of the equation determines whether the graph opens up or down? How do you know?

   c. **Attend to precision.** What are the coordinates of the lowest point on graph A? What are the coordinates of the highest point on graph B? How do the coordinates of these points relate to the equations of the graphs?

6. Continue to work with your group. Consider graphs C and D and their equations.
   a. Describe the relationship between the graphs.

   b. What part of the equation determines whether the graph opens to the right or left? How do you know?

   c. What are the coordinates of the leftmost point on graph C? What are the coordinates of the rightmost point on graph D? How do the coordinates of these points relate to the equations of the graphs?

My Notes

DISCUSSION GROUP TIP

As you share ideas for Items 5 and 6 in your group, ask your group members or your teacher for clarification of any language, terms, or concepts that you do not understand.

MATH TIP

A graph is said to open upward when both ends of the graph point up. A graph is said to open downward when both ends of the graph point down.

The vertex of a graph that opens upward is the *minimum* of the graph, and is its lowest point. The vertex of a graph that opens downward is the *maximum* of the graph, and is its highest point.

My Notes

Check Your Understanding

7. Which equation does the graph at right represent? Explain your answer.
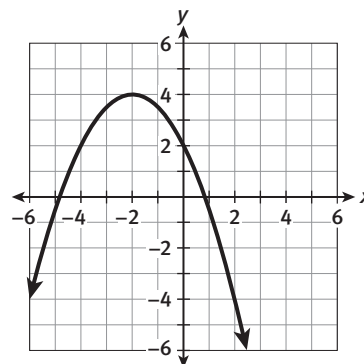
   A. $y = -\frac{1}{2}(x+2)^2 - 4$

   B. $y = -\frac{1}{2}(x+2)^2 + 4$

   C. $y = -\frac{1}{2}(x-2)^2 + 4$

8. **Construct viable arguments.** Which of the equations in Item 1 represent functions? Explain your reasoning.

9. Consider the equation $x = -2(y+4)^2 - 1$. Without graphing the equation, tell which direction its graph opens. Explain your reasoning.

MATH TERMS

A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line.

The fixed point is called the **focus**.

The fixed line is called the **directrix**.

MATH TIP

The distance between two points (x_1, y_1) and (x_2, y_2) is given by
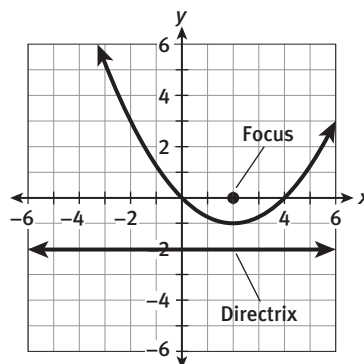
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

MATH TIP

The distance between a point and a horizontal line is the length of the vertical segment with one endpoint at the point and one endpoint on the line.

The graphs shown at the beginning of this lesson are all parabolas. A **parabola** can be defined as the set of points that are the same distance from a point called the **focus** and a line called the **directrix**.

10. The focus of graph A, shown below, is (2, 0), and the directrix is the horizontal line $y = -2$.

   a. The point $(-2, 3)$ is on the parabola. Find the distance between this point and the focus.

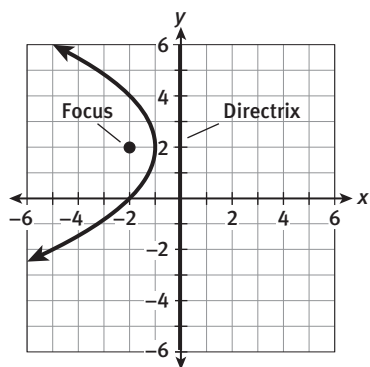**b.** Find the distance between the point (−2, 3) and the directrix.

c. Reason quantitatively. Compare your answers in parts a and b. What do you notice?

11. The focus of graph D, shown below, is $(-2, 2)$, and the directrix is the vertical line $x = 0$.

a. The point $(-2, 4)$ is on the parabola. Show that this point is the same distance from the focus as from the directrix.
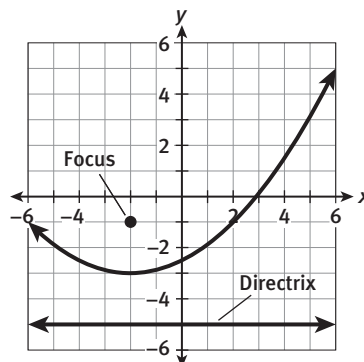
MATH TIP

The distance between a point and a vertical line is the length of the horizontal segment with one endpoint at the point and one endpoint on the line.

b. The point $(-5, -2)$ is also on the parabola. Show that this point is the same distance from the focus as from the directrix.

My Notes

The focus of the parabola shown below is $(-2, -1)$, and the directrix is the line $y = -5$.

MATH TERMS

The **axis of symmetry** is a line that divides the parabola into two congruent halves. The axis of symmetry passes through the focus and is perpendicular to the directrix.

The **vertex** is the point on the parabola that lies on the axis of symmetry. The vertex is the midpoint of the segment connecting the focus and the directrix.

12. a. Draw and label the *axis of symmetry* on the graph above. What is the equation of the axis of symmetry?

b. Explain how you identified the axis of symmetry of the parabola.

13. a. Draw and label the *vertex* on the graph above. What are the coordinates of the vertex?

b. Explain how you identified the vertex of the parabola.
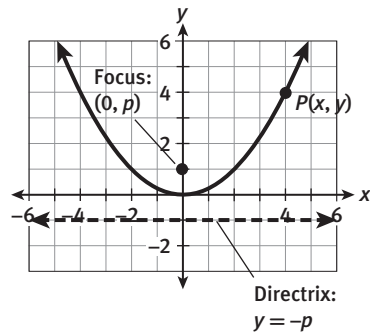
c. What is another way you could have identified the vertex?

My Notes

You can use what you have learned about parabolas to derive a general equation for a parabola whose vertex is located at the origin. Start with a parabola that has a vertical axis of symmetry, a focus of $(0, p)$, and a directrix of $y = -p$. Let $P(x, y)$ represent any point on the parabola.

14. Write, but do not simplify, an expression for the distance from point P to the focus.

15. Write, but do not simplify, an expression for the distance from point P to the directrix.

16. **Make use of structure.** Based on the definition of a parabola, the distance from point P to the focus is the same as the distance from point P to the directrix. Set your expressions from Items 14 and 15 equal to each other, and then solve for y.
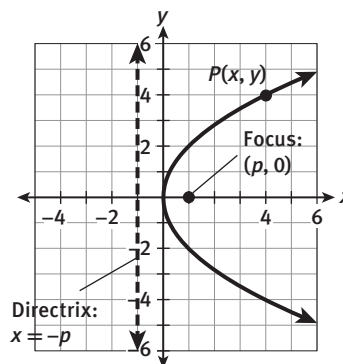
MATH TIP

In Item 16, start by squaring each side of the equation to eliminate the square root symbols. Next, simplify each side and expand the squared terms.

My Notes

17. What is the general equation for a parabola with its vertex at the origin, a focus of $(0, p)$, and a directrix of $y = -p$?

Check Your Understanding

18. See the diagram at right. Derive the general equation of a parabola with its vertex at the origin, a horizontal axis of symmetry, a focus of $(p, 0)$, and a directrix of $x = -p$. Solve the equation for x.

19. **Model with mathematics.** The vertex of a parabola is at the origin and its focus is $(0, -3)$. What is the equation of the parabola? Explain your reasoning.

20. A parabola has a focus of $(3, 4)$ and a directrix of $x = -1$. Answer each question about the parabola, and explain your reasoning.

   a. What is the axis of symmetry?

   b. What is the vertex?

   c. In which direction does the parabola open?

MATH TIP

A parabola always opens toward the focus and away from the directrix.

You can also write general equations for parabolas that do not have their vertex at the origin. You will derive these equations later in this activity.

	Vertical Axis of Symmetry	Horizontal Axis of Symmetry
Vertex	(h, k)	(h, k)
Focus	$(h, k + p)$	$(h + p, k)$
Directrix	horizontal line $y = k - p$	vertical line $x = h - p$
Equation	$y = \dfrac{1}{4p}(x - h)^2 + k$	$x = \dfrac{1}{4p}(y - k)^2 + h$

21. **Reason quantitatively.** Use the given information to write the
    equation of each parabola.
    a. axis of symmetry: $y = 0$; vertex: $(0, 0)$; directrix: $x = \frac{1}{2}$

> **MATH TIP**
>
> You may find it helpful to make a
> quick sketch of the information
> you are given.

   b. vertex: $(3, 4)$; focus: $(3, 6)$

   c. vertex: $(-2, 1)$; directrix: $y = 4$

   d. focus: $(-4, 0)$; directrix: $x = 4$
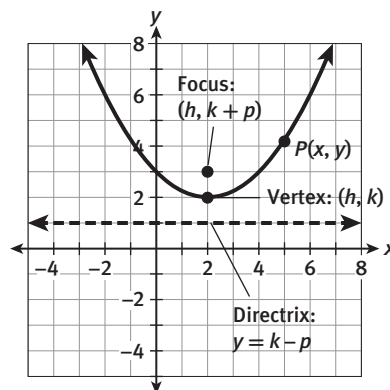
   e. opens up; focus: $(5, 7)$; directrix: $y = 3$

My Notes

Check Your Understanding

22. See the diagram at right. Derive the general equation of a parabola with its vertex at (h, k), a vertical axis of symmetry, a focus of $(h, k + p)$, and a directrix of $y = k - p$. Solve the equation for y.
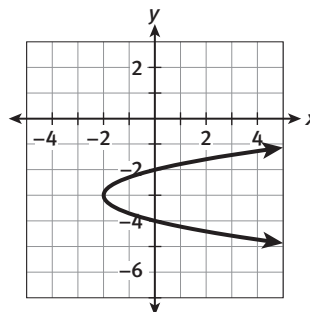
23. Construct viable arguments. Can you determine the equation of a parabola if you know only its axis of symmetry and its vertex? Explain.

24. The equation of a parabola is $x = \dfrac{1}{8}(y - 2)^2 + 1$. Identify the vertex, axis of symmetry, focus, and directrix of the parabola.

LESSON 10-1 PRACTICE

25. Which equation does the graph at right represent?
A. $x = -2(y + 3)^2 - 2$
B. $x = 2(y + 3)^2 - 2$
C. $y = -2(x + 3)^2 - 2$
D. $y = 2(x + 3)^2 - 2$

26. Graph the parabola given by the equation $y = \dfrac{1}{4}(x + 3)^2 - 4$.

27. Make sense of problems. The focus of a parabola is $(0, 2)$, and its directrix is the vertical line $x = -6$. Identify the axis of symmetry, the vertex, and the direction the parabola opens.

Use the given information to write the equation of each parabola.

28. vertex: $(0, 0)$; focus: $\left(0, -\dfrac{1}{2}\right)$

29. focus: $(4, 0)$; directrix: $x = -4$

30. opens to the left; vertex: $(0, 5)$; focus: $(-5, 5)$

31. axis of symmetry: $x = 3$; focus: $(3, -1)$; directrix: $y = -7$

32. vertex: $(-2, 4)$; directrix: $x = -3$

Learning Targets:

- Explain why three points are needed to determine a parabola.
- Determine the quadratic function that passes through three given points on a plane.

> **SUGGESTED LEARNING STRATEGIES:** Create Representations, Quickwrite, Questioning the Text, Create Representations, Identify a Subtask

Recall that if you are given any two points on the coordinate plane, you can write the equation of the line that passes through those points. The two points are said to determine the line because there is only one line that can be drawn through them.

Do two points on the coordinate plane determine a parabola? To answer this question, work through the following items.

1. Follow these steps to write the equation of a quadratic function whose graph passes through the points $(2, 0)$ and $(5, 0)$.

   a. Write a quadratic equation in standard form with the solutions $x = 2$ and $x = 5$.

   b. Replace 0 in your equation from part a with y to write the corresponding quadratic function.

   c. Use substitution to check that the points $(2, 0)$ and $(5, 0)$ lie on the function's graph.

2. **a. Use appropriate tools strategically.** Graph your quadratic function from Item 1 on a graphing calculator.

   b. On the same screen, graph the quadratic functions $y = 2x^2 - 14x + 20$ and $y = -x^2 + 7x - 10$.

> **MATH TIP**
>
> To review writing a quadratic equation when given its solutions, see Lesson 7-3.

My Notes

c. Describe the graphs. Do all three parabolas pass through the points (2, 0) and (5, 0)?

3. **Reason abstractly.** Do two points on the coordinate plane determine a parabola? Explain.

Three points in the coordinate plane that are not on the same line determine a parabola given by a quadratic function. If you are given three noncollinear points on the coordinate plane, you can write the equation of the quadratic function whose graph passes through them.

Consider the quadratic function whose graph passes through the points (1, 2), (3, 0), and (5, 6).

4. Write an equation by substituting the coordinates of the point (1, 2) into the standard form of a quadratic function, $y = ax^2 + bx + c$.

5. Write a second equation by substituting the coordinates of the point (3, 0) into the standard form of a quadratic function.

6. Write a third equation by substituting the coordinates of the point (5, 6) into the standard form of a quadratic function.

7. Use your equations from Items 4–6 to write a system of three equations in the three variables *a, b,* and *c*.

8. Use substitution or Gaussian elimination to solve your system of equations for *a, b,* and *c*.

MATH TIP

To review solving a system of three equations in three variables, see Lesson 3-2.

9. Now substitute the values of *a, b,* and *c* into the standard form of a quadratic function.

10. **Model with mathematics.** Graph the quadratic function to confirm that it passes through the points (1, 2), (3, 0), and (5, 6).

Check Your Understanding

11. Describe how to write the equation of a quadratic function whose graph passes through three given points.

12. a. What happens when you try to write the equation of the quadratic function that passes through the points $(0, 4)$, $(2, 2)$, and $(4, 0)$?
   b. What does this result indicate about the three points?

13. a. Reason quantitatively. The graph of a quadratic function passes through the point $(2, 0)$. The vertex of the graph is $(-2, -16)$. Use symmetry to identify another point on the function's graph. Explain how you determined your answer.
   b. Write the equation of the quadratic function.

LESSON 10-2 PRACTICE

Write the equation of the quadratic function whose graph passes through each set of points.

14. $(-3, 2)$, $(-1, 0)$, $(1, 6)$ **15.** $(-2, -5)$, $(0, -3)$, $(1, 4)$

16. $(-1, -5)$, $(1, -9)$, $(4, 0)$ **17.** $(-3, 7)$, $(0, 4)$, $(1, 15)$

18. $(1, 0)$, $(2, -7)$, $(5, -16)$ **19.** $(-2, -11)$, $(-1, -12)$, $(1, 16)$

20. The table below shows the first few terms of a sequence. This sequence can be described by a quadratic function, where $f(n)$ represents the nth term of the sequence. Write the quadratic function that describes the sequence.

Term Number, n	1	2	3	4	5
Term of Sequence, $f(n)$	2	6	12	20	30

21. A quadratic function $A(s)$ gives the area in square units of a regular hexagon with a side length of s units.
   a. Use the data in the table below to write the equation of the quadratic function.

Side Length, s	2	4	6
Area, $A(s)$	$6\sqrt{3}$	$24\sqrt{3}$	$54\sqrt{3}$

   b. Attend to precision. To the nearest square centimeter, what is the area of a regular hexagon with a side length of 8 cm?

Learning Targets:
- Find a quadratic model for a given table of data.
- Use a quadratic model to make predictions.

> **SUGGESTED LEARNING STRATEGIES:** Think Aloud, Discussion Groups, Create Representations, Interactive Word Wall, Quickwrite, Close Reading, Predict and Confirm, Look for a Pattern, Group Presentation

A model rocketry club placed an altimeter on one of its rockets. An altimeter measures the altitude, or height, of an object above the ground. The table shows the data the club members collected from the altimeter before it stopped transmitting a little over 9 seconds after launch.

Model Rocket Test

Time Since Launch (s)	0	1	2	3	4	5	6	7	8	9
Height (m)	0	54	179	255	288	337	354	368	378	363

1. Predict the height of the rocket 12 seconds after launch. Explain how you made your prediction.

2. **Model with mathematics.** Make a scatter plot of the data on the coordinate grid below.

Model Rocket Test

> **CONNECT TO PHYSICS**
>
> A model rocket is not powerful enough to escape Earth's gravity. The maximum height that a model rocket will reach depends in part on the weight and shape of the rocket, the amount of force generated by the rocket motor, and the amount of fuel the motor contains.

My Notes

3. Enter the rocket data into a graphing calculator. Enter the time data as List 1 (L1) and the height data as List 2 (L2). Then use the calculator to perform a linear regression on the data. Write the equation of the linear model that results from the regression. Round coefficients and constants to the nearest tenth.

4. Use a dashed line to graph the linear model from Item 3 on the coordinate grid showing the rocket data.

5. a. Attend to precision. To the nearest meter, what height does the linear model predict for the rocket 12 seconds after it is launched?

b. How does this prediction compare with the prediction you made in Item 1?

6. Construct viable arguments. Do you think the linear model is a good model for the rocket data? Justify your answer.

MATH TIP

A calculator may be able to generate a linear model for a data set, but that does not necessarily mean that the model is a good fit or makes sense in a particular situation.

MATH TERMS

Quadratic regression is the process of determining the equation of a quadratic function that best fits the given data.

A linear regression is the process of finding a linear function that best fits a set of data. A *quadratic regression* is the process of finding a quadratic function that best fits a set of data. The steps for performing a quadratic regression on a graphing calculator are similar to those for performing a linear regression.

7. Use these steps to perform a quadratic regression for the rocket data.
   - Check that the data set is still entered as List 1 and List 2.
   - Press [STAT] to select the Statistics menu. Then move the cursor to highlight the Calculate (CALC) submenu.
   - Select 5:QuadReg to perform a quadratic regression on the data in Lists 1 and 2. Press [ENTER].
   - The calculator displays the values of *a*, *b*, and *c* for the standard form of the quadratic function that best fits the data.

   Write the equation of the quadratic model that results from the regression. Round coefficients and constants to the nearest tenth.

> **TECHNOLOGY TIP**
>
> You can graph the equation from a quadratic regression by using these steps: After selecting **5:QuadReg** as described at the left, do not press [ENTER]. Instead, press [VARS] to select the VARS menu. Then move the cursor to highlight the Y-VARS submenu. Select **1:Function**. Then select **1:Y1**. Press [ENTER]. The equation from the quadratic regression is now assigned to Y1. You can press [GRAPH] to view the graph of the equation.

8. Graph the quadratic model from Item 7 on the coordinate grid showing the rocket data.

9. **Construct viable arguments.** Contrast the graph of the linear model with the graph of the quadratic model. Which model is a better fit for the data?

10. **a.** To the nearest meter, what height does the quadratic model predict for the rocket 12 seconds after it is launched?

    b. How does this prediction compare with the prediction you made in Item 1?

11. **Reason quantitatively.** Use the quadratic model to predict when the rocket will hit the ground. Explain how you determined your answer.

Check Your Understanding

12. **Make sense of problems.** Most model rockets have a parachute or a similar device that releases shortly after the rocket reaches its maximum height. The parachute helps to slow the rocket so that it does not hit the ground with as much force. Based on this information, do you think your prediction from Item 11 is an underestimate or an overestimate if the rocket has a parachute? Explain.

13. **a.** Could you use a graphing calculator to perform a quadratic regression on three data points? Explain.
    b. How closely would the quadratic model fit the data set in this situation? Explain.
    c. How would your answers to parts a and b change if you knew that the three points lie on the same line?

LESSON 10-3 PRACTICE

Tell whether a linear model or a quadratic model is a better fit for each data set. Justify your answer, and give the equation of the better model.

14.

x	10	12	14	16	18	20	22	24
y	19	15	13	11	9	9	10	11

15.

x	2	4	6	8	10	12	14	16
y	10	22	26	35	45	50	64	66

The tables show time and height data for two other model rockets.

Rocket A	Time (s)	0	1	2	3	4	5	6	7
	Height (m)	0	54	179	255	288	337	354	368

Rocket B	Time (s)	0	1	2	3	4	5	6	7
	Height (m)	0	37	92	136	186	210	221	229

16. **Use appropriate tools strategically.** Use a graphing calculator to perform a quadratic regression for each data set. Write the equations of the quadratic models. Round coefficients and constants to the nearest tenth.

17. Use your models to predict which rocket had a greater maximum height. Explain how you made your prediction.

18. Use your models to predict which rocket hit the ground first and how much sooner. Explain how you made your prediction.
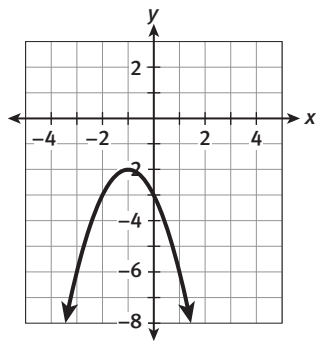
ACTIVITY 10 PRACTICE

Write your answers on notebook paper.
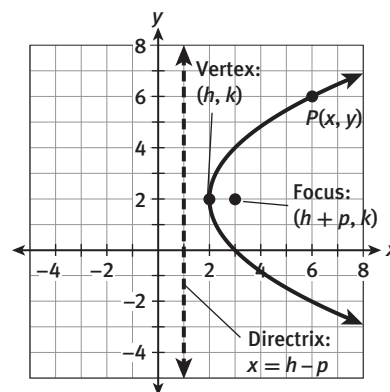**Show your work.**

Lesson 10-1

Use the parabola shown in the graph for Items 1 and 2.

1. What is the equation of the parabola?
   A. $y = -(x-1)^2 - 2$ **B.** $y = -(x+1)^2 - 2$
   C. $y = (x-1)^2 - 2$ **D.** $y = (x+1)^2 + 2$

2. The focus of the parabola is $\left(-1, -\frac{9}{4}\right)$, and the directrix is the line $y = -\frac{7}{4}$. Show that the point $(-2, -3)$ on the parabola is the same distance from the focus as from the directrix.

3. Graph the parabola given by the equation $x = \frac{1}{2}(y-3)^2 + 3$.

4. Identify the following features of the parabola given by the equation $y = \frac{1}{8}(x-4)^2 + 3$.
   a. vertex   **b.** focus
   c. directrix   **d.** axis of symmetry
   e. direction of opening

5. Describe the relationships among the vertex, focus, directrix, and axis of symmetry of a parabola.

6. The focus of a parabola is $(3, -2)$, and its directrix is the line $x = -5$. What are the vertex and the axis of symmetry of the parabola?

For Items 7–11, use the given information to write the equation of each parabola.

7. vertex: $(0, 0)$; focus: $(0, 5)$

8. vertex: $(0, 0)$; directrix: $x = -3$

9. vertex: $(2, 2)$; axis of symmetry: $y = 2$; focus: $(1, 2)$

10. opens downward; vertex: $(-1, -2)$; directrix: $y = -1$

11. focus: $(-1, 3)$; directrix: $x = -5$

12. Use the diagram below to help you derive the general equation of a parabola with its vertex at (h, k), a horizontal axis of symmetry, a focus of $(h + p, k)$, and a directrix of $x = h - p$. Solve the equation for x.

Lesson 10-2

Write the equation of the quadratic function whose graph passes through each set of points.

13. $(-3, 0), (-2, -3), (2, 5)$

14. $(-2, -6), (1, 0), (2, 10)$

15. $(-5, -3), (-4, 0), (0, -8)$

16. $(-3, 10), (-2, 0), (0, -2)$

17. $(1, 0), (4, 6), (7, -6)$

18. $(-2, -9), (-1, 0), (1, -12)$

19. Demonstrate that the points $(-8, 0)$ and $(6, 0)$ do not determine a unique parabola by writing the equations of two different parabolas that pass through these two points.

20. **a.** The graph of a quadratic function passes through the point $(7, 5)$. The vertex of the graph is $(3, 1)$. Use symmetry to identify another point on the function's graph. Explain your answer.

   b. Write the equation of the quadratic function.

Lesson 10-3

Tell whether a linear model or a quadratic model is a better fit for each data set. Justify your answer and give the equation of the better model.

21.

x	0	2	4	6	8	10	12	14
y	17	29	40	45	59	63	76	88

22.

x	2	4	6	8	10	12	14	16
y	15	9	5	2	6	7	16	22

The stopping distance of a vehicle is the distance the vehicle travels between the time the driver recognizes the need to stop and the time the vehicle comes to a stop. The table below shows how the speed of two vehicles affects their stopping distances.

Speed (mi/h)	Stopping distance (ft)	
	Car	Truck
10	27	28
15	44	47
20	63	69
25	85	95
30	109	123
35	135	155
40	164	190

23. Use a graphing calculator to perform a quadratic regression on the data for each vehicle. Write the equations of the quadratic models. Round coefficients and constants to the nearest thousandth.

24. Use your models to predict how much farther it would take the truck to stop from a speed of 50 mi/h than it would the car.

25. Suppose the truck is 300 ft from an intersection when the light at the intersection turns yellow. If the truck's speed is 60 mi/h when the driver sees the light change, will the driver be able to stop without entering the intersection? Explain how you know.

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

26. A shoe company tests different prices of a new type of athletic shoe at different stores. The table shows the relationship between the selling price and the monthly revenue per store the company made from selling the shoes.

Selling Price ($)	Monthly Revenue per Store ($)
80	9680
90	10,520
100	11,010
110	10,660
120	10,400
130	9380

   a. Use a graphing calculator to determine the equation of a quadratic model that can be used to predict y, the monthly revenue per store in dollars when the selling price is x dollars. Round values to the nearest tenth.

   b. Is a quadratic model a good model for the data set? Explain.

   c. Use your model to determine the price at which the company should sell the shoes to generate the greatest revenue.