Congruence Transformations and Triangle Congruence — ACTIVITY 11

Truss Your Judgment

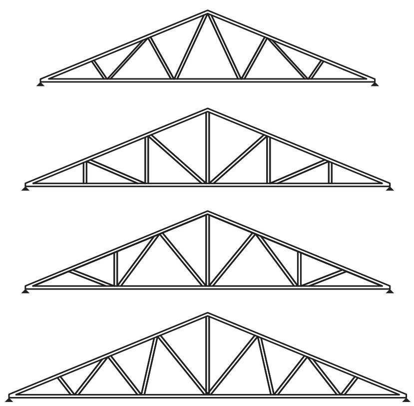
Lesson 11-1 Congruent Triangles

Learning Targets:

- Use the fact that congruent triangles have congruent corresponding parts.
- Determine unknown angle measures or side lengths in congruent triangles.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing, Use Manipulatives, Think-Pair-Share

Greg Carpenter works for the Greene Construction Company. The company is building a new recreation hall, and the roof of the hall will be supported by triangular trusses, like the ones shown below.



Each of the trusses contains pairs of congruent triangles. Greg's boss tells him that his first job will be to determine the side lengths and angle measures in the triangles that make up one of the trusses.

My Notes

CONNECT TO CAREERS

Triangles are often used in construction for roof and floor trusses because of their strength and rigidity. Each angle of a triangle is held solidly in place by its opposite side. That means the angles will not change when pressure is applied—unlike other shapes.

MATH TIP

Congruent triangles are triangles that have the same size and shape. More precisely, you have seen that two triangles are congruent if and only if one can be obtained from the other by a sequence of rigid motions.

Common Core State Standards for Activity 11

HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

ACTIVITY 11

Guided

Activity Standards Focus

In Activity 11, students explore congruence using a transformational approach. They investigate triangle congruence criteria, including SSS, SAS, SSA, AAS, and HL. After writing proofs for each of the triangle congruence criteria, students apply them to solve real-world problems.

Lesson 11-1

PLAN

Materials

- Ruler
- Protractor
- Tracing paper, patty paper, or acetate; or coffee stirrers, stapler, and scissors

Pacing: 1 class period

Chunking the Lesson

#1–3 Example A

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Determine whether or not each transformation is a rigid motion.

1. a reflection across a line [*rigid motion*]
2. a rotation about a point [*rigid motion*]
3. a dilation by a scale factor of 3 [*not a rigid motion*]

ELL Support

To begin the unit, all students should have an understanding of the properties of a *truss*. A truss is used to support the roof of a structure. It is in the shape of a triangle and is usually built from wood or steel.

Introduction Close Reading, Activating Prior Knowledge Have students read the introductory paragraphs in their groups. Elicit from students that a truss is made up of smaller triangles. Point out in the Connect to Careers signal box that triangles are strong and rigid because their angles and sides do not change under pressure.

1–3 Activating Prior Knowledge, Visualization, Discussion Groups

Students begin this activity by using what they know about congruence and rigid motions to identify relationships between parts of two congruent triangles. If students have difficulty getting started, remind them how they learned in Activity 10 that if two figures are congruent, then this means that one can be mapped to the other with a sequence of rigid motions. To help students make use of structure, ask the class as a group to study the diagram and identify what kind of transformation appears to map $\triangle ABC$ to $\triangle DEF$ [*a reflection across a vertical line between vertices C and F*]. Encourage students to share ideas about how they could use what they have previously learned about reflections to help identify the images of each side and angle of $\triangle ABC$.

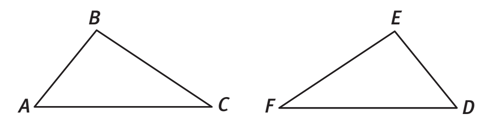
Developing Math Language

The new vocabulary word for this lesson is *corresponding parts*, which are the pairs of matching sides and matching angles identified when one figure is mapped to another. Students may already be familiar with the concept of corresponding parts, but this might be the first time they encounter a definition in terms of rigid motions. Guide students to understand that when figures are congruent, their corresponding parts are also congruent, because there is a sequence of rigid motions that maps each pair of corresponding sides and angles to each other.

My Notes

Greg wonders, "If I know that two triangles are congruent, and I know the side lengths and angle measures in one triangle, do I have to measure all the sides and angles in the other triangle?"

Greg begins by examining two triangles from a truss. According to the manufacturer, the two triangles are congruent.



1. Because the two triangles are congruent, can one triangle be mapped onto the other? If yes, what are the criteria for the mapping?

Each triangle can be mapped onto the other triangle by a sequence of rigid motions.

2. Suppose you use a sequence of rigid motions to map $\triangle ABC$ to $\triangle DEF$. Find the image of each of the following under this sequence of transformations.

$\overline{AB} \rightarrow$ **$\overline{DE}$** $\quad \overline{BC} \rightarrow$ **$\overline{EF}$** $\quad \overline{AC} \rightarrow$ **$\overline{DF}$**

$\angle A \rightarrow$ **$\angle D$** $\quad \angle B \rightarrow$ **$\angle E$** $\quad \angle C \rightarrow$ **$\angle F$**

3. Make use of structure. What is the relationship between $\overline{AB}$ and $\overline{DE}$? What is the relationship between $\angle B$ and $\angle E$? How do you know?

$\overline{AB} \cong \overline{DE}$ and $\angle B \cong \angle E$ because there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$ and a sequence of rigid motions that maps $\angle B$ to $\angle E$.

MATH TERMS

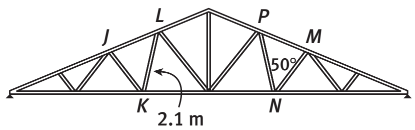
Corresponding parts result from a one-to-one matching of sides and angles from one figure to another. Congruent triangles have three pairs of congruent sides and three pairs of congruent angles.

The triangles from the truss that Greg examined illustrate an important point about congruent triangles. In congruent triangles, corresponding pairs of sides are congruent and corresponding pairs of angles are congruent. These are ***corresponding parts***.

When you write a congruence statement like $\triangle ABC \cong \triangle DEF$, you write the vertices so that corresponding parts are in the same order. So, you can conclude from this statement that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

Example A

For the truss shown below, Greg knows that $\triangle JKL \cong \triangle MNP$.



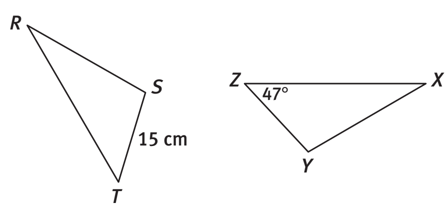
Greg wants to know if there are any additional lengths or angle measures that he can determine.

Since $\triangle JKL \cong \triangle MNP$, $\overline{KL} \cong \overline{NP}$. This means $KL = NP$, so $NP = 2.1$ m.

Also, since $\triangle JKL \cong \triangle MNP$, $\angle K \cong \angle N$. This means $m\angle K = m\angle N$, so $m\angle K = 50°$.

Try These A

In the figure, $\triangle RST \cong \triangle XYZ$. Find each of the following, if possible.



a. $m\angle X$
not possible

b. YZ
15 cm

c. $m\angle T$
47°

d. XZ
not possible

e. Both $\triangle JKL$ and $\triangle MNP$ are equilateral triangles in which the measure of each angle is 60°. Can you tell whether or not $\triangle JKL \cong \triangle MNP$? Explain.
No. All equilateral triangles have the same angle measures, but can have different side measures. The corresponding sides will always be proportional, but may not be equal. Therefore, the triangles may not be congruent.

My Notes

MATH TIP

Two line segments are congruent if and only if they have the same length. Two angles are congruent if and only if they have the same measure.

ACTIVITY 11 Continued

Example A Discussion Groups, Note Taking, Summarizing, Group Presentations Have students work in small groups to work through and discuss Example A and the Try These items. A key takeaway from this section is that even if you know some of the side and angle measures for two congruent triangles, it is not always possible to determine all of the measures. Encourage students to discuss what measures can be determined with the information given, as well as what additional information would be required to find the other measures. Have each group assign a member to take notes during the discussion. After the items have been completed, have each group summarize what they learned about determining measures of congruent triangles and share their insights with their classmates.

Differentiating Instruction

It may be beneficial for some students to explore congruence in the truss structure using hands-on strategies. For example, students could use patty paper to trace the truss diagram. Then, by turning and folding the paper, they could identify rotations and reflections that map various triangles onto each other. Encourage students to label the vertices so that they more easily identify corresponding parts.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to corresponding parts of congruent triangles. When students answer Item 7, you may also want to have them explain how they arrived at their answers.

Answers

4. Yes. Congruent triangles have side lengths with the same measures, and therefore the sum of their side lengths will be the same.
5. No. Congruent triangles have the same angle measures. Since all angles of an acute triangle are acute angles, and since one angle of a right triangle is a right angle, the angles of the two triangles do not have the same measures, and therefore are not congruent.
6. a. $\overline{CD}$
   b. $\overline{DA}$
   c. $\angle DCA$
   d. $\angle CAD$
7. a. $\overline{HJ}$
   b. $\overline{PR}$
   c. $\angle J$
   d. $\angle P$

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 11-1 PRACTICE

8. 18 in.
9. $\angle D = 39^\circ$
10. 73 in.
11. Yes. Since $\triangle XYZ$ is congruent to $\triangle TUV$, corresponding side lengths have the same measure. In a congruence statement corresponding parts are in the same position, so $\overline{XY}$ corresponds to $\overline{TU}$. Since $\overline{XY}$ is the longest side in $\triangle XYZ$, $\overline{TU}$ is the longest side in $\triangle TUV$.

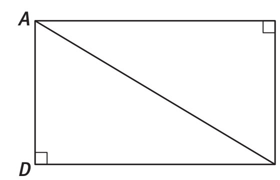
ADAPT

Check students' answers to the Lesson Practice to ensure that they understand basic concepts related to finding measures of corresponding parts of congruent figures. Remind students that the triangle congruence statement can help them identify the corresponding vertices.

My Notes

Check Your Understanding

4. If two triangles are congruent, can you conclude that they have the same perimeter? Why or why not?
5. Is it possible to draw two congruent triangles so that one triangle is an acute triangle and one triangle is a right triangle? Why or why not?
6. Rectangle *ABCD* is divided into two congruent right triangles by diagonal $\overline{AC}$.



Fill in the blanks to show the congruent sides and angles.

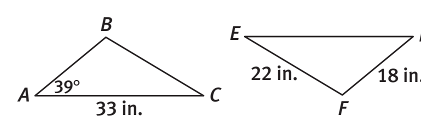
a. $\overline{AB} \cong$ ______ b. $\overline{BC} \cong$ ______
c. $\angle BAC \cong$ ______ d. $\angle ACB \cong$ ______

7. $\triangle PQR \cong \triangle GHJ$. Complete the following.

a. $\overline{QR} \cong$ ______ b. $\overline{GJ} \cong$ ______
c. $\angle R \cong$ ______ d. $\angle G \cong$ ______

LESSON 11-1 PRACTICE

In the figure, $\triangle ABC \cong \triangle DFE$.



8. Find the length of $\overline{AB}$.
9. Find the measure of all angles in $\triangle DEF$ that it is possible to find.
10. What is the perimeter of $\triangle DEF$? Explain how you know.
11. **Construct viable arguments.** Suppose $\triangle XYZ \cong \triangle TUV$ and that $\overline{XY}$ is the longest side of $\triangle XYZ$. Is it possible to determine which side of $\triangle TUV$ is the longest? Explain.

Learning Targets:
- Develop criteria for proving triangle congruence.
- Determine which congruence criteria can be used to show that two triangles are congruent.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Use Manipulatives, Think-Pair-Share

As you have seen, congruent triangles have six pairs of congruent corresponding parts. The converse of this statement is also true. That is, if two triangles have three pairs of congruent corresponding sides and three pairs of congruent corresponding angles, the triangles are congruent.

Greg's boss asks him to check that two triangles in a truss are congruent. Greg wonders, "Must I measure and compare all six parts of both triangles?" He decides that a shortcut will allow him to conclude that two triangles are congruent without checking all six pairs of corresponding parts.

1. Greg begins by checking just one pair of corresponding parts of the two triangles.
   a. In your group, each student should draw a triangle that has a side that is 2 inches long. The other two sides can be any measure. Draw the triangles on acetate or tracing paper.
   Check students' work

   b. To check whether two triangles are congruent, place the sheets of acetate on the desk. If the triangles are congruent, you can use a sequence of translations, reflections, and rotations to map one triangle onto the other.

   Are all of the triangles congruent to each other? Why or why not?
   The triangles are not congruent. No sequence of rigid motions will map one triangle to another.

   c. Cite your results from part b to prove or disprove this statement: "If one part of a triangle is congruent to a corresponding part of another triangle, then the triangles must be congruent."
   Students' triangles should serve as counterexamples to disprove the statement.

My Notes

MATH TIP

A *counterexample* is a single example that shows that a statement is false.

Lesson 11-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1	#2	#3
#4	#5–6	#7

Example A
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Complete each statement if $\triangle HJK \cong \triangle QRS$.

1. $\overline{JK} \cong$ _____ [$\overline{RS}$]
2. $\angle Q \cong$ _____ [$\angle H$]
3. $\overline{HK} \cong$ _____ [$\overline{QS}$]

1 Paraphrasing, Use Manipulatives, Create Representations, Debriefing

In this item, students are led to the conclusion that one pair of congruent parts is not sufficient to prove that two triangles are congruent. To get started, suggest that students paraphrase in mathematical terms what Greg actually wants to know—for example, "If one part of a triangle is congruent to a corresponding part of another triangle, does this mean that the triangles are congruent? If it does, then I know that all the other corresponding parts are congruent too, and I don't have to do any more measuring." Remind students that a single counterexample is all that is needed to prove that Greg's reasoning is incorrect. The triangles students draw in Part a will serve as counterexamples to Greg's reasoning that one pair of corresponding parts is sufficient to prove that two triangles are congruent. Encourage students who are having difficulty to create a second counterexample by drawing two different triangles, each having an angle measure of 50°.

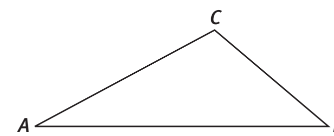
2 Paraphrasing, Marking the Text, Identify a Subtask In this item, students are now led to discover that two pairs of congruent parts are not sufficient to prove that two triangles are congruent. This time, they are asked to draw triangles with two sides congruent to corresponding sides of $\triangle ABC$ and determine if the triangles are congruent. Again, suggest that students paraphrase in mathematical terms what Greg actually wants to know—for example, "If two parts of a triangle are congruent to the two corresponding parts of another triangle, does this mean that the triangles are congruent and I won't have to measure the other parts?"

As in Item 1, the triangles students draw will serve as counterexamples to Greg's new line of reasoning that two pairs of corresponding parts are sufficient to prove that two triangles are congruent. Note that there are many possible triangles that could be drawn. It is important that students label the vertices of their triangles and understand the concept of corresponding parts.

My Notes

Now Greg wonders if checking two pairs of corresponding parts suffices to show that two triangles are congruent.

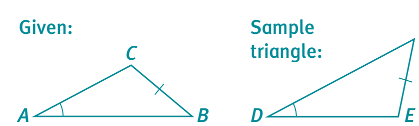
2. Greg starts by considering $\triangle ABC$ below.



a. Draw triangles that each have one side congruent to $\overline{AB}$ and another side congruent to $\overline{AC}$. Use transformations to check whether every triangle is congruent to $\triangle ABC$. Explain your findings.
Every such triangle is *not* congruent to $\triangle ABC$. The angle that corresponds to $\angle A$ can vary between 0 and 180 degrees.

b. Draw triangles that each have an angle congruent to $\angle A$ and an adjacent side congruent to $\overline{AB}$. Is every such triangle congruent to $\triangle ABC$? Explain.
No. The side that corresponds to $\overline{AC}$ can have any length.

c. Draw triangles that each have an angle congruent to $\angle A$ and an opposite side congruent to $\overline{CB}$. Is every such triangle congruent to $\triangle ABC$? Explain. **No; see the drawing below.**



d. Draw triangles that each have an angle congruent to $\angle A$ and an angle congruent to $\angle B$. Is every such triangle congruent to $\triangle ABC$? Explain.
No. These triangles all have the same shape but may have different sizes.

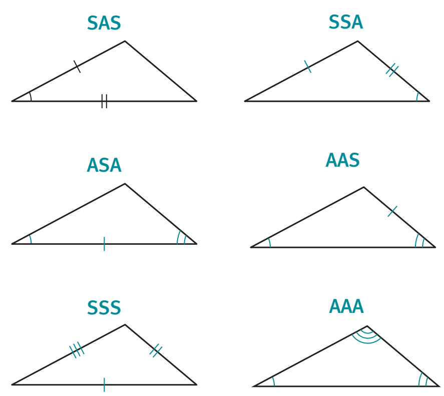
e. Consider the statement: "If two parts of one triangle are congruent to the corresponding parts in a second triangle, then the triangles must be congruent." Prove or disprove this statement. Cite the triangles you constructed.
Disprove. Students' triangles serve as counterexamples.

My Notes

Greg decides that he must have at least three pairs of congruent parts in order to conclude that two triangles are congruent. In order to work more efficiently, he decides to make a list of all possible combinations of three congruent parts.

3. Greg uses *A* as an abbreviation to represent angles and *S* to represent sides. For example, if Greg writes *SAS*, it represents two sides and the included angle, as shown in the first triangle below. Here are the combinations in Greg's list: SAS, SSA, ASA, AAS, SSS, and AAA.

a. Mark each triangle below to illustrate the combinations in Greg's list.
Answers may vary. Sample answers.



b. Are there any other combinations of three parts of a triangle? If so, is it necessary for Greg to add these to his list? Explain.
SAA and ASS. It is not necessary to add these to the list since SAA is the same as AAS and ASS is the same as SSA.

ACTIVITY 11 Continued

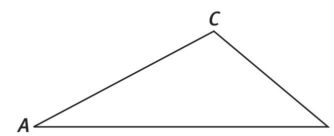
3 Shared Reading, Debriefing, Think-Pair-Share, Quickwrite In this item, it is important that students understand how to mark a particular combination of angles and sides. For example, note the difference between the triangle marked for *SAS* and the one marked for *SSA*. In Part a, students should mark the appropriate parts of each triangle and use Think-Pair-Share to compare their answers with others in their group before moving on to Part b. In Part b, students should recognize that the other combinations are ASS and SAA. However, ASS and SSA, and SAA and AAS are the same combinations. Therefore, ASS and SAA do not have to be added to the list.

4 Debriefing, Use Manipulatives
Provide students with materials for copying angles and segments. There are many different materials that can be used: tracing or patty paper, acetate, and coffee stirrers are a few suggestions. To use acetate or tracing paper, each student will need six separate pieces. Students should carefully copy the given figures, one per piece, onto the paper or acetate. They should then use the figure numbers to label each figure. To use coffee stirrers, students cut a coffee stirrer to the same length as a segment. To copy an angle, students put two coffee stirrers together to form an angle the same size as those in the figures shown, and then staple the stirrers together at the vertex. Reinforce the understanding that when copying an angle, the lengths of the sides of the angle may vary, but the degree measure of the angle must remain constant.

My Notes

Now Greg wants to find out *which* pairs of congruent corresponding parts guarantee congruent triangles.

4. Three segments congruent to the sides of $\triangle ABC$ and three angles congruent to the angles of $\triangle ABC$ are given in Figures 1–6, shown below.



a. If needed, use manipulatives supplied by your teacher to recreate the six figures given below. **Check students' figures**

Figure 1

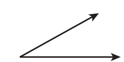


Figure 3

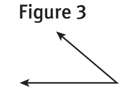


Figure 2

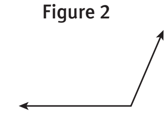


Figure 4 ______________________

Figure 5 ________________

Figure 6 ____________

b. Identify which of the figures in part a is congruent to each of the parts of $\triangle ABC$.

$\angle A$: **Figure 1** $\angle B$: **Figure 3**

$\angle C$: **Figure 2** $\overline{AB}$: **Figure 4**

$\overline{CB}$: **Figure 6** $\overline{AC}$: **Figure 5**

Lesson 11-2
Congruence Criteria

ACTIVITY 11
continued

My Notes

5. For each combination in Greg's list in Item 3, choose three appropriate triangle parts from Item 4. Each student should create a triangle using these parts. Then use transformations to check whether every such triangle is congruent to $\triangle ABC$. Use the table to organize your results.

Combination	Name the Three Figures Used by Listing the Figure Numbers	Is Every Such Triangle Congruent to $\triangle ABC$?
SSS	4, 5, 6	Yes
SAS	4, 1, 5 5, 2, 6 6, 3, 4	Yes
ASA	1, 5, 2 2, 6, 3 3, 4, 1	Yes
AAS	1, 2, 6 1, 3, 6 2, 1, 4 2, 3, 4 3, 1, 5 3, 2, 5	Yes
AAA	1, 2, 3	No
SSA	1, 5, 6 2, 6, 4 3, 4, 5 4, 5, 2 5, 6, 3 6, 4, 1	No

6. Express regularity in repeated reasoning. Compare your results from Item 5 with those of students in other groups. Then list the different combinations that seem to guarantee a triangle congruent to $\triangle ABC$. These combinations are called ***triangle congruence criteria***.
SSS, SAS, ASA, AAS

7. Do you think there is an AAA triangle congruence ***criterion***? Why or why not?
No. We have shown that it is possible for two triangles to have three pairs of congruent corresponding angles but not be congruent.

ACADEMIC VOCABULARY

A ***criterion*** (plural: *criteria*) is a standard or rule on which a judgment can be based. Criteria exist in every subject area. For example, a scientist might use a set of criteria to determine whether a sample of water is safe for human consumption.

ACTIVITY 11 Continued

5 Discussion Groups, Create Representations, Use Manipulatives
In order to investigate the combinations of three congruent parts, it is necessary for students to build the triangles using the triangle parts they created in Item 4. Students should list the combinations SSS, SAS, ASA, AAS, AAA, and SSA in the table in any order. Next, they should investigate the combinations by attempting to build a triangle that is not congruent to $\triangle ABC$, but that has three corresponding congruent parts. It is especially important for students to understand the need for the parts to be both congruent and corresponding. If students are having trouble completing the table in a timely manner, consider using the Discussion Groups strategy by dividing the class into six expert groups, one for each combination. Each expert group should work only on one combination. Students should then be regrouped into groups of at least six students, one from each of the expert groups, to share what they learned and complete the table.

6–7 Interactive Word Wall, Debriefing, Think-Pair-Share
Students compare their results in the table in Item 5 with members of their group and the class. The four combinations that the students should identify as seeming to guarantee congruent triangles are SSS, SAS, ASA, and AAS. Debriefing is especially important here to make sure all students have identified the four triangle congruence criteria before moving on.

Example A Debriefing Students should identify which triangle congruence criterion applies. They must identify some congruent parts that are not marked in the figures. Students may not be familiar with the arrow notation along the sides of the figure in Try These Item b. Explain that the arrows indicate that the sides are parallel.

ELL Support

Understanding and using the terms *criterion* and its plural *criteria* may be challenging for most students, in part because the plural is not formed in the usual way (by adding an *s*). Reinforce for ELL students that *criterion* and *criteria* are the singular and plural forms of the same word.

My Notes

MATH TIP

You can use theorems about vertical angles, midpoints, bisectors, and parallel lines that are cut by a transversal to identify additional congruent parts that may not be marked in the figure.

Greg realizes that it is not necessary to check all six pairs of corresponding parts to determine if two triangles are congruent. The triangle congruence criteria can be used as "shortcuts" to show that two triangles are congruent.

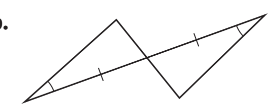
Example A

For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a.

Since three pairs of corresponding sides are congruent, the triangles are congruent by SSS.

b.

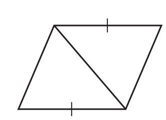


Although they are not marked as such, the vertical angles in the figure are congruent. The triangles are congruent by ASA.

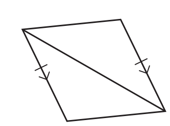
Try These A

For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a. none



b. SAS



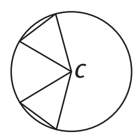
My Notes

Check Your Understanding

8. Two triangles each have two sides that are 8 feet long and an included angle of 50°. Must the two triangles be congruent? Why or why not?

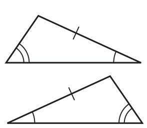
9. Two equilateral triangles each have a side that is 5 cm long. Is it possible to conclude whether or not the triangles are congruent? Explain.

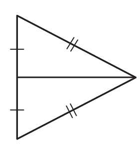
10. The figure shows a circle and two triangles. For both triangles, two vertices are points on the circle and the other vertex is the center of the circle. What information would you need in order to prove that the triangles are congruent?

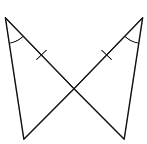


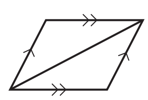
LESSON 11-2 PRACTICE

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

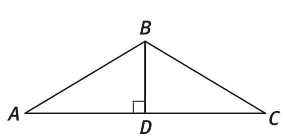
11.


12.


13.


14.


15. Make sense of problems. What one additional piece of information do you need, if any, in order to conclude that $\triangle ABD \cong \triangle CBD$? Is there more than one set of possible information? Explain.



CONNECT TO AP

The free-response items on the AP Calculus exam will often ask you to *justify* your answer. Such justifications should follow the same rules of deductive reasoning as the proofs in this geometry course.

ACTIVITY 11 Continued

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to triangle congruence criteria. When students answer Item 10, you may also want to have them explain how they arrived at their answer.

Answers

8. Yes. Two corresponding sides and the corresponding included angle are congruent, so the triangles are congruent by SAS congruence.

9. Yes. Sample answer. An equilateral triangle has three sides with the same measure. If one side of an equilateral triangle measures 5 cm, then all sides measure 5 cm. Because the two triangles both have sides measuring 5 cm, the triangles are congruent by SSS congruence.

10. If the distance between the two vertices on the circle are the same for both triangles, then the triangles are congruent by SSS congruence. Also, if the vertex at the center of the circle has the same measure for both triangles, then the triangles are congruent by SAS congruence.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand basic concepts related to triangle congruence criteria. Remind students that there is no SSA triangle congruence criterion.

LESSON 11-2 PRACTICE

11. none
12. SSS
13. ASA
14. ASA
15. Three possible additional pieces of information can prove $\triangle ABD \cong \triangle CBD$. In $\triangle ABD$ and $\triangle CBD$, we know that $\angle ADB$ is congruent to $\angle CDB$, since $\overline{AC}$ is perpendicular to $\overline{BD}$, therefore $\angle ADB$ and $\angle CDB$ are both right angles. Side BD is a common side of both triangles, so $\overline{BD}$ is congruent to $\overline{BD}$. Since an angle and the adjacent side are congruent in the triangles, then the following can prove the two triangles congruent: If $\angle DBA$ and $\angle DBC$ are congruent, the triangles are congruent by ASA. If sides $\overline{AD}$ and $\overline{CD}$ are congruent, the triangles are congruent by SAS. If $\angle C$ is congruent to $\angle A$, the triangles are congruent by AAS.

Lesson 11-3

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2 #3
#4 #5 Example A
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Determine if there is enough information to tell whether the triangles in each pair are congruent and, if they are, by what congruence criterion.

1. Two triangles each have two sides that are 3 inches long. [*not enough information*]
2. Two triangles each have two sides that are 3 inches long and an included angle of 60°. [*congruent; SAS*]
3. Two triangles each have two sides that are 3 inches long and one side that is 4 inches long. [*congruent; SSS*]

1 Close Reading, Marking the Text, Think Aloud, Discussion Groups In Item 1, students justify the SSS triangle congruence criterion by starting with two triangles that have three pairs of congruent corresponding sides and using a sequence of rigid motions to map one triangle to the other. Although the SSS, SAS, and ASA congruence criteria are postulates and therefore do not have to be proven, students may benefit from understanding why the criteria work based on a demonstration that follows from the definition of congruence based on rigid motions. The argument in Item 1 contains numerous steps and may be difficult to follow, so encourage students to read closely and mark the diagrams to keep track of congruent corresponding parts. Parts a through c set up the problem by having students analyze the diagram to determine exactly what information is given. In Part d, students draw a new figure that maps $\overline{AB}$ to $\overline{DE}$ but does not map point C to point F. Because the rest of the proof depends on drawing this diagram correctly, have students share their diagrams in the course of the group discussion and help students whose diagrams are incorrect to draw the correct diagram. As students complete the remainder of the steps in the proof, have them focus their attention to the use of the Perpendicular Bisector Theorem in Part g, as applying it is a common strategy for showing that two segments are congruent.

My Notes

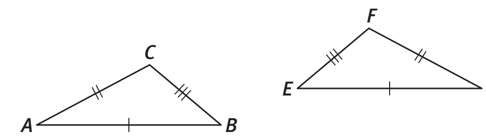
Learning Targets:

- Prove that congruence criteria follow from the definition of congruence.
- Use the congruence criteria in simple proofs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think Aloud, Discussion Groups, Visualization

You can use the SSS, SAS, or ASA congruence criteria as shortcuts to show that two triangles are congruent. In order to prove *why* these criteria work, you must show that they follow from the definition of congruence in terms of rigid motions.

1. To justify the SSS congruence criterion, consider the two triangles below.

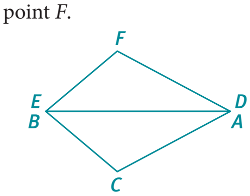


a. What given information is marked in the figure?
$\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$

b. Based on the definition of congruence, what do you need to do in order to show that $\triangle ABC \cong \triangle DEF$? **Show that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.**

c. It is given that $\overline{AB} \cong \overline{DE}$. What does this tell you?
There is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$.

d. Draw a figure to show the result of the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Assume that this sequence of rigid motions does *not* map point C to point F.

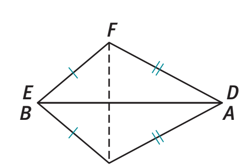


MATH TIP

You are asked to assume that the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$ does not map C to F. If it did map C to F, you would have found a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ and the proof would be complete!

e. Which points coincide in your drawing? Which segments coincide?
Points A and D coincide; points B and E coincide; $\overline{AB}$ and $\overline{DE}$ coincide.

f. Mark the line segments that you know are congruent in the figure below.



My Notes

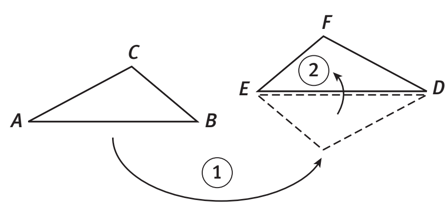
g. Based on the figure, how is $\overline{ED}$ related to $\overline{FC}$? Why?
$\overline{ED}$ is the perpendicular bisector of $\overline{FC}$. This is because E is equidistant from F and C, so it must lie on the perpendicular bisector of $\overline{FC}$ by the Perpendicular Bisector Theorem. Similarly, D must lie on the perpendicular bisector of $\overline{FC}$. Therefore, $\overline{ED}$ is the perpendicular bisector of $\overline{FC}$.

h. Consider the reflection of $\triangle ABC$ across $\overline{ED}$. What is the image of $\triangle ABC$? How do you know?
The image of $\triangle ABC$ is $\triangle DEF$. Points A and B are fixed under this reflection since they lie on the line of reflection. Point C maps to point F since the line of reflection is the perpendicular bisector of $\overline{FC}$.

MATH TIP

The Perpendicular Bisector Theorem states that a point lies on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the segment.

The argument in Item 1 shows that a sequence of rigid motions maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions is a rotation that maps $\overline{AB}$ to $\overline{DE}$, followed by a reflection across $\overline{ED}$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.



2. In the proof, is it important to know exactly which rigid motions map $\overline{AB}$ to $\overline{DE}$? Explain.
Although it is not necessary to specify the exact sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, doing so is a useful strategy. It shows that such a rigid motion does exist.

3. **Attend to precision.** How is the definition of a reflection used in the proof?
The definition of a reflection across a line ℓ states that the reflection maps ℓ to ℓ, and that any point P that is not on ℓ is mapped to P' such that ℓ is the perpendicular bisector of $\overline{PP'}$. In the proof, line ℓ is $\overline{ED}$ and we show that it is the perpendicular bisector of $\overline{FC}$.

ACTIVITY 11 Continued

2–3 Summarizing, Levels of Questions, Activating Prior Knowledge, Debriefing In Items 2 and 3, students reflect on the proof they just completed. They come to understand that on a universal level, it is only important that there exists a sequence of rigid motions that maps one triangle to the other, but it is not necessary to specify each one. On a more specific level, however, students can greatly benefit by identifying and analyzing each transformation in the sequence to understand exactly how it is used in the proof. This benefit is not only useful for understanding the proof at hand but also for developing a collection of strategies to be used in future proofs. For example, in Item 3, students revisit the basic definition of a reflection and translate it to apply to this particular proof.

Teacher to Teacher

As students become more proficient with writing proofs, they gain experience with various techniques and strategies that may not be available to the novice proof writer. To beginners, drawing the diagram that maps $\overline{AB}$ to $\overline{DE}$ might not seem like the next logical step in the argument. Because diagrams in which triangles share a side are often used in proofs, it is important to ensure that students have a complete understanding of why this step is taken.

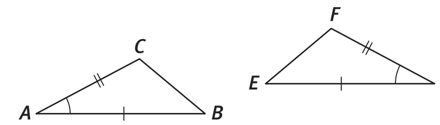
4 Activating Prior Knowledge, Create Representations, Debriefing In Item 4, students continue the process of justifying the postulates for triangle congruence criteria, using the same two triangles as in Item 1. Students begin the proof in the same way as when they proved the SSS criterion: by drawing a diagram that maps $\overline{AB}$ to $\overline{DE}$. This time, however, they are proving SAS, so in addition to showing that a sequence of rigid motions maps two pairs of sides to each other, they will have to find a way to show that both included angles are mapped to each other as well. Point out that the same technique is used to draw a diagram with the triangles forming a quadrilateral as was used in the previous proof. Ask students why they think it is useful to draw the diagram this way. Guide them to understand that since they are trying to show congruency by a sequence of rigid motions, placing the triangles so that one pair of sides forms a line of reflection makes it easy to use a reflection as one of the rigid motions in the sequence.

My Notes

MATH TIP

Your proof should use all three pieces of given information that you identified in part a. If you find that you are not using all of these facts, you may be missing an element of the proof.

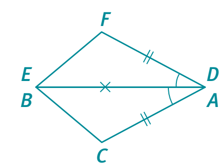
4. To justify the SAS congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

$\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Draw a figure at the right to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does *not* map point C to point F.



c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? Why?

The image of $\overline{AC}$ is $\overline{DF}$ because $\overline{ED}$ is the angle bisector of $\angle CDF$.

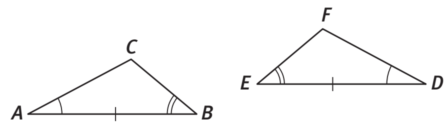
e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point C is point F? Explain.

Yes. Since the image of $\overrightarrow{AC}$ is $\overrightarrow{DF}$, the image of point C must lie on $\overrightarrow{DF}$. However, $\overline{AC} \cong \overline{DF}$, so the image of point C must be the same distance from point D as point F. That means the image of point C is point F.

The argument in Item 4 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, followed by the reflection across $\overline{ED}$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

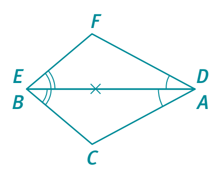
My Notes

5. To justify the ASA congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps AB to DE. Draw a figure at the left to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does *not* map point C to point F.



c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? What is the image of $\overline{BC}$? Why?
The image of $\overline{AC}$ is $\overline{DF}$ because $\overline{ED}$ is the angle bisector of $\angle CDF$.
The image of $\overline{BC}$ is $\overline{EF}$ because $\overline{ED}$ is the angle bisector of $\angle CEF$.

e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point C is point F? Explain.
Yes. Since the image of $\overline{AC}$ is $\overline{DF}$, the image of point C must lie on $\overline{DF}$. Since the image of $\overline{BC}$ is $\overline{EF}$, the image of point C must lie on $\overline{EF}$. The only point that lies on both $\overline{DF}$ and $\overline{EF}$ is point F. So the image of point C must be point F.

ACTIVITY 11 Continued

5 Activating Prior Knowledge, Create Representations, Look for a Pattern, Graphic Organizer In Item 5, students conclude their proofs of the three triangle congruence criteria by justifying ASA. Again, the two triangles are the same as in the proofs of SSS and SAS, and the students begin by drawing the same diagram that maps $\overline{AB}$ to $\overline{DE}$. By this point, students should be recognizing a pattern as to how a diagram can be drawn in order to use a reflection as one of the transformations in a sequence of rigid motions to prove congruency of two triangles. Suggest that if they haven't done so already, students create some sort of graphic organizer to record useful strategies for future proofs.

Example A Visualization, Close Reading, Summarizing, Debriefing

Students now use the triangle congruence postulates to solve a problem that requires proof. Review with students the real-world details of the situation and have them study the diagram. Ask students how Greg was able to determine the information that is given [*by measuring*]. Also point out the use of a definition of a midpoint as reason #2. Explain that using a geometric definition as a reason in a proof is a common strategy. Conclude the example by having students summarize what they have learned so far about triangle congruence criteria and how they can be used to solve problems.

Universal Access

A common error students make when proving triangles congruent is not stating that a side is congruent to itself by the Reflexive Property of Congruence. Although such a statement may seem redundant, it is mandatory when proving triangles congruent by any criteria in which the side is included.

Try These A

a. Yes.

Statements	Reasons
1. $\overline{JX} \cong \overline{KX}$	**1.** Given
2. $\overline{LX} \perp \overline{JK}$	**2.** Given
3. $\angle LXK$ and $\angle LXJ$ are right angles.	**3.** Perpendicular lines intersect to form right angles.
4. $\angle LXK \cong \angle LXJ$	**4.** All right angles are congruent.
5. $\overline{LX} \cong \overline{LX}$	**5.** Congruence is reflexive.
6. $\triangle JXL \cong \triangle KXL$	**6.** SAS Congruence Postulate

b. No. Only two parts of the triangles can be proven congruent; an angle and a side. Since $\overline{LX}$ bisects $\angle JLK$, $\angle XLJ$ and $\angle XLK$ are congruent by the definition of angle bisection, and $\overline{LX} \cong \overline{LX}$ because congruence is reflexive. We would need to be able to prove that sides *LJ* and *LK* are congruent in order to use the SAS Congruence Postulate, or prove that $\angle LXJ$ and $\angle LXK$ are congruent in order to use the ASA Congruence Postulate, or prove that $\angle J$ and $\angle K$ are congruent in order to use the AAS Congruence Postulate.

ACTIVITY 11
continued

My Notes

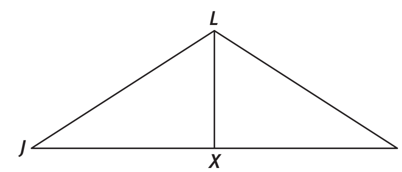
Lesson 11-3
Proving and Applying the Congruence Criteria

The argument in Item 5 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, followed by the reflection across $\overline{ED}$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

Now you can use the SSS, SAS, and ASA congruence criteria to prove that triangles are congruent.

Example A

Greg knows that point *X* is the midpoint of $\overline{JK}$ in the truss shown below. He also makes measurements and finds that $\overline{JL} \cong \overline{KL}$. He must prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$.



Given: *X* is the midpoint of $\overline{JK}$; $\overline{JL} \cong \overline{KL}$.

Prove: $\triangle JXL \cong \triangle KXL$

Statements	Reasons
1. *X* is the midpoint of $\overline{JK}$.	**1.** Given
2. $\overline{JX} \cong \overline{KX}$	**2.** Definition of midpoint
3. $\overline{JL} \cong \overline{KL}$	**3.** Given
4. $\overline{LX} \cong \overline{LX}$	**4.** Congruence is reflexive.
5. $\triangle JXL \cong \triangle KXL$	**5.** SSS

Try These A

a. Suppose that Greg knew instead that $\overline{LX}$ was perpendicular to $\overline{JK}$ and suppose he made measurements to find that $\overline{JX} \cong \overline{KX}$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

b. Suppose that Greg knew instead that $\overline{LX}$ bisects $\angle JLK$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

Check Your Understanding

6. In the Example, can Greg conclude that $\angle J \cong \angle K$? Why or why not?

7. Draw a figure that contains two triangles. Provide given information that would allow you to prove that the triangles are congruent by the ASA congruence criterion.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts proving that triangles are congruent.

Answers

6. Yes. The two triangles have been proven congruent in the example, and since $\angle J$ and $\angle K$ are corresponding angles in the congruent triangles, the conclusion can be made that the two angles are congruent.

7. Check students' drawings and statements. Students must be able to use the drawing and statements to prove that two angles and the included side of one triangle are congruent to two angles and the included side of the other triangle.

Lesson 11-3
Proving and Applying the Congruence Criteria

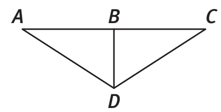
ACTIVITY 11
continued

My Notes

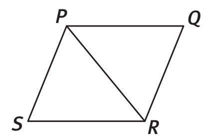
LESSON 11-3 PRACTICE

For Items 8–10, write each proof as a two-column proof.

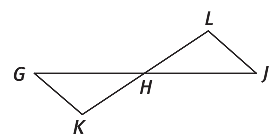
8. Given: $\overline{AD} \cong \overline{CD}$;
$\angle ADB \cong \angle CDB$
Prove: $\triangle ADB \cong \triangle CDB$



9. Given: $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$
Prove: $\triangle PQR \cong \triangle RSP$

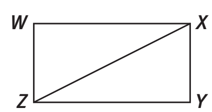


10. Given: $\angle K \cong \angle L$;
$\overline{KH} \cong \overline{LH}$
Prove: $\triangle GKH \cong \triangle JLH$



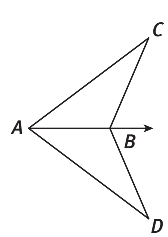
11. Critique the reasoning of others. A student wrote the proof shown below. Critique the student's work and correct any errors he or she may have made.

Given: $\overline{WX} \cong \overline{YZ}$;
$\overline{ZW} \cong \overline{XY}$
Prove: $\triangle ZWX \cong \triangle XYZ$



Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$	**1.** Given
2. $\overline{ZW} \cong \overline{XY}$	**2.** Given
3. $\angle W$ and $\angle Y$ are right angles.	**3.** Given
4. $\angle W \cong \angle Y$	**4.** All right angles are congruent.
5. $\Delta ZWX \cong \Delta XYZ$	**5.** SAS

12. Model with mathematics. A graphic designer made a logo for an airline, as shown below. The designer made sure that $\overline{AB}$ bisects $\angle CAD$ and that $\overline{AC} \cong \overline{AD}$. Can the designer prove that $\triangle ABC \cong \triangle ABD$? Why or why not?



ACTIVITY 11 Continued

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 11-3 PRACTICE

8.

Statements	Reasons
1. $\overline{AD} \cong \overline{CD}$	**1.** Given
2. $\angle ADB \cong \angle CDB$	**2.** Given
3. $\overline{BD} \cong \overline{BD}$	**3.** Congruence is reflexive.
4. $\triangle ADB \cong \triangle CDB$	**4.** SAS Congruence Postulate

9.

Statements	Reasons
1. $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$	**1.** Given
2. $\overline{PQ} \cong \overline{RS}$	**2.** Transitive Property
3. $\overline{QR} \cong \overline{SP}$	**3.** Transitive Property
4. $\overline{PR} \cong \overline{PR}$	**4.** Congruence is reflexive.
5. $\triangle PQR \cong \triangle RSP$	**5.** SSS Congruence Postulate

10.

Statements	Reasons
1. $\angle K \cong \angle L$	**1.** Given
2. $\overline{KH} \cong \overline{LH}$	**2.** Given
3. $\angle LHJ \cong \angle GHJ$	**3.** Vertical angles are congruent.
4. $\triangle GKH \cong \triangle JLH$	**4.** ASA Congruence Postulate

11. The student incorrectly assumed that $\angle W$ and $\angle Y$ are right angles and wrote that statement as a given statement. Then the student concluded that $\angle W \cong \angle Y$ and found the triangles congruent by SAS. The student should have proven the triangles congruent by SSS after proving side $\overline{XZ} \cong \overline{XZ}$.

The correct proof is shown:

Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$	**1.** Given
2. $\overline{ZW} \cong \overline{XY}$	**2.** Given
3. $\overline{XZ} \cong \overline{XZ}$	**3.** Congruence is reflexive.
4. $\triangle ZWX \cong \triangle XYZ$	**4.** SSS Congruence Postulate

12. Yes. $\overline{AC} \cong \overline{AD}$ is given. Since $\overline{AB}$ bisects $\angle CAD$, $\angle CAB$ and $\angle DAB$ are congruent, and $\overline{AB} \cong \overline{AB}$ because congruence is reflexive. Therefore, the triangles can be proven congruent by the SAS Congruence Postulate.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand basic concepts related to using triangle congruence postulates to prove that two triangles are congruent. Encourage students to mark their drawings to show given or otherwise known congruent corresponding sides and angles.

Lesson 11-4

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2 #3

#4–5 #6

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Use the Pythagorean Theorem to find the missing side of each right triangle.

1. leg: 6 inches; leg: 8 inches [*10 in.*]
2. leg: 5 feet; hypotenuse: 13 feet [*12 ft*]
3. leg: 3 meters; hypotenuse: 4 meters [$\sqrt{7}\,m$]

1 Activating Prior Knowledge, Identify a Subtask In Item 1, students explore how triangle congruence criteria can be used in the coordinate plane. Remind students that while rulers and protractors are used to measure distance off the coordinate plane, they can use the Distance Formula in the situation. If students are having difficulty, work through the first task of finding the distance between $B(-2, 1)$ and $D(-5, -2)$. Using the Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ gives $\sqrt{(-5-(-2))^2 + (-2-1)^2} = \sqrt{(-3)^2 + (-3)^2}$. This can be simplified as $\sqrt{18}$, or $3\sqrt{2}$.

2 Discussion Groups, Predict and Confirm In Item 2, students begin reasoning about the fourth congruence criterion, AAS. Point out that while SSS, SAS, and ASA are all postulates and do not need to be proven, AAS is a theorem, and therefore requires proof. Encourage the class to begin discussing how they are going to prove the AAS congruence criterion. Ask them to predict how the proof of AAS may be different from the proofs of SSS, SAS, and ASA.

My Notes

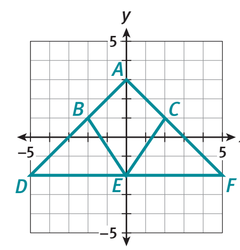
Learning Targets:

- Apply congruence criteria to figures on the coordinate plane.
- Prove the AAS criterion and develop the HL criterion.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing

You can use the triangle congruence criteria on the coordinate plane.

1. Reason quantitatively. Greg's boss hands him a piece of graph paper that shows the plans for a truss. Greg's boss asks him if he can prove that $\triangle DBE$ is congruent to $\triangle FCE$.



a. Use the distance formula to find each length.

$BD =$ $3\sqrt{2}$ $CF =$ $3\sqrt{2}$

$DE =$ 5 $FE =$ 5

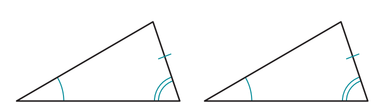
$BE =$ $\sqrt{13}$ $CE =$ $\sqrt{13}$

b. Can Greg use this information to prove that $\triangle DBE \cong \triangle FCE$? Explain.

Yes; $\triangle DBE \cong \triangle FCE$ by the SSS congruence criterion.

2. In Item 5 of Lesson 11-2, you discovered that SSS, SAS, and ASA are not the only criteria for proving two triangles are congruent. You also discovered that there is an AAS congruence criterion. What does the AAS congruence criterion state? Mark the triangles below to illustrate the statement.

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and nonincluded side of another triangle, then the triangles are congruent.



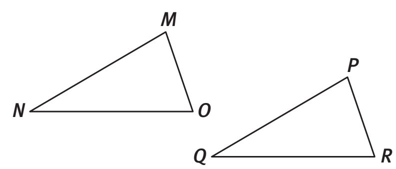
Lesson 11-4
Extending the Congruence Criteria

My Notes

3. The proof of the AAS congruence criterion follows from the other congruence criteria. Complete the statements in the proof of the AAS congruence criterion below.

Given: $\triangle MNO$ and $\triangle PQR$ with $\angle N \cong \angle Q$, $\angle O \cong \angle R$, and $\overline{MO} \cong \overline{PR}$

Prove: $\triangle MNO \cong \triangle PQR$



Statements	Reasons
1. $\triangle MNO$ and $\triangle PQR$	**1.** Given
2. $m\angle M + m\angle N + m\angle O = 180°$, $m\angle P + m\angle Q + m\angle R = 180°$	**2.** The sum of the measures of the angles of a triangle is 180°.
3. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle Q + m\angle R$	**3.** Transitive Property of Equality
4. $\angle N \cong \angle Q$; $\angle O \cong \angle R$	**4.** Given
5. $m\angle N = m\angle Q$; $m\angle O = m\angle R$	**5.** Definition of congruent angles
6. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle N + m\angle O$	**6.** Substitution Property of Equality
7. $m\angle M = m\angle P$	**7.** Subtraction Property of Equality
8. $\angle M \cong \angle P$	**8.** Definition of congruent angles
9. $\overline{MO} \cong \overline{PR}$	**9.** Given
10. $\triangle MNO \cong \triangle PQR$	**10.** ASA

ACTIVITY 11 Continued

3 Think-Pair-Share, Close Reading, Marking the Text, Self Revision/Peer Revision In Item 3, students complete the reasons in a two-column proof of the AAS congruence criterion. Since the proof follows from the other congruence criteria, students may feel as if they have seen it before and therefore gloss over the details. Caution against this response as this proof is actually quite different from the others. Encourage students to closely read all statements in the proof and to mark the text and diagram freely, making copies if necessary. If students are having difficulties, suggest that they pair up, review what has been completed so far, and devise a strategy to complete the proof. Remind students that there may be more than one way to express some reasons. For example, one way to express reason #2 is as a sentence: "The sum of the measures of the angles of a triangle is 180°." Another way is to simply write "Triangle Sum Theorem."

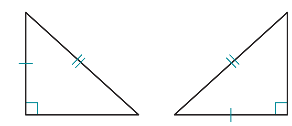
4–5 Think-Pair-Share, Activating Prior Knowledge, Look for a Pattern
Each of the pairs of triangles should be marked SSA; however, only those shown in Items 4a and 4c are congruent triangles. Encourage students to look for patterns as they try to determine what the congruent triangles have in common. If students are having difficulty finding the common characteristic, ask them to think about ways that triangles can be classified.

My Notes

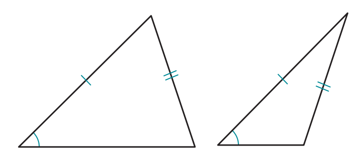
Lesson 11-4
Extending the Congruence Criteria

4. Below are pairs of triangles in which congruent parts are marked. For each pair of triangles, name the angle and side combination that is marked and tell whether the triangles *appear* to be congruent.

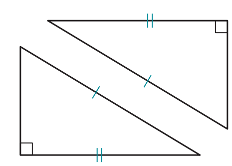
a. SSA; yes



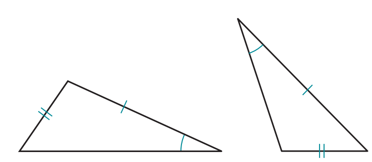
b. SSA; no



c. SSA; yes



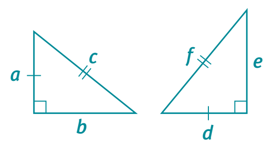
d. SSA; no



5. We know that in general SSA does not always determine congruence of triangles. However, for two of the cases in Item 4 the triangles appear to be congruent. What do the congruent pairs of triangles have in common?
They are right triangles.

My Notes

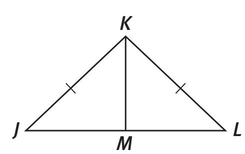
6. In a right triangle, we refer to the correspondence SSA shown in Items 4a and 4c as *hypotenuse-leg* (HL). Write a convincing argument in the space below to prove that HL will ensure that right triangles are congruent.
Answers may vary. Using the Pythagorean Theorem, the third sides of the triangles can be shown to be congruent. If the third sides are congruent, then the triangles are congruent by SSS or SAS. The triangles from Item 4a can be labeled as shown below.



From the figure it follows that $c^2 = a^2 + b^2$ and $f^2 = d^2 + e^2$. Since $c = f$, $c^2 = f^2$. Therefore, $a^2 + b^2 = d^2 + e^2$. Also, $a = d$, so $a^2 + b^2 = a^2 + e^2$ and $b^2 = e^2$, which means $b = e$ and the triangles can be proved congruent by SSS or SAS.

Check Your Understanding

7. Is it possible to prove that $\triangle LKM \cong \triangle JKM$ using the HL congruence criterion? If not, what additional information do you need?



8. Do you think there is a leg-leg congruence criterion for right triangles? If so, what does the criterion say? If not, why not? Review your answers. Be sure to check that you have described the situation with specific details, included the correct mathematical terms to support your reasoning, and that your sentences are complete and grammatically correct.

6 Debriefing, Group Presentation

Students should recognize that since the triangles are right triangles and two pairs of sides are congruent, the Pythagorean Theorem can be used to prove the third pair of sides congruent. In this way, the triangles are congruent by SSS. Thus, HL is a valid way to prove two *right* triangles are congruent.

Teacher to Teacher

To help students review what they've learned in this activity, present the following scenario: Greg and his boss, John, want to discuss the findings about the trusses for the new recreation hall, but John is out of town. So, John asked Greg to e-mail a report to him, summarizing his findings. Included in the report could be a brief description of how Greg arrived at his triangle congruence criteria and how he will use the criteria in the recreational hall trusses.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to the AAS and HL congruence criteria.

Answers

7. No. It would have to be proven that $\angle KMJ$ and $\angle KML$ are right angles.

8. A leg-leg congruence of a right triangle can prove two right triangles congruent, but a leg-leg congruence is not necessary, since it is really the same as a hypotenuse-leg congruence. If the two legs of one triangle are known to be congruent to the two legs of another triangle, then the hypotenuse are also congruent.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 11-4 PRACTICE

9. Check students' triangles. Using the Distance Formula, $AB = \sqrt{13}$, $BC = \sqrt{5}$, and $CA = 2\sqrt{5}$; $DE = \sqrt{13}$, $EF = \sqrt{5}$, and $FD = 2\sqrt{5}$. Since $AB = DE$, $BC = EF$, and $CA = FD$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$, by definition of congruent segments. Therefore, $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate.
10. HL
11. AAS
12. none
13. none

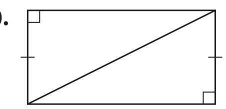
ADAPT

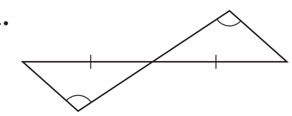
Check students' answers to the Lesson Practice to ensure that they understand basic concepts related to the HL congruence criterion and the application of congruence criteria to figures on the coordinate plane. Remind students that just because two triangles look congruent that does not mean that they are. To prove congruence one must show that all of the conditions of one of the triangle congruence criteria are met.

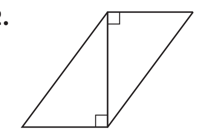
LESSON 11-4 PRACTICE

9. **Construct viable arguments.** On a coordinate plane, plot triangles ABC and DEF with vertices $A(-3, -1)$, $B(-1, 2)$, $C(1, 1)$, $D(3, -4)$, $E(1, -1)$, and $F(-1, -2)$. Then prove $\triangle ABC \cong \triangle DEF$.

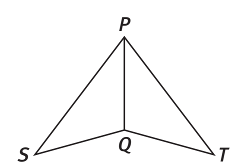
For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

10. 

11. 

12. 

13. $\overline{PQ}$ bisects $\angle SPT$.

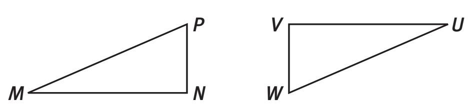


ACTIVITY 11 PRACTICE

Write your answers on notebook paper. Show your work.

1. $\triangle FIX \cong \triangle TOP$. Complete the following.
   a. Name three pairs of corresponding vertices.
   b. Name three pairs of corresponding sides.
   c. Name three pairs of corresponding angles.
   d. Is it correct to say that $\triangle POT \cong \triangle XIF$? Why or why not?
   e. Is it correct to say that $\triangle IFX \cong \triangle PTO$? Why or why not?

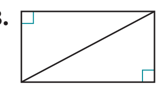
In the figure, $\triangle MNP \cong \triangle UVW$. Use the figure for Items 2–4.



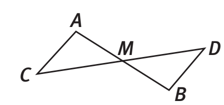
2. Write six congruence statements about line segments and angles in the figure.
3. Suppose $MN = 14$ cm and $m\angle U = 24°$. What other side lengths or angle measures can you determine? Why?
4. Suppose $MN = 2PN$ and $VW = 8$ in. Find MN.
5. If $\triangle MAP \cong \triangle TON$, $m\angle M = 80°$, and $m\angle N = 50°$, name four congruent angles.
6. $\triangle ABC \cong \triangle DEF$, $AB = 15$, $BC = 20$, $AC = 25$, and $FE = 3x - 7$. Find x.
7. $\triangle MNO \cong \triangle PQR$, $m\angle N = 57°$, $m\angle P = 64°$ and $m\angle O = 5x + 4$. Find x and $m\angle R$.

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

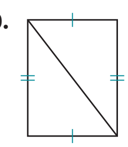
8.



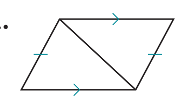
9. M bisects $\overline{AB}$ and $\overline{CD}$.



10.



11.



ACTIVITY 11 Continued

ACTIVITY PRACTICE

1. a. $\angle F$ and $\angle T$, $\angle I$ and $\angle O$, $\angle X$ and $\angle P$
   b. $\overline{FI}$ and $\overline{TO}$, $\overline{IX}$ and $\overline{OP}$, $\overline{XF}$ and $\overline{PT}$
   c. $\angle FIX$ and $\angle TOP$, $\angle IXF$ and $\angle OPT$, $\angle XFI$ and $\angle PTO$
   d. Yes. Since $\triangle FIX \cong \triangle TOP$, $\angle P$ corresponds with $\angle X$, $\angle O$ corresponds with $\angle I$, and $\angle T$ corresponds with $\angle F$, so it is correct to say that $\triangle POT \cong \triangle XIF$.
   e. No. Since $\triangle FIX \cong \triangle TOP$, $\angle I$ and $\angle P$ do not correspond, and $\angle X$ and $\angle O$ do not correspond, so it is not correct to say that $\triangle IFX \cong \triangle PTO$.
2. $\angle M$ and $\angle U$, $\angle N$ and $\angle V$, $\angle P$ and $\angle W$, $\overline{MN}$ and $\overline{UV}$, $\overline{NP}$ and $\overline{VW}$, $\overline{PM}$ and $\overline{WU}$
3. $\angle M$ and $\overline{UV}$ because $\angle U$ is congruent to $\angle M$, and $\overline{UV}$ is congruent to $\overline{MN}$.
4. 16 in.
5. $\angle A$, $\angle P$, $\angle O$ and $\angle N$
6. $x = 9$
7. $x = 11$, $m\angle R = 59°$
8. none
9. SAS
10. SSS
11. none
12.

Statements	Reasons
1. $\overline{EF}$ bisects $\angle GEH$	1. Given
2. $\angle GEF \cong \angle HEF$	2. Def. of $\angle$ bisector
3. $\overline{EF} \cong \overline{EF}$	3. $\cong$ is reflexive
4. $\overline{EF}$ bisects $\angle GFH$	4. Given
5. $\angle GFE \cong \angle HFE$	5. Def. of $\angle$ bisector
6. $\triangle EGF \cong \triangle EHF$	6. ASA

13.

Statements	Reasons
1. $\overline{PQ} \cong \overline{SR}$	1. Given
2. $\overline{PQ}$ is $\parallel$ to $\overline{SR}$	2. Given
3. $\angle PQS \cong \angle RSQ$	3. Alt. int. $\angle$ of $\parallel$ lines are $\cong$
4. $\overline{QS} \cong \overline{QS}$	4. $\cong$ is reflexive
5. $\triangle PQS \cong \triangle RSQ$	5. SAS

14.

Statements	Reasons
1. M is mdpt. of $\overline{AB}$	**1.** Given
2. $\overline{AM} \cong \overline{BM}$	**2.** Def. of mdpt.
3. $\angle AMD \cong \angle BMC$	**3.** Vert. $\angle$ are $\cong$
4. M is mdpt. of $\overline{CD}$	**4.** Given
5. $\overline{MD} \cong \overline{MC}$	**5.** Def. of mdpt.
6. $\triangle AMD \cong \triangle BMC$	**6.** SAS

15.

Statements	Reasons
1. $\overline{UV}$ is $\parallel$ to $\overline{WX}$	**1.** Given
2. $\angle VUY \cong \angle WXY$ and $\angle UVY \cong \angle XWY$	**2.** Alt. int. $\angle$ of $\parallel$ lines are $\cong$
3. $\overline{UV} \cong \overline{WX}$	**3.** Given
4. $\triangle UVY \cong \triangle XWY$	**4.** ASA

Students may choose to prove the triangles congruent using AAS Congruence Postulate. They can use the given side and prove either $\angle VUY \cong \angle WXY$ or $\angle UVY \cong \angle XWY$ using alternate interior angles of parallel lines, and they can also prove $\angle UYV \cong \angle XYW$, since they are vertical angles.

16. Check students' drawings. Students should include given information that allows them to prove two angles and a nonincluded side of one triangle congruent to two angles and a nonincluded side of the other triangle.

17. a. Using the Distance Formula, $AB = \sqrt{26}$ and $CD = \sqrt{26}$, so $\overline{AB} \cong \overline{CD}$. $AD = \sqrt{29}$ and $CB = \sqrt{29}$, so $\overline{AD} \cong \overline{CB}$. $\overline{BD} \cong \overline{BD}$ is reflexive. Therefore, $\triangle ABD \cong \triangle CDB$ by SSS.

b. Yes. $\angle A$ and $\angle C$ are corresponding angles of $\cong \triangle$; therefore, $\angle A \cong \angle C$.

18. a. In each triangle, each of the two shorter sides is $\sqrt{10}$ and the longest side is $2\sqrt{5}$. Since $(\sqrt{10})^2 + (\sqrt{10})^2 = (2\sqrt{5})^2$, the triangles are right with hypotenuses $\overline{PT}$ and $\overline{RT}$. Therefore, $\angle Q$ and $\angle S$ are right angles, so $\overline{RS}$ and $\overline{PQ}$ are $\perp$ to $\overline{QS}$.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

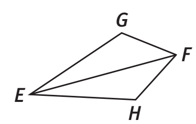
ACTIVITY 11 *continued*

Congruence Transformations and Triangle Congruence

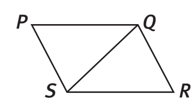
Truss Your Judgment

In Items 12–15, write each proof as a two-column proof.

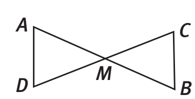
12. Given: $\overline{EF}$ bisects $\angle GEH$;
$\overline{EF}$ bisects $\angle GFH$.
Prove: $\triangle EGF \cong \triangle EHF$



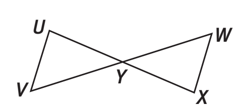
13. Given: $\overline{PQ}$ is parallel to $\overline{SR}$;
$\overline{PQ} \cong \overline{SR}$.
Prove: $\triangle PQS \cong \triangle RSQ$



14. Given: M is the midpoint of $\overline{AB}$;
M is the midpoint of $\overline{CD}$.
Prove: $\triangle AMD \cong \triangle BMC$

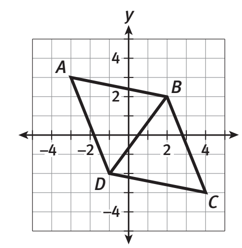


15. Given: $\overline{UV}$ is parallel to $\overline{WX}$;
$\overline{UV} \cong \overline{WX}$.
Prove: $\triangle UVY \cong \triangle XWY$

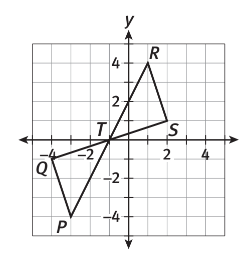


16. Draw a figure that includes two triangles. Provide given information that would allow you to prove that the triangles are congruent by the AAS Congruence Postulate.

17. a. Explain why $\triangle ABD \cong \triangle CDB$.
b. Can you conclude that $\angle A \cong \angle C$? Why or why not?



18. a. Explain why $\overline{RS}$ and $\overline{PQ}$ are perpendicular to $\overline{QS}$.
b. Explain how to use the SAS Congruence Postulate to show that $\triangle PQT \cong \triangle RST$.



MATHEMATICAL PRACTICES
Look For and Make Use of Structure

19. The opposite sides of a rectangle are congruent. Describe three different ways you could show that a diagonal divides a rectangle into two congruent triangles.

18. b. Sample answer: Use the Distance Formula to find that $\overline{RT}$ and $\overline{PT}$ are the same lengths and are therefore $\cong$. Use vertical angles to show that $\angle PTQ$ and $\angle RTS$ are $\cong$. Use the Distance Formula to find that $\overline{QT}$ and $\overline{ST}$ are $\cong$.

19. (1) Use the two opposite $\cong$ sides of the rectangle, along with the diagonal (reflexive) to prove SSS congruence.
(2) Use the right $\angle$ opposite the diagonal, and the pair of $\cong$ sides of the rectangle to prove SAS congruence.
(3) Use the diagonal (reflexive) as the $\cong$ hypotenuse and use one of the $\cong$ sides to prove HL congruence.

Truss Your Judgment

Lesson 11-1 Congruent Triangles

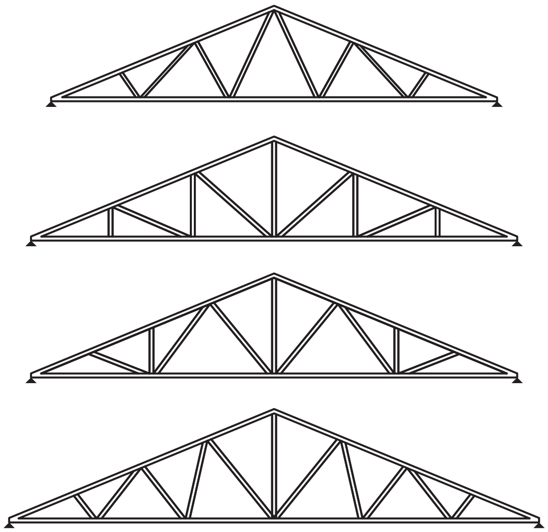
My Notes

Learning Targets:

- Use the fact that congruent triangles have congruent corresponding parts.
- Determine unknown angle measures or side lengths in congruent triangles.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing, Use Manipulatives, Think-Pair-Share

Greg Carpenter works for the Greene Construction Company. The company is building a new recreation hall, and the roof of the hall will be supported by triangular trusses, like the ones shown below.



Each of the trusses contains pairs of congruent triangles. Greg's boss tells him that his first job will be to determine the side lengths and angle measures in the triangles that make up one of the trusses.

CONNECT TO CAREERS

Triangles are often used in construction for roof and floor trusses because of their strength and rigidity. Each angle of a triangle is held solidly in place by its opposite side. That means the angles will not change when pressure is applied—unlike other shapes.

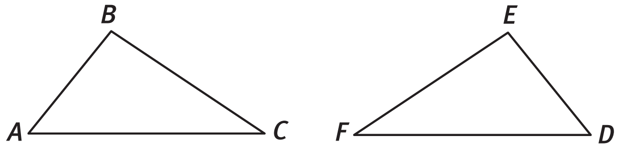
MATH TIP

Congruent triangles are triangles that have the same size and shape. More precisely, you have seen that two triangles are congruent if and only if one can be obtained from the other by a sequence of rigid motions.

My Notes

Greg wonders, "If I know that two triangles are congruent, and I know the side lengths and angle measures in one triangle, do I have to measure all the sides and angles in the other triangle?"

Greg begins by examining two triangles from a truss. According to the manufacturer, the two triangles are congruent.



1. Because the two triangles are congruent, can one triangle be mapped onto the other? If yes, what are the criteria for the mapping?

2. Suppose you use a sequence of rigid motions to map $\triangle ABC$ to $\triangle DEF$. Find the image of each of the following under this sequence of transformations.

$\overline{AB} \rightarrow$ ________ $\overline{BC} \rightarrow$ ________ $\overline{AC} \rightarrow$ ________

$\angle A \rightarrow$ ________ $\angle B \rightarrow$ ________ $\angle C \rightarrow$ ________

3. Make use of structure. What is the relationship between $\overline{AB}$ and $\overline{DE}$? What is the relationship between $\angle B$ and $\angle E$? How do you know?

MATH TERMS

Corresponding parts result from a one-to-one matching of sides and angles from one figure to another. Congruent triangles have three pairs of congruent sides and three pairs of congruent angles.

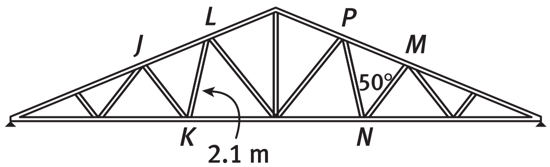
The triangles from the truss that Greg examined illustrate an important point about congruent triangles. In congruent triangles, corresponding pairs of sides are congruent and corresponding pairs of angles are congruent. These are ***corresponding parts***.

When you write a congruence statement like $\triangle ABC \cong \triangle DEF$, you write the vertices so that corresponding parts are in the same order. So, you can conclude from this statement that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

My Notes

Example A

For the truss shown below, Greg knows that $\triangle JKL \cong \triangle MNP$.



Greg wants to know if there are any additional lengths or angle measures that he can determine.

Since $\triangle JKL \cong \triangle MNP$, $\overline{KL} \cong \overline{NP}$. This means $KL = NP$, so $NP = 2.1$ m.

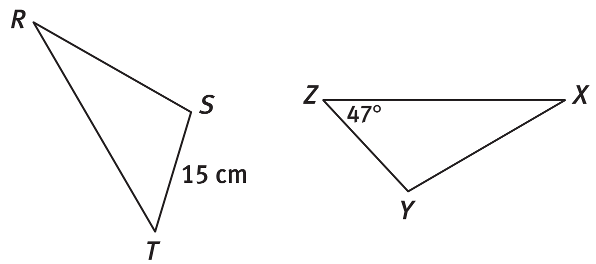
Also, since $\triangle JKL \cong \triangle MNP$, $\angle K \cong \angle N$. This means $m\angle K = m\angle N$, so $m\angle K = 50°$.

MATH TIP

Two line segments are congruent if and only if they have the same length. Two angles are congruent if and only if they have the same measure.

Try These A

In the figure, $\triangle RST \cong \triangle XYZ$. Find each of the following, if possible.



a. $m\angle X$

b. YZ

c. $m\angle T$

d. XZ

e. Both $\triangle JKL$ and $\triangle MNP$ are equilateral triangles in which the measure of each angle is 60°. Can you tell whether or not $\triangle JKL \cong \triangle MNP$? Explain.

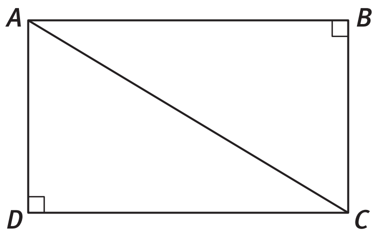
My Notes

Check Your Understanding

4. If two triangles are congruent, can you conclude that they have the same perimeter? Why or why not?

5. Is it possible to draw two congruent triangles so that one triangle is an acute triangle and one triangle is a right triangle? Why or why not?

6. Rectangle *ABCD* is divided into two congruent right triangles by diagonal $\overline{AC}$.



Fill in the blanks to show the congruent sides and angles.

a. $\overline{AB} \cong$ ________ **b.** $\overline{BC} \cong$ ________

c. $\angle BAC \cong$ ________ **d.** $\angle ACB \cong$ ________

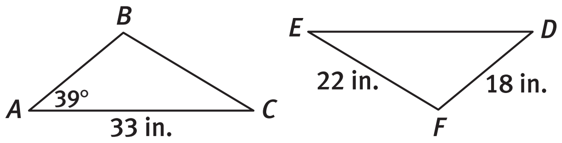
7. $\triangle PQR \cong \triangle GHJ$. Complete the following.

a. $\overline{QR} \cong$ ________ **b.** $\overline{GJ} \cong$ ________

c. $\angle R \cong$ ________ **d.** $\angle G \cong$ ________

LESSON 11-1 PRACTICE

In the figure, $\triangle ABC \cong \triangle DFE$.



8. Find the length of $\overline{AB}$.

9. Find the measure of all angles in $\triangle DEF$ that it is possible to find.

10. What is the perimeter of $\triangle DEF$? Explain how you know.

11. Construct viable arguments. Suppose $\triangle XYZ \cong \triangle TUV$ and that $\overline{XY}$ is the longest side of $\triangle XYZ$. Is it possible to determine which side of $\triangle TUV$ is the longest? Explain.

My Notes

Learning Targets:

- Develop criteria for proving triangle congruence.
- Determine which congruence criteria can be used to show that two triangles are congruent.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Use Manipulatives, Think-Pair-Share

As you have seen, congruent triangles have six pairs of congruent corresponding parts. The converse of this statement is also true. That is, if two triangles have three pairs of congruent corresponding sides and three pairs of congruent corresponding angles, the triangles are congruent.

Greg's boss asks him to check that two triangles in a truss are congruent. Greg wonders, "Must I measure and compare all six parts of both triangles?" He decides that a shortcut will allow him to conclude that two triangles are congruent without checking all six pairs of corresponding parts.

1. Greg begins by checking just one pair of corresponding parts of the two triangles.

a. In your group, each student should draw a triangle that has a side that is 2 inches long. The other two sides can be any measure. Draw the triangles on acetate or tracing paper.

b. To check whether two triangles are congruent, place the sheets of acetate on the desk. If the triangles are congruent, you can use a sequence of translations, reflections, and rotations to map one triangle onto the other.

Are all of the triangles congruent to each other? Why or why not?

c. Cite your results from part b to prove or disprove this statement: "If one part of a triangle is congruent to a corresponding part of another triangle, then the triangles must be congruent."

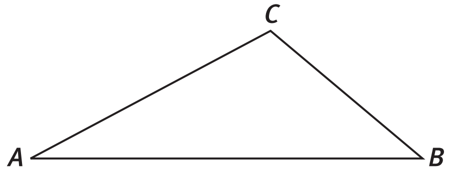
MATH TIP

A *counterexample* is a single example that shows that a statement is false.

My Notes

Now Greg wonders if checking two pairs of corresponding parts suffices to show that two triangles are congruent.

2. Greg starts by considering $\triangle ABC$ below.



a. Draw triangles that each have one side congruent to $\overline{AB}$ and another side congruent to $\overline{AC}$. Use transformations to check whether every triangle is congruent to $\triangle ABC$. Explain your findings.

b. Draw triangles that each have an angle congruent to $\angle A$ and an adjacent side congruent to $\overline{AB}$. Is every such triangle congruent to $\triangle ABC$? Explain.

c. Draw triangles that each have an angle congruent to $\angle A$ and an opposite side congruent to $\overline{CB}$. Is every such triangle congruent to $\triangle ABC$? Explain.

d. Draw triangles that each have an angle congruent to $\angle A$ and an angle congruent to $\angle B$. Is every such triangle congruent to $\triangle ABC$? Explain.

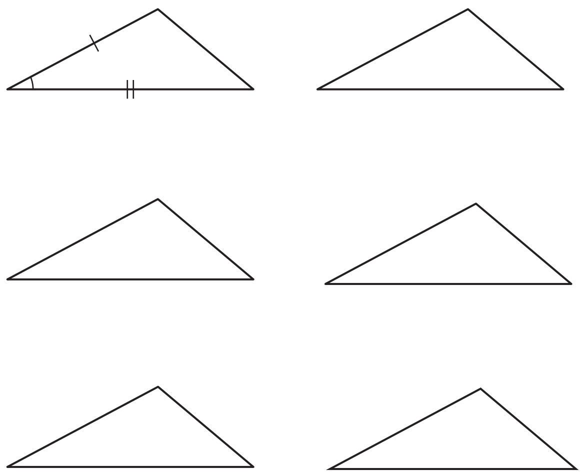
e. Consider the statement: "If two parts of one triangle are congruent to the corresponding parts in a second triangle, then the triangles must be congruent." Prove or disprove this statement. Cite the triangles you constructed.

My Notes

Greg decides that he must have at least three pairs of congruent parts in order to conclude that two triangles are congruent. In order to work more efficiently, he decides to make a list of all possible combinations of three congruent parts.

3. Greg uses *A* as an abbreviation to represent angles and *S* to represent sides. For example, if Greg writes *SAS*, it represents two sides and the included angle, as shown in the first triangle below. Here are the combinations in Greg's list: SAS, SSA, ASA, AAS, SSS, and AAA.

a. Mark each triangle below to illustrate the combinations in Greg's list.

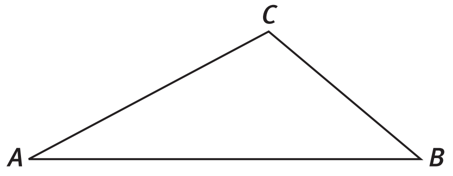


b. Are there any other combinations of three parts of a triangle? If so, is it necessary for Greg to add these to his list? Explain.

My Notes

Now Greg wants to find out *which* pairs of congruent corresponding parts guarantee congruent triangles.

4. Three segments congruent to the sides of $\triangle ABC$ and three angles congruent to the angles of $\triangle ABC$ are given in Figures 1–6, shown below.



a. If needed, use manipulatives supplied by your teacher to recreate the six figures given below.

Figure 1

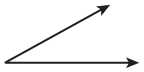


Figure 3

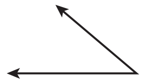


Figure 2

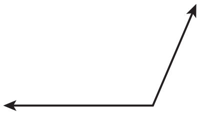


Figure 4 ______________________________

Figure 5 _____________________

Figure 6 _______________

b. Identify which of the figures in part a is congruent to each of the parts of $\triangle ABC$.

$\angle A$: _______________ $\angle B$: _______________

$\angle C$: _______________ $\overline{AB}$: _______________

$\overline{CB}$: _______________ $\overline{AC}$: _______________

My Notes

5. For each combination in Greg's list in Item 3, choose three appropriate triangle parts from Item 4. Each student should create a triangle using these parts. Then use transformations to check whether every such triangle is congruent to $\triangle ABC$. Use the table to organize your results.

Combination	Name the Three Figures Used by Listing the Figure Numbers	Is Every Such Triangle Congruent to $\triangle ABC$?

6. Express regularity in repeated reasoning. Compare your results from Item 5 with those of students in other groups. Then list the different combinations that seem to guarantee a triangle congruent to $\triangle ABC$. These combinations are called ***triangle congruence criteria***.

7. Do you think there is an AAA triangle congruence ***criterion***? Why or why not?

ACADEMIC VOCABULARY

A ***criterion*** (plural: *criteria*) is a standard or rule on which a judgment can be based. Criteria exist in every subject area. For example, a scientist might use a set of criteria to determine whether a sample of water is safe for human consumption.

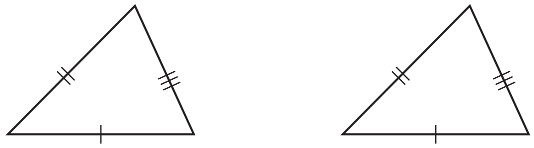
My Notes

Greg realizes that it is not necessary to check all six pairs of corresponding parts to determine if two triangles are congruent. The triangle congruence criteria can be used as "shortcuts" to show that two triangles are congruent.

Example A

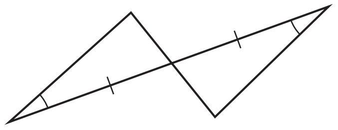
For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a.



Since three pairs of corresponding sides are congruent, the triangles are congruent by SSS.

b.



Although they are not marked as such, the vertical angles in the figure are congruent. The triangles are congruent by ASA.

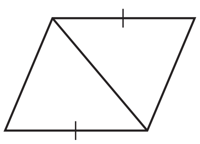
MATH TIP

You can use theorems about vertical angles, midpoints, bisectors, and parallel lines that are cut by a transversal to identify additional congruent parts that may not be marked in the figure.

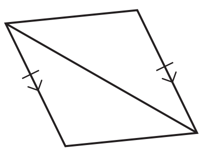
Try These A

For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a.



b.



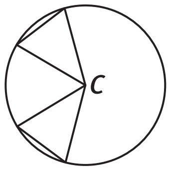
My Notes

Check Your Understanding

8. Two triangles each have two sides that are 8 feet long and an included angle of 50°. Must the two triangles be congruent? Why or why not?

9. Two equilateral triangles each have a side that is 5 cm long. Is it possible to conclude whether or not the triangles are congruent? Explain.

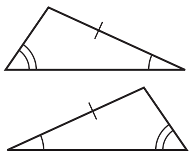
10. The figure shows a circle and two triangles. For both triangles, two vertices are points on the circle and the other vertex is the center of the circle. What information would you need in order to prove that the triangles are congruent?



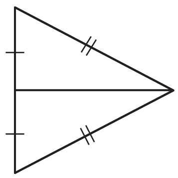
LESSON 11-2 PRACTICE

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

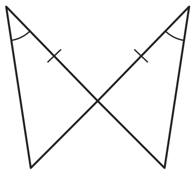
11.



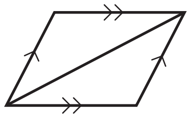
12.



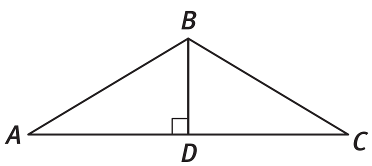
13.



14.



15. **Make sense of problems.** What one additional piece of information do you need, if any, in order to conclude that $\triangle ABD \cong \triangle CBD$? Is there more than one set of possible information? Explain.



CONNECT TO AP

The free-response items on the AP Calculus exam will often ask you to *justify* your answer. Such justifications should follow the same rules of deductive reasoning as the proofs in this geometry course.

My Notes

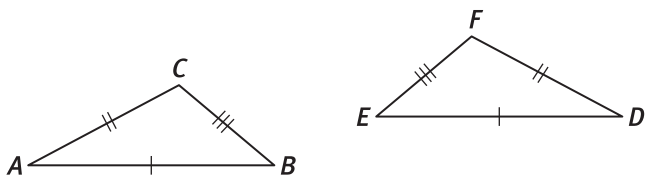
Learning Targets:

- Prove that congruence criteria follow from the definition of congruence.
- Use the congruence criteria in simple proofs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think Aloud, Discussion Groups, Visualization

You can use the SSS, SAS, or ASA congruence criteria as shortcuts to show that two triangles are congruent. In order to prove *why* these criteria work, you must show that they follow from the definition of congruence in terms of rigid motions.

1. To justify the SSS congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

b. Based on the definition of congruence, what do you need to do in order to show that $\triangle ABC \cong \triangle DEF$?

c. It is given that $\overline{AB} \cong \overline{DE}$. What does this tell you?

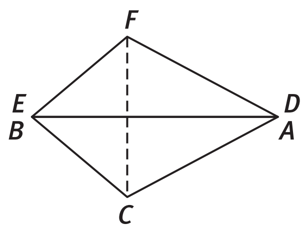
d. Draw a figure to show the result of the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Assume that this sequence of rigid motions does *not* map point C to point F.

MATH TIP

You are asked to assume that the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$ does not map C to F. If it did map C to F, you would have found a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ and the proof would be complete!

e. Which points coincide in your drawing? Which segments coincide?

f. Mark the line segments that you know are congruent in the figure below.



My Notes

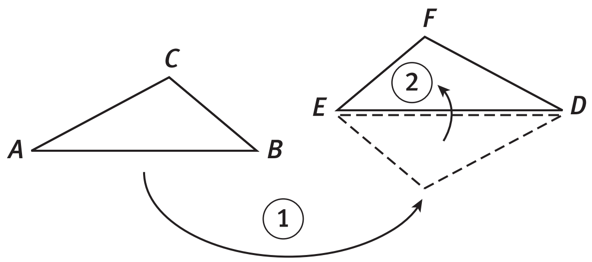
g. Based on the figure, how is $\overline{ED}$ related to $\overline{FC}$? Why?

h. Consider the reflection of $\triangle ABC$ across $\overline{ED}$. What is the image of $\triangle ABC$? How do you know?

MATH TIP

The Perpendicular Bisector Theorem states that a point lies on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the segment.

The argument in Item 1 shows that a sequence of rigid motions maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions is a rotation that maps $\overline{AB}$ to $\overline{DE}$, followed by a reflection across $\overline{ED}$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

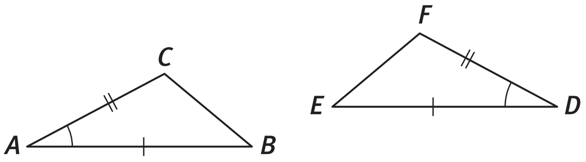


2. In the proof, is it important to know exactly which rigid motions map $\overline{AB}$ to $\overline{DE}$? Explain.

3. Attend to precision. How is the definition of a reflection used in the proof?

My Notes

4. To justify the SAS congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Draw a figure at the right to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does *not* map point C to point F.

MATH TIP

Your proof should use all three pieces of given information that you identified in part a. If you find that you are not using all of these facts, you may be missing an element of the proof.

c. Mark the line segments and angles that you know are congruent in your figure.

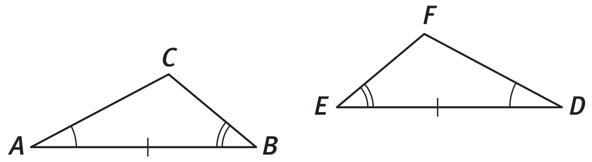
d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? Why?

e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point C is point F? Explain.

The argument in Item 4 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, followed by the reflection across $\overline{ED}$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

My Notes

5. To justify the ASA congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Draw a figure at the left to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does *not* map point C to point F.

c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? What is the image of $\overline{BC}$? Why?

e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point C is point F? Explain.

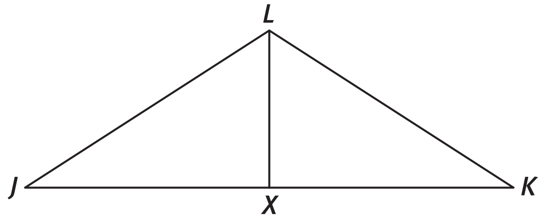
My Notes

The argument in Item 5 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, followed by the reflection across $\overline{ED}$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

Now you can use the SSS, SAS, and ASA congruence criteria to prove that triangles are congruent.

Example A

Greg knows that point X is the midpoint of $\overline{JK}$ in the truss shown below. He also makes measurements and finds that $\overline{JL} \cong \overline{KL}$. He must prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$.



Given: X is the midpoint of $\overline{JK}$; $\overline{JL} \cong \overline{KL}$.

Prove: $\triangle JXL \cong \triangle KXL$

Statements	Reasons
1. X is the midpoint of $\overline{JK}$.	**1.** Given
2. $\overline{JX} \cong \overline{KX}$	**2.** Definition of midpoint
3. $\overline{JL} \cong \overline{KL}$	**3.** Given
4. $\overline{LX} \cong \overline{LX}$	**4.** Congruence is reflexive.
5. $\triangle JXL \cong \triangle KXL$	**5.** SSS

Try These A

a. Suppose that Greg knew instead that $\overline{LX}$ was perpendicular to $\overline{JK}$ and suppose he made measurements to find that $\overline{JX} \cong \overline{KX}$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

b. Suppose that Greg knew instead that $\overline{LX}$ bisects $\angle JLK$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

Check Your Understanding

6. In the Example, can Greg conclude that $\angle J \cong \angle K$? Why or why not?

7. Draw a figure that contains two triangles. Provide given information that would allow you to prove that the triangles are congruent by the ASA congruence criterion.

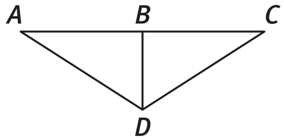
My Notes

LESSON 11-3 PRACTICE

For Items 8–10, write each proof as a two-column proof.

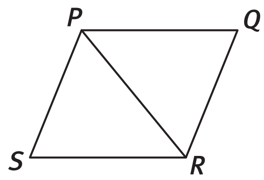
8. Given: $\overline{AD} \cong \overline{CD}$;
$\angle ADB \cong \angle CDB$

Prove: $\triangle ADB \cong \triangle CDB$



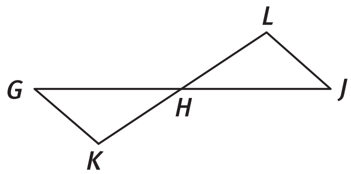
9. Given: $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$

Prove: $\triangle PQR \cong \triangle RSP$



10. Given: $\angle K \cong \angle L$;
$\overline{KH} \cong \overline{LH}$

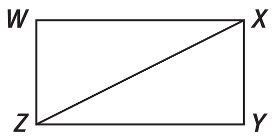
Prove: $\triangle GKH \cong \triangle JLH$



11. Critique the reasoning of others. A student wrote the proof shown below. Critique the student's work and correct any errors he or she may have made.

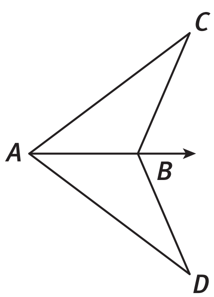
Given: $\overline{WX} \cong \overline{YZ}$;
$\overline{ZW} \cong \overline{XY}$

Prove: $\triangle ZWX \cong \triangle XYZ$



Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$	**1.** Given
2. $\overline{ZW} \cong \overline{XY}$	**2.** Given
3. $\angle W$ and $\angle Y$ are right angles.	**3.** Given
4. $\angle W \cong \angle Y$	**4.** All right angles are congruent.
5. $\Delta ZWX \cong \Delta XYZ$	**5.** SAS

12. Model with mathematics. A graphic designer made a logo for an airline, as shown below. The designer made sure that $\overrightarrow{AB}$ bisects $\angle CAD$ and that $\overline{AC} \cong \overline{AD}$. Can the designer prove that $\triangle ABC \cong \triangle ABD$? Why or why not?



My Notes

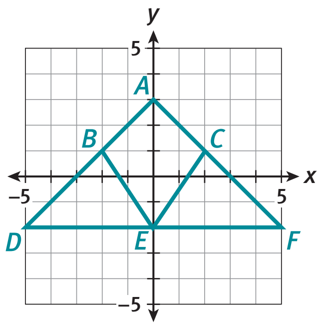
Learning Targets:

- Apply congruence criteria to figures on the coordinate plane.
- Prove the AAS criterion and develop the HL criterion.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing

You can use the triangle congruence criteria on the coordinate plane.

1. **Reason quantitatively.** Greg's boss hands him a piece of graph paper that shows the plans for a truss. Greg's boss asks him if he can prove that $\triangle DBE$ is congruent to $\triangle FCE$.



   a. Use the distance formula to find each length.

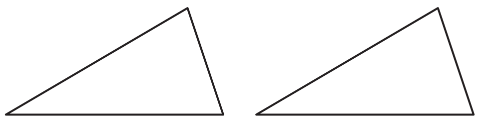
   $BD =$ ________ $CF =$ ________

   $DE =$ ________ $FE =$ ________

   $BE =$ ________ $CE =$ ________

   b. Can Greg use this information to prove that $\triangle DBE \cong \triangle FCE$? Explain.

2. In Item 5 of Lesson 11-2, you discovered that SSS, SAS, and ASA are not the only criteria for proving two triangles are congruent. You also discovered that there is an AAS congruence criterion. What does the AAS congruence criterion state? Mark the triangles below to illustrate the statement.

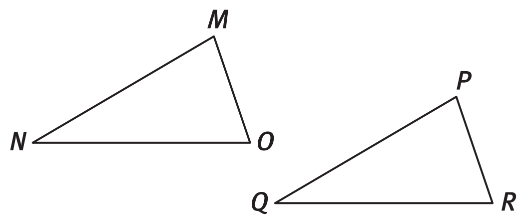


My Notes

3. The proof of the AAS congruence criterion follows from the other congruence criteria. Complete the statements in the proof of the AAS congruence criterion below.

Given: $\triangle MNO$ and $\triangle PQR$ with $\angle N \cong \angle Q$, $\angle O \cong \angle R$, and $\overline{MO} \cong \overline{PR}$

Prove: $\triangle MNO \cong \triangle PQR$

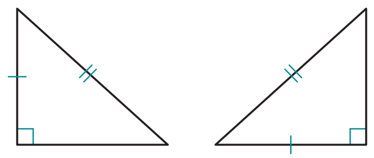


Statements	Reasons
1. $\triangle MNO$ and $\triangle PQR$	**1.**
2. $m\angle M + m\angle N + m\angle O = 180°$, $m\angle P + m\angle Q + m\angle R = 180°$	**2.**
3. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle Q + m\angle R$	**3.**
4. $\angle N \cong \angle Q$; $\angle O \cong \angle R$	**4.**
5. $m\angle N = m\angle Q$; $m\angle O = m\angle R$	**5.**
6. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle N + m\angle O$	**6.**
7. $m\angle M = m\angle P$	**7.**
8. $\angle M \cong \angle P$	**8.**
9. $\overline{MO} \cong \overline{PR}$	**9.**
10. $\triangle MNO \cong \triangle PQR$	**10.**

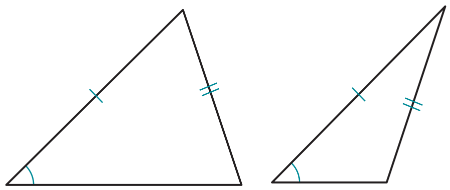
My Notes

4. Below are pairs of triangles in which congruent parts are marked. For each pair of triangles, name the angle and side combination that is marked and tell whether the triangles *appear* to be congruent.

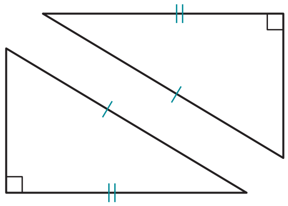
a.



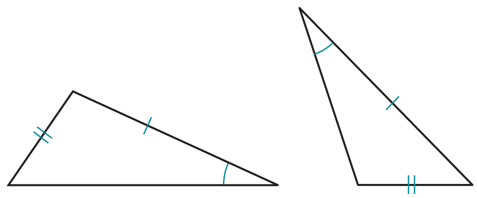
b.



c.



d.

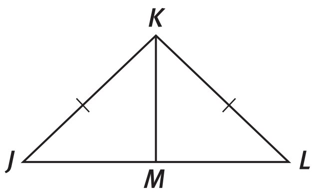


5. We know that in general SSA does not always determine congruence of triangles. However, for two of the cases in Item 4 the triangles appear to be congruent. What do the congruent pairs of triangles have in common?

6. In a right triangle, we refer to the correspondence SSA shown in Items 4a and 4c as *hypotenuse-leg* (HL). Write a convincing argument in the space below to prove that HL will ensure that right triangles are congruent.

Check Your Understanding

7. Is it possible to prove that $\triangle LKM \cong \triangle JKM$ using the HL congruence criterion? If not, what additional information do you need?



8. Do you think there is a leg-leg congruence criterion for right triangles? If so, what does the criterion say? If not, why not? Review your answers. Be sure to check that you have described the situation with specific details, included the correct mathematical terms to support your reasoning, and that your sentences are complete and grammatically correct.

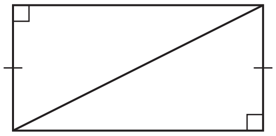
My Notes

LESSON 11-4 PRACTICE

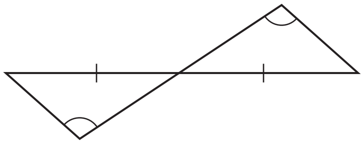
9. Construct viable arguments. On a coordinate plane, plot triangles ABC and DEF with vertices $A(-3, -1)$, $B(-1, 2)$, $C(1, 1)$, $D(3, -4)$, $E(1, -1)$, and $F(-1, -2)$. Then prove $\triangle ABC \cong \triangle DEF$.

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

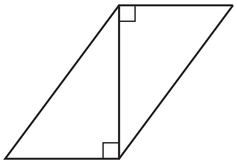
10.



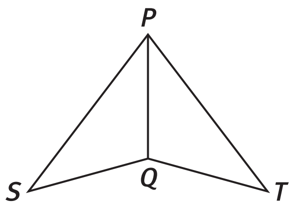
11.



12.



13. $\overline{PQ}$ bisects $\angle SPT$.

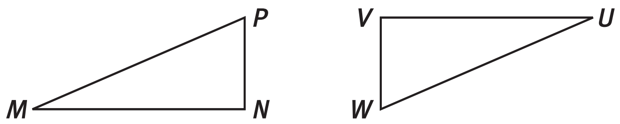


ACTIVITY 11 PRACTICE

Write your answers on notebook paper.
Show your work.

1. $\triangle FIX \cong \triangle TOP$. Complete the following.
   a. Name three pairs of corresponding vertices.
   b. Name three pairs of corresponding sides.
   c. Name three pairs of corresponding angles.
   d. Is it correct to say that $\triangle POT \cong \triangle XIF$? Why or why not?
   e. Is it correct to say that $\triangle IFX \cong \triangle PTO$? Why or why not?

In the figure, $\triangle MNP \cong \triangle UVW$. Use the figure for Items 2–4.



2. Write six congruence statements about line segments and angles in the figure.

3. Suppose $MN = 14$ cm and $m\angle U = 24°$. What other side lengths or angle measures can you determine? Why?

4. Suppose $MN = 2PN$ and $VW = 8$ in. Find MN.

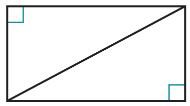
5. If $\triangle MAP \cong \triangle TON$, $m\angle M = 80°$, and $m\angle N = 50°$, name four congruent angles.

6. $\triangle ABC \cong \triangle DEF$, $AB = 15$, $BC = 20$, $AC = 25$, and $FE = 3x - 7$. Find x.

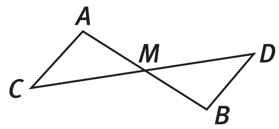
7. $\triangle MNO \cong \triangle PQR$, $m\angle N = 57°$, $m\angle P = 64°$ and $m\angle O = 5x + 4$. Find x and $m\angle R$.

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

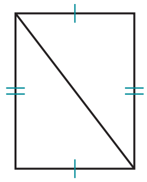
8.



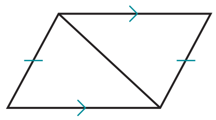
9. M bisects $\overline{AB}$ and $\overline{CD}$.



10.

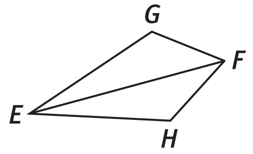


11.

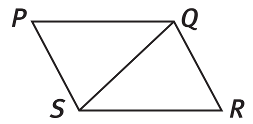


In Items 12–15, write each proof as a two-column proof.

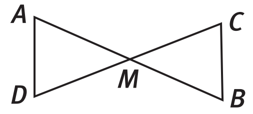
12. Given: $\overline{EF}$ bisects $\angle GEH$;
$\overline{EF}$ bisects $\angle GFH$.
Prove: $\triangle EGF \cong \triangle EHF$



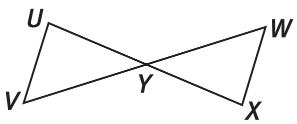
13. Given: $\overline{PQ}$ is parallel to $\overline{SR}$;
$\overline{PQ} \cong \overline{SR}$.
Prove: $\triangle PQS \cong \triangle RSQ$



14. Given: M is the midpoint of $\overline{AB}$;
M is the midpoint of $\overline{CD}$.
Prove: $\triangle AMD \cong \triangle BMC$

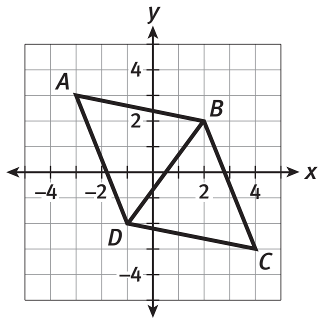


15. Given: $\overline{UV}$ is parallel to $\overline{WX}$;
$\overline{UV} \cong \overline{WX}$.
Prove: $\triangle UVY \cong \triangle XWY$

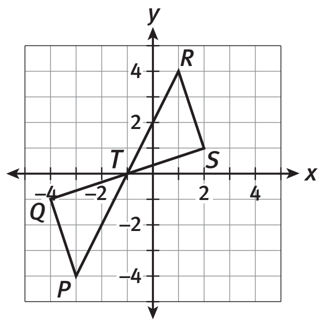


16. Draw a figure that includes two triangles. Provide given information that would allow you to prove that the triangles are congruent by the AAS Congruence Postulate.

17. a. Explain why $\triangle ABD \cong \triangle CDB$.
b. Can you conclude that $\angle A \cong \angle C$? Why or why not?



18. a. Explain why $\overline{RS}$ and $\overline{PQ}$ are perpendicular to $\overline{QS}$.
b. Explain how to use the SAS Congruence Postulate to show that $\triangle PQT \cong \triangle RST$.



MATHEMATICAL PRACTICES
Look For and Make Use of Structure

19. The opposite sides of a rectangle are congruent. Describe three different ways you could show that a diagonal divides a rectangle into two congruent triangles.