Lesson 6-1 Slope and Rate of Change

Understanding Linear Functions

Take a Dive

Activity 6

Guided

Activity Standards Focus

In this activity, students explore linear functions, including slope and rate of change, direct variation, and the different forms of linear equations. They also consider functions that are not linear, including indirect variation. They develop an understanding of when the slope of a line is positive, negative, zero, or undefined, and they develop fluency in writing linear equations in various forms and rewriting linear equations in a different form.

My Notes

ACADEMIC VOCABULARY

An expedition is a journey or voyage that has a specific purpose, such as studying a coral reef.

Common Core State Standards for Activity 6

HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Learning Targets:

- Understand the connection between rate of change and slope of a linear function.
- Identify functions that do not have a constant rate of change and understand that these functions are not linear.
- Find the slope of a line and understand when the slope is positive, negative, zero, or undefined.

SUGGESTED LEARNING STRATEGIES: Close Reading, Summarizing, Sharing and Responding, Discussion Groups, Construct an Argument, Identify a Subtask

Margo is a marine biologist. She is preparing to go on a diving expedition to study a coral reef. As she loads the boat with the supplies she will need, she uses a ramp like the one shown in the following diagram:

1. Notice the terms rise and run in the diagram. What do you think these terms mean in this context?

   Answers may vary. Rise represents the vertical movement, and run represents the horizontal movement.

Consider the line in the graph below:

Vertical change can be represented as a change in y, and horizontal change can be represented by a change in x.

2. What is the vertical change between:
   a. points A and B?  3 units
   b. points A and C?  6 units
   c. points C and D?  3 units

Bell-Ringer Activity

Ask students to imagine they are helping to design a playground. They should think about the types of straight slides they would design for preschoolers, elementary-age children, and middle school-age students. Then sketch a slide for each age group. Students describe in writing how the three slides are different.

Introduction, 1 Close Reading, Summarizing, Paraphrasing, Sharing and Responding

Have students examine the diagram of the ramp. Have students connect the words rise and run to everyday contexts to underscore the vertical component of “rise” and the horizontal component of “run.” Ask them to explain in their own words how the length of the rise and run affect the steepness of the ramp. Allow students to share responses to Item 1. All responses should be valued.

2–6 Discussion Groups, Vocabulary Organizer, Interactive Word Wall, Debriefing

Monitor groups carefully to be sure students study the graph carefully and understand the difference between horizontal and vertical. Be sure students are counting the number of squares to find the distance from the beginning of a segment to the end of a segment.
2–6 (continued) In Item 4, students should write the ratios in lowest terms to facilitate the idea that the slope between any two points on a line is constant. Add the math term slope to the classroom Word Wall. Create with students a spider vocabulary organizer with the term slope in the center. Each spoke of the organizer should show a different representation of slope and may be added to at any time during the activity or unit as new representations are explored. Students may choose to add diagrams or pictures to help with memory.

Developing Math Language
A solid understanding of the term slope is crucial for students as they continue their study of mathematics. Draw lines with various slopes. Have students brainstorm words that describe the differences among the lines: slant, incline, pitch, tilt, and so on. Invite students who speak other languages to add words from their languages to the list.

Explain to students that mathematics uses the word slope to describe the differences among the lines. Ask students to draw three lines with different slopes.

7 Look for a Pattern, Think-Pair-Share Ask students to compare the diagram of the ramp at the beginning of the lesson to the graph before Item 2. This is an excellent opportunity for formative assessment of students’ understanding of the concepts of rise and run. Allow students time to add the ratio rise to their vocabulary organizer.

Example A Marking the Text, Visualization, Interactive Word Wall
Students may have difficulty understanding the subscripts of the coordinate notation of the points. Explain that each point on a line has unique x- and y-coordinates, and mathematicians use this notation to differentiate the coordinates of separate points. Point 1 has coordinates \((x_1, y_1)\), point 2 has coordinates \((x_2, y_2)\), and so on.

Common Core State Standards for Activity 6 (continued)

- HSF-BF.A.1 Write a function that describes a relationship between two quantities.
- HSF-LE.A.1.A: Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.
- HSF-LE.A.1.B Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
Lesson 6-1
Slope and Rate of Change

Example A
Use the slope formula to determine the slope of a line that passes through the points (5, 6) and (−1, 4).
Let \((x_1, y_1) = (5, 6)\) and \((x_2, y_2) = (−1, 4)\). So,

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 6}{-1 - 5} = \frac{-2}{-6} = \frac{1}{3}
\]

The slope of the line is \(\frac{1}{3}\).

Try These A
a. Use the slope formula to determine the slope of a line that passes through the points (6, 2) and (8, 6).

\[
m = \frac{2 - 6}{8 - 6} = \frac{-4}{2} = -2
\]

b. Use the slope formula to determine the slope of a line that passes through the points (−4, 0) and (3, −1).

\[
m = \frac{-1 - 0}{3 - (-4)} = \frac{-1}{7}
\]

8. Compute the slope of the same line described in Example A, but let \((x_1, y_1) = (-1, 4)\) and \((x_2, y_2) = (5, 6)\). Show your work.

\[
m = \frac{4 - 6}{5 - (-1)} = \frac{-2}{6} = \frac{1}{3}
\]

9. What do you notice about the slope computed in Example A and the slope computed in Item 8?

They are the same.

10. Reason abstractly. What does your answer to Item 9 tell you about choosing which point is \((x_1, y_1)\) and which point is \((x_2, y_2)\)?

It doesn’t matter which point you use as \((x_1, y_1)\) and which point you use as \((x_2, y_2)\). The slope is the same.

11. Critique the reasoning of others. Anthony computed the slope of the line that passes through the points (4, 3) and (−2, 1). His calculation is shown below:

\[
m = \frac{3 - 1}{-2 - 4} = \frac{2}{-6} = -\frac{1}{3}
\]

Is Anthony’s calculation correct? Explain why or why not.

No, Anthony’s reasoning is not correct. He switched the order of the \(x\)-coordinates when computing the run. His calculation should be

\[
m = \frac{3 - 1}{4 - (-2)} = \frac{2}{6} = \frac{1}{3} \text{ or } m = \frac{1 - 3}{-2 - 4} = \frac{-2}{-6} = \frac{1}{3}.
\]

The rate of change for a function is the ratio of the change in \(y\), the dependent variable, to the change in \(x\), the independent variable.

ACTIVITY 6
Understanding Linear Functions

Universal Access
Support students by graphing the points in Example A and then connecting the points with a line. Label the point (5, 6) as “Point 1: \((x_1, y_1)\),” and label the point (−1, 4) as “Point 2: \((x_2, y_2)\).”

8–10 Critique Reasoning, Discussion Groups, Use Manipulatives, Debriefing
For Item 8, plot the two points of Example A on the coordinate grid but switch the labeling of Point 1 and Point 2. Draw the line that connects the two points and emphasize the positive slope shown on the graph. Explain that the slope is positive no matter which point is used first in the slope formula. Also, students should understand that they should get the same value if the order of the points is switched in the slope formula.

Have students compare the slope calculations for Example A and for Item 8. Explain that they should be the same. In the calculation for Example A, the change in \(y\) and the change in \(x\) are both negative, so the slope is positive. For Item 8, the change in \(x\) and the change in \(y\) are both positive, so the slope is also positive.

11 Sharing and Responding, Debriefing
The error presented in this item is a common student error, so ensure that all students can identify the mistake. Point out that the first numbers in the denominator and numerator (in this example, −2 and 3) should be the coordinates of one of the points. Since they are not, we know that the calculation is incorrect.

Differentiating Instruction
Support students who have trouble correctly substituting into the slope formula. Write the formula on the board, using red for the numerator and blue for the denominator. Have students copy the formula including the color coding. Then have them list two ordered pairs, circling the \(x\)-coordinates with blue and the \(y\)-coordinates with red. Have them substitute the values into the formula and simplify it. Caution students that when they choose a \(y\)-coordinate to use as \(y_1\), they must use the \(x\)-coordinate from the same point as \(x_1\).
Lesson 6-1
Slope and Rate of Change

Check Your Understanding

Debrief this lesson by encouraging students to connect the slope formula with the ratios determined from a graph as they work.

Answers

12. \( m = \frac{6 - 9}{8 - 4} = \frac{-3}{4} \)

13. \( m = \frac{-10 - (-3)}{9 - (-3)} = \frac{-7}{12} \)

14. To find the slope when given a graph, find the rise over run between two points on the line.

15. To find the slope when given two points on the line, find the difference in the \( y \)-values and write this over the difference in the \( x \)-values in a ratio.

16 Discussion Groups, Create Representations, Look for a Pattern, Debriefing

Before they begin working, ask students to consider whether the amount of money Aliyah has will increase or decrease as the number of books she buys increases. Again, monitor group discussions carefully to ensure choice of input values appropriate to the context. Debrief by asking students to compare and contrast this problem situation with the line analyzed earlier in the lesson. Students should make the connection between money being spent, a negative rate of change, and a line with a negative slope.

Differentiating Instruction

Extend students’ understanding of slope by having them compare the graphs of functions with different slopes. Give students the example of different-sized beverages:

Beverages
Small: $0.50 Medium: $1 Large: $2

For each size of beverage, have students write a function modeling how much \( x \) beverages would cost. Have them create a table and a graph for each. Finally, ask students to describe in writing how the graphs of \( f(x) = 0.5x \) and \( h(x) = 2x \) are each different from the graph of the parent function, \( g(x) = x \).

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16. Aliyah has saved $375. She wants to buy books that cost $3 each.

a. Write a function \( f(x) \) for the amount of money that Aliyah still has if she buys \( x \) books.

\[ f(x) = 375 - 3x \]

b. Make an input/output table of ordered pairs and then graph the function.

<table>
<thead>
<tr>
<th>Number of Books, ( x )</th>
<th>Money Remaining, ( f(x) ) (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>375</td>
</tr>
<tr>
<td>25</td>
<td>300</td>
</tr>
<tr>
<td>50</td>
<td>225</td>
</tr>
<tr>
<td>75</td>
<td>150</td>
</tr>
<tr>
<td>100</td>
<td>75</td>
</tr>
<tr>
<td>125</td>
<td>0</td>
</tr>
</tbody>
</table>

---

c. Does the function have a constant rate of change? If so, what is it?

Yes; \(-3\)

d. What is the slope of the line that you graphed?

Coordinates used may vary, but students should get slope \( = -3 \).

e. Describe the relationship between the slope of the line, the rate of change, and the equation of the line.

The slope is the same as the coefficient of \( x \) in the equation of the line. The slope is also equal to the rate of change.
Lesson 6-1
Slope and Rate of Change

f. Describe the meaning of the slope within the context of Aliyah’s savings.
   The slope of this line tells how much the amount of Aliyah’s savings changes for each piece of wood purchased.

g. How does this slope differ from the other slopes that you have seen in this activity?
   Answers may vary. This slope is negative, and the others were positive.

Check Your Understanding

17. The constant rate of change of a function is $-5$. Describe the graph of the function as you look at it from left to right.

18. Does the table represent data with a constant rate of change? Justify your answer.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
</tr>
</tbody>
</table>

19. The table below represents a function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>62</td>
</tr>
<tr>
<td>-6</td>
<td>34</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>47</td>
</tr>
</tbody>
</table>

a. Determine the rate of change between the points $(-8, 62)$ and $(-6, 34)$.
\[
\frac{\Delta y}{\Delta x} = \frac{34 - 62}{-6 - (-8)} = \frac{-28}{2} = -14
\]
b. Determine the rate of change between the points $(-1, -1)$ and $(1, -1)$.
\[
\frac{\Delta y}{\Delta x} = \frac{-1 - (-1)}{1 - (-1)} = \frac{0}{2} = 0
\]
c. Construct viable arguments. Is this a linear function? Justify your answer.
   No; the rate of change is not constant.
20 Look for a Pattern, Think-Pair-Share, Sharing and Responding

As students complete Item 20, they make connections between the rise and fall of the graph of a line and the slope of the line. Students should notice that the graph of a line with a positive slope rises from left to right and that the graph of a line with a negative slope falls from left to right.

As students complete parts c and d, use the slope ratios from the graphs to be sure they understand why a horizontal line has a slope of 0, and why a vertical line has an undefined slope. As students share their findings with the whole class, encourage the use of the term undefined rather than the phrase no slope when describing the slope of a vertical line. Students often confuse no slope with zero slope. The consistent use of the term undefined will help minimize confusion.

**LESSON 6-1 PRACTICE**

20. Determine the slopes of the lines shown.

\[ m = \frac{1}{3} \quad \frac{2}{2} \quad m = 5 \]

\[ m = -3 \quad m = -2 \quad m = -\frac{3}{4} \]

21 Think-Pair-Share, Summarizing, Vocabulary Organizer

After students have summarized their findings in the chart, allow time to add the information to students' vocabulary organizers for slope from the previous lessons. Encourage students to add visual representations along with verbal descriptions on their vocabulary organizers.

**LESSON 6-1 PRACTICE**

25. Connor; April has found the difference in the x-values over the difference in the y-values instead of the difference in the y-values over the difference in the x-values.

26. a. \( f(x) = 50 + 15x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>95</td>
</tr>
<tr>
<td>7</td>
<td>155</td>
</tr>
<tr>
<td>9</td>
<td>185</td>
</tr>
<tr>
<td>12</td>
<td>230</td>
</tr>
</tbody>
</table>

Check students' graphs.

b. \( \frac{15}{1}, m = 15 \)

c. \( \frac{15}{1}, m = 15 \)

d. The slope of the line is the same as the cost of each trip to the museum, $15.
Lesson 6-1
Slope and Rate of Change

21. Express regularity in repeated reasoning. Summarize your findings in Item 20. Tell whether the slopes of the lines described in the table below are positive, negative, 0, or undefined.

<table>
<thead>
<tr>
<th>Up from Left to Right</th>
<th>Down from Left to Right</th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>negative</td>
<td>0</td>
<td>undefined</td>
</tr>
</tbody>
</table>

Check Your Understanding

22. Suppose you are given several points on the graph of a function. Without graphing, how could you determine whether the function is linear?

23. How can you tell from a graph if the slope of a line is positive or negative?

24. Describe a line having an undefined slope. Why is the slope undefined?

LESSON 6-1 PRACTICE

25. Critique the reasoning of others. Connor determines the slope between (−2, 4) and (3, −3) by calculating \(\frac{4-(-3)}{-2-3}\). April determines the slope by calculating \(\frac{3-(-2)}{3-4}\). Explain whose reasoning is correct.

26. The art museum charges an initial membership fee of $50.00. For each visit the museum charges $15.00.
   a. Write a function \(f(x)\) for the total amount charged for \(x\) trips to the museum.
   b. Make a table of ordered pairs and then graph the function.
   c. What is the rate of change? What is the slope of the line?
   d. How does the slope of this line relate to the number of museum visits?

27. Make use of structure. Sketch a line for each description.
   a. The line has a positive slope.
   b. The line has a negative slope.
   c. The line has a slope of 0.

28. Are the points (12, 11), (2, 7), (5, 9), and (1, 5) part of the same linear function? Explain.

Check Your Understanding

As you debrief this lesson, focus on the idea that, for a linear function, the ratios of the change in \(y\) to the change in \(x\) between any two points on the line are equivalent. Students should also be able to describe lines with positive, negative, zero, and undefined slopes.

Answers

22. If the rate of change or the slope is not constant, the function is not linear.

23. Look at the graph from left to right. If the line goes up, it has a positive slope. If the line goes down, it has a negative slope.

24. A vertical line has an undefined slope. It is undefined because, for a vertical line, all points have the same \(x\)-coordinate. This means that the denominator in the slope formula \(\frac{x_2-x_1}{y_2-y_1}\) will always be 0 and division by 0 is undefined.

ASSESS

Students’ answers to the Lesson Practice items will provide a formative assessment of their understanding of slope and rate of change, and of students’ ability to apply their learning. Short-cycle formative assessment items for Lesson 6-1 are also available in the Assessment section on SpringBoard Digital.

Refer back to the graphic organizer the class created when unpacking Embedded Assessment 1. Ask students to use the graphic organizer to identify the concepts or skills they learned in this lesson.

ADAPT

Check students’ answers to the Lesson Practice to ensure that they understand slope and rate of change and are ready to transition to exploring direct and indirect variation. Students who may need more practice with the concepts in this lesson can practice finding the slope of lines drawn on the coordinate plane by counting rise and run, then relating this to the slope formula.

See the Activity Practice on page 107 and the Additional Unit Practice in the Teacher Resources on SpringBoard Digital for additional problems for this lesson.

You may wish to use the Teacher Assessment Builder on SpringBoard Digital to create custom assessments or additional practice.
Learning Targets:
- Recognize that direct variation is an example of a linear function.
- Write, graph, and analyze a linear model for a real-world situation.
- Distinguish between direct variation and indirect variation.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Interactive Word Wall, Marking the Text, Sharing and Responding, Discussion Groups

Margo is loading the boat with supplies she will need for her diving expedition. Each box is 10 inches high.

1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Height of the Stack (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
</tr>
</tbody>
</table>

2. Write a function to represent the data in the table and graph above.
   \[ f(x) = 10x \]
   or
   \[ y = 10x \]

3. What do \( f(x) \), \( y \), and \( x \) represent in your equation from Item 2?
   - \( f(x) \) represents the height of the stack and \( x \) represents the number of boxes.

4. The number of boxes is directly proportional to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.
   \[ \frac{1 \text{ box}}{10 \text{ in.}} = \frac{12 \text{ boxes}}{?} \]
   the height is 120 inches, or 10 ft.
Lesson 6-2
Direct and Indirect Variation

When two values are directly proportional, there is a **direct variation**. In terms of stacking boxes, the height of the stack varies directly as the number of boxes.

5. Using variables $x$ and $y$ to represent the two values, you can say that $y$ varies directly as $x$. Explain this statement.

   **Answers may vary.** $y$ is the height of the stack and $x$ is the number of boxes, so “$y$ varies directly as $x$” means that the height of the stack varies as the number of boxes changes.

6. Direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient $k$ is the **constant of variation**.
   a. Consider your answer to Item 2. What is the constant of variation in your function?
      
      10
   
   b. Why do you think the coefficient is called the constant of variation?
      
      The height constantly varies by 10 with each addition of another box.
   
   c. **Reason quantitatively.** Explain why the value of $k$ cannot be equal to 0.
      
      **Answers may vary.** If $k = 0$, then $y = 0x = 0$, which means that the value of $y$ will always be 0 and will not vary.
   
   d. Write an equation for finding the constant of variation by solving the equation $y = kx$ for $k$.
      
      $k = \frac{y}{x}$

7. a. Interpret the meaning of the point $(0, 0)$ in your table and graph.
      
      A stack of 0 boxes has no height.
   
   b. True or false? Explain your answer. “The graphs of all direct variations are lines that pass through the point $(0, 0)$.”
      
      True; explanations may vary. In an equation of the form $y = kx$, when $x = 0$, then $y = 0$.
   
   c. Identify the slope and $y$-intercept in the graph of the stacking boxes.
      
      $m = 10$; $y$-intercept = $(0, 0)$
   
   d. Describe the relationship between the constant of variation and the slope.
      
      The constant of variation and the slope are equal.
Differentiating Instruction

Extend students’ understanding of direct variations by having them examine a linear function that is not a direct variation. Change the problem about the boxes by saying that the boxes are stacked on a platform that is 5 centimeters high. Have them generate an equation that gives the height of the stack of boxes including the platform. Instruct them to make a table and a graph to show the height with various numbers of boxes. Then have them analyze the table and graph to determine whether or not it is a direct variation. They should consider both the ratio of height to boxes as well as the $y$-intercept of the equation.

Check Your Understanding

As you debrief the lesson, continue to emphasize that, in a direct variation, the ratio of $y : x$ is constant, and the graph passes through the point $(0, 0)$. Use the vocabulary introduced in this lesson as you discuss the problems.

Answers

8. a. No; the graph is not a line through the origin.
   b. Yes; the graph is a line through the origin.
   c. Yes; the values can be described by the equation $y = 6x$, which is in the form $y = kx$, where $k = 6$.
   d. No; the ratio $y : x$ is not a constant value for these ordered pairs.
   e. Yes; the equation is in the form $y = kx$.
   f. No; the equation cannot be written in the form $y = kx$.

Huan is stacking identical boxes on a pallet. The table below shows the height from the floor to the top of the boxes.

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
</tr>
<tr>
<td>7</td>
<td>41</td>
</tr>
<tr>
<td>10</td>
<td>56</td>
</tr>
</tbody>
</table>
Lesson 6-2
Direct and Indirect Variation

9. Make a graph of the data.

10. Write an equation that gives the height, \( h \), of a stack of \( n \) boxes, including the pallet. Explain what the numbers in the equation represent.
   \[ h = 5n + 6; \] The height of each box is 5 inches, and the height of the pallet is 6 inches.

11. Does the function represent direct variation? Explain how you can tell from the graph and from the equation.
   No. The graph does not include \((0, 0)\), so it does not represent direct variation. A direct variation equation has the form \( y = kx \). The equation above includes a constant term, so it does not represent direct variation.

12. Use your equation to find the height of a stack of 16 boxes, including the height of the pallet.
   86 in.

Check Your Understanding

13. The equation \( h = 0.25n + 8.5 \) gives the height \( h \) in inches of a stack of \( n \) paper cups.
   a. What would be the height of 25 cups? Of 50 cups?
   b. Graph this equation. Describe your graph.
Margo is loading the supplies she will need for her experiments. All of these boxes have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.

14. To explore the relationship between length and width, complete the table and make a graph of the points.

<table>
<thead>
<tr>
<th>Width (x)</th>
<th>Length (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

15. How are the lengths and widths in Item 14 related? Write an equation that shows this relationship.

Answers may vary. The product of the length and width must be 40, so look for pairs of factors that have a product of 40; \( xy = 40 \).

16. Use the equation you wrote in Item 15 to write a function to represent the data in the table and graph above.

\[ f(x) = \frac{40}{x} \text{, or } y = \frac{40}{x} \]

17. Describe any patterns that you notice in the table and graph representing your function.

Answers may vary. As the width increases, the length decreases.
Lesson 6-2
Direct and Indirect Variation

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an indirect variation.

18. Recall that direct variation is defined as \( y = kx \), where \( k \neq 0 \) and the coefficient \( k \) is the constant of variation.

a. How would you define indirect variation in terms of \( y, k, \) and \( x? \)
\[ y = \frac{k}{x} \]

b. Are there any limitations on these variables as there are on \( k \) in direct variation? Explain.
\[ k \neq 0, x \neq 0, y \neq 0. \text{Answers may vary. If} \ k = 0, \text{then} \ x \text{can be any number except 0 and} \ y \text{will always be 0. From the graph in Item 1, the values of} \ x \text{and} \ y \text{can get closer and closer to 0 but never equal 0.} \]

c. Write an equation for finding the constant of variation by solving for \( k \) in your answer to part a.
\[ k = xy \]

19. Reason abstractly. Compare and contrast the equations of direct and indirect variation.
Answers may vary. With direct variation you multiply \( k \) by \( x \) to find \( y \). With indirect variation you divide \( k \) by \( x \) to find \( y \).

20. Compare and contrast the graphs of direct and indirect variation.
Answers may vary. With direct variation, as \( x \) increases, \( y \) also increases by a given constant of variation, \( k \). With indirect variation, as \( x \) increases, \( y \) decreases because the constant of variation, \( k \), is divided by \( x \).

MINI-LESSON: Recognizing Variation Equations

Give each student three index cards. On the first card they should write Direct Variation and an equation in the form \( y = kx \). On the second card they should write Indirect Variation and an equation in the form \( y = \frac{k}{x} \). On the third card they should write Neither Direct nor Indirect and write an equation that is neither a direct nor indirect variation. On the back of each card, students should create a table of values that corresponds to the equation on the front of the card. Collect and shuffle the cards. Distribute three cards to each student, table side up. Students should use the table to decide if it is a direct variation, an indirect variation, or neither. If it is a variation, they should determine the constant of variation. They can turn over the card to check the answer.
Check Your Understanding

Students should be able to recognize direct and indirect variations both by their equations and by the shapes of their graphs. As you debrief this lesson, have students explain their reasoning for each answer. Be aware that some students may state that, for example, \( y = \frac{x}{2} \) is an indirect variation because it contains a fraction. Help these students realize that the equation could be rewritten as \( y = \frac{1}{2}x \) since \( x \) is in the numerator. This equation shows a direct variation with constant of variation \( \frac{1}{2} \).

Answers

21. a. direct variation
   b. indirect variation

22. C; The equation is in the form \( y = \frac{k}{x} \), where \( k = 2 \).
   D; The equation can be written in the form \( y = \frac{k}{x} \), where \( k = 2 \).

23. 80

ASSESS

Students' answers to the Lesson Practice items will provide a formative assessment of their understanding of direct and indirect variation, and of students' ability to apply their learning. Short-cycle formative assessment items for Lesson 6-2 are also available in the Assessment section on SpringBoard Digital.

Refer back to the graphic organizer the class created when unpacking Embedded Assessment 1. Ask students to use the graphic organizer to identify the concepts or skills they learned in this lesson as a culmination for the activity.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand direct and indirect variations and are ready to transition to the slope-intercept form of linear equations. Students who may need more practice with the concepts in this lesson should explore various real-life scenarios and identify them as direct variation, indirect variation, or linear but not direct variation.

See the Activity Practice on page 107 and the Additional Unit Practice in the Teacher Resources on SpringBoard Digital for additional problems for this lesson.

You may wish to use the Teacher Assessment Builder on SpringBoard Digital to create custom assessments or additional practice.

LESSON 6-2 PRACTICE

24. 15
25. 9
26. 20 feet
27. $1,875
28. a. 9
   b. 160
Learning Targets:
- Write the equation of a line in slope-intercept form.
- Use slope-intercept form to solve problems.
- Understand that a linear equation can be written in different forms, including slope-intercept, point-slope, and standard form.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Marking the Text, Discussion Groups

To study the coral reef, Margo must dive below the ocean’s surface. When a diver descends in a lake or ocean, pressure is produced by the weight of the water on the diver. As a diver swims deeper into the water, the pressure on the diver’s body increases at a rate of about $1 \text{ atmosphere per 10 meters of depth}$. The table and graph below represent the total pressure, $y$, on a diver given the depth, $x$, under water in meters.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

1. Write an equation describing the relationship between the pressure exerted on a diver and the diver’s depth under water.
   
   $y = 1 + 0.1x$

2. Attend to precision. What is the slope of the line? What are the units of the slope?
   
   $m = 0.1$; The units are atmospheres per meter.

3. What is the $y$-intercept? Explain its meaning in this context.
   
   $(0, 1)$; the amount of pressure on the diver at the surface of the water (when depth $= 0$)

4. Identify the slope and $y$-intercept of the line described by the equation
   
   $y = -2x + 9$.
   
   slope $= -2$; $y$-intercept $= (0, 9)$

CONNECT TO SCIENCE

Pressure is force per unit area. Atmospheric pressure is defined using the unit atmosphere. 1 atm is $14.6956$ pounds per square inch.

MATH TERMS

A linear equation is an equation that can be written in standard form $Ax + By = C$, where $A$, $B$, and $C$ are constants and $A$ and $B$ cannot both be zero.

MATH TIP

Linear equations can be written in several forms.
5. Create a table of values for the equation \( y = -2x + 9 \). Then plot the points and graph the line.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
</tr>
</tbody>
</table>

6. Explain how to find the value of the slope from the table. What is the value of the slope of the line?

Look at two points and find the change in \( y \) and the change in \( x \); write the ratio of the change in \( y \) over the change in \( x \); the slope is \(-2\).

7. Explain how to find the \( y \)-intercept from the table. What is the \( y \)-intercept?

Find the point where \( x = 0 \); the \( y \)-intercept is \((0, 9)\).

8. Explain how to find the value of the slope from the graph. What is the value of the slope?

Find a point and move from it to another point. Find the change in \( y \) and the change in \( x \) and write it as a ratio; \(-2\).

9. Explain how to find the \( y \)-intercept from the graph. What is the \( y \)-intercept?

Find the point where the line intersects the \( y \)-axis. That point is the \( y \)-intercept, \((0, 9)\).

**MINI-LESSON: Using Slope-Intercept Form**

A mini-lesson is available for students who need more practice writing a linear equation in slope-intercept form when given its slope and \( y \)-intercept. The mini-lesson also provides practice in identifying the slope and \( y \)-intercept of a linear equation when given its slope-intercept form.

See the Teacher Resources on SpringBoard Digital for a student page for this mini-lesson.
Lesson 6-3
Slope-Intercept Form

Check Your Understanding

10. What are the slope and $y$-intercept of the line described by the equation $y = -\frac{4}{5}x - 10$?

11. Write the equation in slope-intercept form of the line that is represented by the data in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>9</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

12. Write the equation, in slope-intercept form, of the line with a slope of 4 and a $y$-intercept of (0, 5).

13. Write an equation of the line graphed below.

Monica gets on an elevator in a skyscraper. The elevator starts to move at a rate of $-20$ ft/s. After 6 seconds on the elevator, Monica is 350 feet from the ground floor of the building. Use this information for Items 14–16.

14. The rate of the elevator is negative. What does this mean in the situation? What value in the slope-intercept form of an equation does this rate represent?

The elevator is moving down; slope, or $m$.

15. a. How many feet was Monica above the ground when she got on the elevator? Show how you determined your answer.

$350 + 20(6) = 470$ ft

b. What value in the slope-intercept form does your answer to part a represent?

the $y$-coordinate of the $y$-intercept, or $b$
Lesson 6-3
Slope-Intercept Form

16. Model with mathematics. Write an equation in slope-intercept form for the motion of the elevator since it started to move. What do \( x \) and \( y \) represent?

\[
 y = -20x + 470 ; \quad x \text{ represents the time in seconds since Monica got on the elevator, and } y \text{ represents the height of the elevator above the ground in feet.}
\]

a. What does the \( y \)-intercept represent?

Monica’s original height above the ground in feet

b. Use the equation you wrote to determine, at this rate, how long it will take after Monica enters the elevator for her to exit the elevator on the ground floor. Explain how you found your answer.

At the ground floor, \( y = 0 \). Solve the equation \( 0 = -20x + 470 \) to find \( x = 23.5 \) seconds.

17. Sample answer: Subtract 3\( x \) from both sides: \(-2y = -3x + 16\); divide both sides by \(-2\): \(y = \frac{3}{2}x - 8\).

18. a. \( y = 0.25x + 6.5 \)

b. \( x \) represents the time in days it takes to reach a height of \( y \) inches.

c. \( 11.25 = 0.25x + 6.5; \quad x = 19; 19 \text{ days} \)

Check Your Understanding

Debrief students’ answers to these items to make sure they understand concepts related to writing linear equations in slope-intercept form. Ask students to explain what the slope and \( y \)-intercept of the linear equation in Item 18 represent in the situation.

Answers

17. Write the equation \( 3x - 2y = 16 \) in slope-intercept form. Explain your steps.

18. A flowering plant stands 6.5 inches tall when it is placed under a growing light. Its growth is 0.25 inches per day. Today the plant is 11.25 inches tall.

a. Write an equation in slope-intercept form for the height of the plant since it was placed under the growing light.

b. In your equation, what do \( x \) and \( y \) represent?

c. Use the equation to determine how many days ago the plant was placed under the light.

Two other forms of linear equations are point-slope form and standard form. The table on the next page summarizes the three forms of linear equations and shows the linear function in the following graph written in each form.

 différentiating Instruction

Support students who make errors with signs when writing the point-slope form of an equation by emphasizing that the coordinates of the given point are subtracted from the variables \( x \) and \( y \). Suggest that students circle the subtraction signs when writing the point-slope form.

Mini-Lesson: Using Point-Slope Form

A mini-lesson is available for students who need more practice writing a linear equation in point-slope form when given its slope and the coordinates of a point on the line. The mini-lesson also provides practice in using the point-slope form of a linear equation to identify the slope and the coordinates of a point on the line.

See the Teacher Resources on SpringBoard Digital for a student page for this mini-lesson.
**Lesson 6-3**  
**Slope-Intercept Form**

<table>
<thead>
<tr>
<th>Slope-Intercept Form</th>
<th>Point-Slope Form</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = mx + b )</td>
<td>( y - y_1 = m(x - x_1) )</td>
<td>( Ax + By = C )</td>
</tr>
</tbody>
</table>

where \( m \) is the slope of the line and \((0, b)\) is the \(y\)-intercept  
where \( m \) is the slope of the line and \((x_1, y_1)\) is a point on the line  
where \( A \geq 0 \), \( A \) and \( B \) are not both zero, and \( A \), \( B \), and \( C \) are integers whose greatest common factor is 1

The slope of the line shown is \(-\frac{1}{3}\), and the \(y\)-intercept is \((0, 2)\).

\[ y = -\frac{1}{3}x + 2 \]

The slope of the line shown is \(-\frac{1}{3}\), and the line passes through the point \((3, 1)\).

\[ y - 1 = -\frac{1}{3}(x - 3) \]

Start with slope-intercept form:

\[ y = -\frac{1}{3}x + 2 \]

Multiply by 3 to eliminate the fraction:

\[ 3y = -x + 6 \]

Add \( x \) to both sides so that equation is in standard form:

\[ x + 3y = 6 \]

19. A line has a slope of \(\frac{3}{2}\) and passes through the point \((-4, 5)\).
   a. Write an equation of the line in point-slope form.

\[ y - 5 = \frac{3}{2}(x + 4) \]

b. Write an equation of the line in slope-intercept form.

\[ y = \frac{3}{2}x + 11 \]

c. Write an equation of the line in standard form.

\[ 3x - 2y = -22 \]

**Check Your Understanding**

20. Write the equation \( y = -\frac{5}{3}x - 4 \) in standard form.

21. Write an equation in standard form for the line that is represented by the data in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

20. \( 6x + 5y = -20 \)  
21. \( 2x + y = 5 \)

**MINI-LESSON: Writing Linear Equations in Standard Form**

A mini-lesson is available for students who need more practice writing a linear equation in standard form when given a different form of the equation.

See the Teacher Resources on SpringBoard Digital for a student page for this mini-lesson.
LESSON 6-3 PRACTICE

22. What are the slope, \( m \), and \( y \)-intercept, \((0, b)\), of the line described by the equation \(3x + 6y = 12\)?

23. Write an equation in slope-intercept form for the line that passes through the points \((6, -3)\) and \((0, 2)\).

24. Matt sells used books on the Internet. He has a weekly fee he has to pay for his website. He has graphed his possible weekly earnings, as shown.

   a. What is the weekly fee that Matt pays for his website? How do you know?
   
   b. How much does Matt make for each book sold? How do you know?
   
   c. Write the equation in slope-intercept form for the line in Matt’s graph.
   
   d. How many books does Matt have to sell to make $30 for the week?

25. Write an equation of the line with a slope of 0.25 that passes through the point \((-1, -8)\).

26. Make sense of problems. Keisha bought a discount pass at a movie theater. It entitles her to a special discounted admission price for every movie she sees. Keisha wrote an equation that gives the total cost \(y\) of seeing \(x\) movies. In standard form, the equation is \(7x - 2y = -31\).

   a. What was the cost of the pass?
   
   b. What is the discounted admission price for each movie?
Understanding Linear Functions
Take a Dive

ACTIVITY 6 PRACTICE
Answer each item. Show your work.

Lesson 6-1
1. Find $\Delta x$ and $\Delta y$ for each of the following pairs of points.
   a. $(2, 6), (6, 8)$
   b. $(0, 9), (4, 8)$
   c. $(-3, -3), (7, 10)$
2. Two points on a line are $(-10, 1)$ and $(5, -5)$. If the $y$-coordinate of another point on the line is $-3$, what is the $x$-coordinate?

For Items 3–5, determine the slope of the line that passes through each pair of points.
3. $(-4, 11)$ and $(1, -9)$
4. $(-10, -3)$ and $(-5, 1)$
5. $(-2, -7)$ and $(-8, -4)$
6. Are the three points $(2, 3), (5, 6)$, and $(0, -2)$ on the same line? Explain.
7. Which of the following pairs of points lies on a line with a slope of $-\frac{3}{5}$?
   a. $(4, 0), (-2, 10)$
   b. $(4, 2), (10, 4)$
   c. $(-4, -10), (0, -2)$
   d. $(10, -2), (0, 4)$
8. Juan earns $7$ per hour plus $20$ per week making picture frames.
   a. Write a function $g(x)$ for Juan’s total earnings if he works $x$ hours in one week.
   b. Without graphing the function, determine the slope.
   c. Describe the meaning of the slope within the context of Juan’s job.
9. Which of the following is not a linear function?
   a. $(4, -6), (7, -12), (8, -14), (10, -18), (2, -2)$
   b. $(-2, -6), (1, 0), (4, -30), (0, 2), (7, -96)$
   c. $(-4, 9), (0, 7), (2, 6), (6, 4), (8, 3)$
   d. $(2, 18), (6, 50), (-3, -22), (0, 2), (3, 26)$
10. The slope of a line is 0. It passes through the point $(-3, 4)$. Identify two other points on the line. Justify your answers.

Lesson 6-2
11. The value of $y$ varies directly as $x$ and $y = 125$ when $x = 25$. What is the value of $y$ when $x = 2$?
12. Which is the graph of a direct variation?

   ![Graph Options]

   A. ![Graph A]
   B. ![Graph B]
   C. ![Graph C]
   D. ![Graph D]

13. The amount of gas left in the gas tank of a car varies indirectly with the number of miles driven. There are 9 gallons of gas left after 24 miles. How much gas is left after the car is driven 120 miles?
Jeremy collected the following data on stacking chairs. Use the data for Items 14 and 15.

14. Write a linear function that models the data.

15. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack? Justify your answer.

Lesson 6-3

16. Write the equation of a line in slope-intercept form that has a slope of $-8$ and a $y$-intercept of $(0, 3)$.

17. Find the slope and the $y$-intercept of the line whose equation is $-5x + 3y - 8 = 0$.

18. Which of the following is the slope-intercept form of the equation of the line in the graph?

A. $y = \frac{5}{3}x + 3$
B. $y = \frac{3}{5}x + 5$
C. $y = \frac{5}{3}x + 3$
D. $y = \frac{5}{3}x + 5$

Understanding Linear Functions

Take a Dive

19. What is the equation in point-slope form of the line that passes through the points $(2, -3)$ and $(-5, 0)$?

20. David is ordering tea from an online store. Black tea costs $0.80 per ounce and green tea costs $1.20 per ounce. He plans to spend a total of $12 on the two types of tea.
   a. Write an equation that represents the different amounts of black tea, $x$, and green tea, $y$, that David can buy.
   b. Graph the equation.
   c. What is the $x$-intercept? What does it represent?
   d. Suppose David decides to buy 10 ounces of black tea. How many ounces of green tea will he buy?

21. Which is a true statement about the line $x - 4y = 8$?
   A. The $x$-intercept of the line is $(2, 0)$.
   B. The $y$-intercept of the line is $(0, 2)$.
   C. The slope of the line is $\frac{1}{4}$.
   D. The line passes through the origin.

Mathematical Practices

Construct Viable Arguments and Critique the Reasoning of Others

22. Aidan stated that for any value of $b$, the line $y = 2x + b$ has the same slope as the line that passes through $(2, 5)$ and $(-1, -1)$. Do you agree with Aidan? Explain why or why not.
Learning Targets:
- Understand the connection between rate of change and slope of a linear function.
- Identify functions that do not have a constant rate of change and understand that these functions are not linear.
- Find the slope of a line and understand when the slope is positive, negative, zero, or undefined.

SUGGESTED LEARNING STRATEGIES: Close Reading, Summarizing, Sharing and Responding, Discussion Groups, Construct an Argument, Identify a Subtask

Margo is a marine biologist. She is preparing to go on a diving expedition to study a coral reef. As she loads the boat with the supplies she will need, she uses a ramp like the one shown in the following diagram:

1. Notice the terms rise and run in the diagram. What do you think these terms mean in this context?

Consider the line in the graph below:

Vertical change can be represented as a change in y, and horizontal change can be represented by a change in x.

2. What is the vertical change between:
   a. points A and B?  
   b. points A and C?  
   c. points C and D?
3. What is the horizontal change between:
   a. points A and B?
   b. points A and C?
   c. points C and D?

The ratio of the vertical change to the horizontal change determines the slope of the line.

\[
\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}
\]

4. Find the slope of the segment of the line connecting:
   a. points A and B
   b. points A and C
   c. points C and D

5. What do you notice about the slope of the line in Items 4a, 4b, and 4c?

6. What does your answer to Item 5 indicate about points on a line?

7. Slope is sometimes referred to as \textit{rise over run}. Explain how the ratio \textit{rise over run} relates to the ratios for finding slope mentioned above.

The slope \( m \) of a line can be calculated numerically using any two points \((x_1, y_1)\) and \((x_2, y_2)\) on the line. The vertical change, \( \Delta y \), of the line through these two points is \( y_2 - y_1 \) or \( y_1 - y_2 \). The horizontal change, \( \Delta x \), of the line through these two points is \( x_2 - x_1 \) or \( x_1 - x_2 \). Note that the first \( x \)-coordinate of the denominator of the slope formula is from the same ordered pair as the first \( y \)-coordinate of the numerator.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}
\]
Lesson 6-1
Slope and Rate of Change

Example A
Use the slope formula to determine the slope of a line that passes through the points (5, 6) and (−1, 4).

Let \((x_1, y_1) = (5, 6)\) and \((x_2, y_2) = (−1, 4)\). So,

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 6}{−1 - 5} = \frac{-2}{-6} = \frac{1}{3}
\]

The slope of the line is \(\frac{1}{3}\).

Try These A

a. Use the slope formula to determine the slope of a line that passes through the points (6, 2) and (8, 6).

b. Use the slope formula to determine the slope of a line that passes through the points (−4, 0) and (3, −1).

8. Compute the slope of the same line described in Example A, but let \((x_1, y_1) = (−1, 4)\) and \((x_2, y_2) = (5, 6)\). Show your work.

9. What do you notice about the slope computed in Example A and the slope computed in Item 8?

10. Reason abstractly. What does your answer to Item 9 tell you about choosing which point is \((x_1, y_1)\) and which point is \((x_2, y_2)\)?

11. Critique the reasoning of others. Anthony computed the slope of the line that passes through the points (4, 3) and (−2, 1). His calculation is shown below:

\[
m = \frac{3 - 1}{−2 - 4} = \frac{2}{−6} = \frac{1}{3}
\]

Is Anthony’s calculation correct? Explain why or why not.

The rate of change for a function is the ratio of the change in \(y\), the dependent variable, to the change in \(x\), the independent variable.
Lesson 6-1
Slope and Rate of Change

16. Aliyah has saved $375. She wants to buy books that cost $3 each.
   
   a. Write a function \( f(x) \) for the amount of money that Aliyah still has if she buys \( x \) books.

   b. Make an input/output table of ordered pairs and then graph the function.

   \[
   \begin{array}{c|c}
   \text{Number of Books, } x & \text{Money Remaining, } f(x) \text{ (dollars)} \\
   \hline
   & \\
   & \\
   & \\
   & \\
   & \\
   \end{array}
   \]

   c. Does the function have a constant rate of change? If so, what is it?

   d. What is the slope of the line that you graphed?

   e. Describe the relationship between the slope of the line, the rate of change, and the equation of the line.

12. Use the slope formula to determine the slope of a line that passes through the points \((4, 9)\) and \((-8, -6)\).

13. Use the slope formula to determine the slope of the line that passes through the points \((-5, -3)\) and \((9, -10)\).

14. Explain how to find the slope of a line from a graph.

15. Explain how to find the slope of a line when given two points on the line.

Check Your Understanding

90 SpringBoard® Integrated Mathematics I, Unit 2 • Linear Functions
Lesson 6-1
Slope and Rate of Change

17. The constant rate of change of a function is $-5$. Describe the graph of the function as you look at it from left to right.

18. Does the table represent data with a constant rate of change? Justify your answer.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
</tr>
</tbody>
</table>

Check Your Understanding

19. The table below represents a function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>62</td>
</tr>
<tr>
<td>-6</td>
<td>34</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>47</td>
</tr>
</tbody>
</table>

a. Determine the rate of change between the points $(-8, 62)$ and $(-6, 34)$.

b. Determine the rate of change between the points $(-1, -1)$ and $(1, -1)$.

c. Construct viable arguments. Is this a linear function? Justify your answer.
20. Determine the slopes of the lines shown.

a.

b.

c.

d.
21. **Express regularity in repeated reasoning.** Summarize your findings in Item 20. Tell whether the slopes of the lines described in the table below are positive, negative, 0, or undefined.

<table>
<thead>
<tr>
<th>Up from Left to Right</th>
<th>Down from Left to Right</th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
</table>

22. Suppose you are given several points on the graph of a function. Without graphing, how could you determine whether the function is linear?

23. How can you tell from a graph if the slope of a line is positive or negative?

24. Describe a line having an undefined slope. Why is the slope undefined?

**Check Your Understanding**

25. Critique the reasoning of others. Connor determines the slope between \((-2, 4)\) and \((3, -3)\) by calculating \(\frac{4-(-3)}{-2-3}\). April determines the slope by calculating \(\frac{3-(-2)}{-3-4}\). Explain whose reasoning is correct.

26. The art museum charges an initial membership fee of $50.00. For each visit the museum charges $15.00.
   a. Write a function \(f(x)\) for the total amount charged for \(x\) trips to the museum.
   b. Make a table of ordered pairs and then graph the function.
   c. What is the rate of change? What is the slope of the line?
   d. How does the slope of this line relate to the number of museum visits?

27. Make use of structure. Sketch a line for each description.
   a. The line has a positive slope.
   b. The line has a negative slope.
   c. The line has a slope of 0.

28. Are the points \((12, 11), (2, 7), (5, 9),\) and \((1, 5)\) part of the same linear function? Explain.
Learning Targets:
- Recognize that direct variation is an example of a linear function.
- Write, graph, and analyze a linear model for a real-world situation.
- Distinguish between direct variation and indirect variation.

SUGGESTED LEARNING STRATEGIES: Create Representations, Interactive Word Wall, Marking the Text, Sharing and Responding, Discussion Groups

Margo is loading the boat with supplies she will need for her diving expedition. Each box is 10 inches high.

### 1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Height of the Stack (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

### 2. Write a function to represent the data in the table and graph above.

### 3. What do $f(x)$, or $y$, and $x$ represent in your equation from Item 2?

### 4. The number of boxes is directly proportional to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.
Lesson 6-2
Direct and Indirect Variation

When two values are directly proportional, there is a **direct variation**. In terms of stacking boxes, the height of the stack varies directly as the number of boxes.

5. Using variables $x$ and $y$ to represent the two values, you can say that $y$ varies directly as $x$. Explain this statement.

6. Direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient $k$ is the **constant of variation**.
   a. Consider your answer to Item 2. What is the constant of variation in your function?

   b. Why do you think the coefficient is called the constant of variation?

   c. **Reason quantitatively.** Explain why the value of $k$ cannot be equal to 0.

   d. Write an equation for finding the constant of variation by solving the equation $y = kx$ for $k$.

7. a. Interpret the meaning of the point $(0, 0)$ in your table and graph.

   b. True or false? Explain your answer. “The graphs of all direct variations are lines that pass through the point $(0, 0)$.”

   c. Identify the slope and $y$-intercept in the graph of the stacking boxes.

   d. Describe the relationship between the constant of variation and the slope.
8. Tell whether the tables, graphs, and equations below represent direct variations. Justify your answers.

a. \[y = 20x\]

b. \[y = 3x + 2\]

c. \[
\begin{array}{c|c}
 x & y \\
2 & 12 \\
4 & 24 \\
6 & 36 \\
\end{array}
\]

d. \[
\begin{array}{c|c}
 x & y \\
2 & 8 \\
4 & 12 \\
6 & 16 \\
\end{array}
\]

e. \[y = 20x\]

Huan is stacking identical boxes on a pallet. The table below shows the height from the floor to the top of the boxes.

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
</tr>
<tr>
<td>7</td>
<td>41</td>
</tr>
<tr>
<td>10</td>
<td>56</td>
</tr>
</tbody>
</table>
Lesson 6-2
Direct and Indirect Variation

9. Make a graph of the data.

10. Write an equation that gives the height, \( h \), of a stack of \( n \) boxes, including the pallet. Explain what the numbers in the equation represent.

11. Does the function represent direct variation? Explain how you can tell from the graph and from the equation.

12. Use your equation to find the height of a stack of 16 boxes, including the height of the pallet.

Check Your Understanding

13. The equation \( h = 0.25n + 8.5 \) gives the height \( h \) in inches of a stack of \( n \) paper cups.
   a. What would be the height of 25 cups? Of 50 cups?
   b. Graph this equation. Describe your graph.
Margo is loading the supplies she will need for her experiments. All of these boxes have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.

14. To explore the relationship between length and width, complete the table and make a graph of the points.

<table>
<thead>
<tr>
<th>Width (x)</th>
<th>Length (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

15. How are the lengths and widths in Item 14 related? Write an equation that shows this relationship.

16. Use the equation you wrote in Item 15 to write a function to represent the data in the table and graph above.

17. Describe any patterns that you notice in the table and graph representing your function.
Lesson 6-2  
Direct and Indirect Variation

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an **indirect variation**.

18. Recall that direct variation is defined as \( y = kx \), where \( k \neq 0 \) and the coefficient \( k \) is the constant of variation.
   
a. How would you define indirect variation in terms of \( y \), \( k \), and \( x \)?

   b. Are there any limitations on these variables as there are on \( k \) in direct variation? Explain.

   c. Write an equation for finding the constant of variation by solving for \( k \) in your answer to part a.

19. **Reason abstractly.** Compare and contrast the equations of direct and indirect variation.

20. Compare and contrast the graphs of direct and indirect variation.
Lesson 6-2
Direct and Indirect Variation

Check Your Understanding

21. Identify each graph as direct variation, indirect variation, neither, or both.
   a. 
   b.

22. Which equations are examples of indirect variation? Justify your answers.
   A. \( y = 2x \)
   B. \( y = \frac{x}{2} \)
   C. \( y = \frac{2}{x} \)
   D. \( xy = 2 \)

23. In the equation \( y = \frac{80}{x} \), what is the constant of variation?

LESSON 6-2 PRACTICE

24. In the equation \( y = 15x \), what is the constant of variation?

25. The value of \( y \) varies directly with \( x \) and the constant of variation is 7. What is the value of \( x \) when \( y = 63 \)?

26. Model with mathematics. The height of a stack of boxes varies directly with the number of boxes. A stack of 12 boxes is 15 feet high. How tall is a stack of 16 boxes?

27. A consultant earns a flat fee of $75 plus $50 per hour for a contracted job. The table shows the consultant’s earnings for the first four hours she works.

<table>
<thead>
<tr>
<th>Hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>$75</td>
<td>$125</td>
<td>$175</td>
<td>$225</td>
<td>$275</td>
</tr>
</tbody>
</table>

The consultant has a 36-hour contract. How much will she earn?

28. Make sense of problems. Assume that \( y \) varies indirectly as \( x \).
   a. If \( y = 6 \) when \( x = 24 \), find \( y \) when \( x = 16 \).
   b. If \( y = 8 \) when \( x = 20 \), find the value of \( k \).
Lesson 6-3
Slope-Intercept Form

Learning Targets:
• Write the equation of a line in slope-intercept form.
• Use slope-intercept form to solve problems.
• Understand that a linear equation can be written in different forms, including slope-intercept, point-slope, and standard form.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Marking the Text, Discussion Groups

To study the coral reef, Margo must dive below the ocean’s surface. When a diver descends in a lake or ocean, pressure is produced by the weight of the water on the diver. As a diver swims deeper into the water, the pressure on the diver’s body increases at a rate of about 1 atmosphere of pressure per 10 meters of depth. The table and graph below represent the total pressure, $y$, on a diver given the depth, $x$, under water in meters.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

1. Write an equation describing the relationship between the pressure exerted on a diver and the diver’s depth under water.

2. **Attend to precision.** What is the slope of the line? What are the units of the slope?

3. What is the $y$-intercept? Explain its meaning in this context.

4. Identify the slope and $y$-intercept of the line described by the equation $y = -2x + 9$. 

**Slope-Intercept Form of a Linear Equation**

$$y = mx + b$$

where $m$ is the slope of the line and $(0, b)$ is the $y$-intercept.

**MATH TERMS**

A linear equation is an equation that can be written in standard form $Ax + By = C$, where $A$, $B$, and $C$ are constants and $A$ and $B$ cannot both be zero.

**MATH TIP**

Linear equations can be written in several forms.
5. Create a table of values for the equation $y = -2x + 9$. Then plot the points and graph the line.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

6. Explain how to find the value of the slope from the table. What is the value of the slope of the line?

7. Explain how to find the $y$-intercept from the table. What is the $y$-intercept?

8. Explain how to find the value of the slope from the graph. What is the value of the slope?

9. Explain how to find the $y$-intercept from the graph. What is the $y$-intercept?
Lesson 6-3
Slope-Intercept Form

Check Your Understanding

10. What are the slope and \( y \)-intercept of the line described by the equation \( y = -\frac{4}{5}x - 10 \)?

11. Write the equation in slope-intercept form of the line that is represented by the data in the table.

\[
\begin{array}{c|c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 & 3 \\
 y & 9 & 7 & 5 & 3 & 1 & -1 \\
\end{array}
\]

12. Write the equation, in slope-intercept form, of the line with a slope of 4 and a \( y \)-intercept of (0, 5).

13. Write an equation of the line graphed below.

Monica gets on an elevator in a skyscraper. The elevator starts to move at a rate of \(-20\) ft/s. After 6 seconds on the elevator, Monica is 350 feet from the ground floor of the building. Use this information for Items 14–16.

14. The rate of the elevator is negative. What does this mean in the situation? What value in the slope-intercept form of an equation does this rate represent?

15. a. How many feet was Monica above the ground when she got on the elevator? Show how you determined your answer.

b. What value in the slope-intercept form does your answer to part a represent?
16. **Model with mathematics.** Write an equation in slope-intercept form for the motion of the elevator since it started to move. What do \( x \) and \( y \) represent?

   a. What does the \( y \)-intercept represent?

   b. Use the equation you wrote to determine, at this rate, how long it will take after Monica enters the elevator for her to exit the elevator on the ground floor. Explain how you found your answer.

**Check Your Understanding**

17. Write the equation \( 3x - 2y = 16 \) in slope-intercept form. Explain your steps.

18. A flowering plant stands 6.5 inches tall when it is placed under a growing light. Its growth is 0.25 inches per day. Today the plant is 11.25 inches tall.

   a. Write an equation in slope-intercept form for the height of the plant since it was placed under the growing light.

   b. In your equation, what do \( x \) and \( y \) represent?

   c. Use the equation to determine how many days ago the plant was placed under the light.

Two other forms of linear equations are point-slope form and standard form. The table on the next page summarizes the three forms of linear equations and shows the linear function in the following graph written in each form.
Lesson 6-3
Slope-Intercept Form

<table>
<thead>
<tr>
<th>Slope-Intercept Form</th>
<th>Point-Slope Form</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = mx + b )</td>
<td>( y - y_1 = m(x - x_1) )</td>
<td>( Ax + By = C )</td>
</tr>
<tr>
<td>where ( m ) is the slope</td>
<td>where ( m ) is the slope of the line and ((x_1, y_1)) is a point on the line</td>
<td>where ( A \geq 0 ), ( A ) and ( B ) are not both zero, and ( A, B, ) and ( C ) are integers whose greatest common factor is 1</td>
</tr>
<tr>
<td>of the line and ((0, b)) is the ( y )-intercept</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The slope of the line shown is \(-\frac{1}{3}\), and the \( y \)-intercept is \((0, 2)\).

\[ y = -\frac{1}{3}x + 2 \]

19. A line has a slope of \(\frac{3}{2}\) and passes through the point \((-4, 5)\).
   a. Write an equation of the line in point-slope form.
   
   
   b. Write an equation of the line in slope-intercept form.
   
   c. Write an equation of the line in standard form.

Check Your Understanding

20. Write the equation \( y = -\frac{6}{5}x - 4 \) in standard form.

21. Write an equation in standard form for the line that is represented by the data in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>9</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>
LESSON 6-3 PRACTICE

22. What are the slope, \( m \), and \( y \)-intercept, \( (0, b) \), of the line described by the equation \( 3x + 6y = 12 \)?

23. Write an equation in slope-intercept form for the line that passes through the points \( (6, -3) \) and \( (0, 2) \).

24. Matt sells used books on the Internet. He has a weekly fee he has to pay for his website. He has graphed his possible weekly earnings, as shown.

   ![Graph of Used Book Internet Business](image)

   a. What is the weekly fee that Matt pays for his website? How do you know?
   b. How much does Matt make for each book sold? How do you know?
   c. Write the equation in slope-intercept form for the line in Matt's graph.
   d. How many books does Matt have to sell to make $30 for the week? Explain.

25. Write an equation of the line with a slope of 0.25 that passes through the point \( (-1, -8) \).

26. **Make sense of problems.** Keisha bought a discount pass at a movie theater. It entitles her to a special discounted admission price for every movie she sees. Keisha wrote an equation that gives the total cost \( y \) of seeing \( x \) movies. In standard form, the equation is \( 7x - 2y = -31 \).
   a. What was the cost of the pass?
   b. What is the discounted admission price for each movie?
ACTIVITY 6 PRACTICE
Answer each item. Show your work.

Lesson 6-1
1. Find $\Delta x$ and $\Delta y$ for each of the following pairs of points.
   a. $(2, 6), (-6, -8)$
   b. $(0, 9), (4, -8)$
   c. $(-3, -3), (7, 10)$
2. Two points on a line are $(-10, 1)$ and $(5, -5)$. If the $y$-coordinate of another point on the line is $-3$, what is the $x$-coordinate?

For Items 3–5, determine the slope of the line that passes through each pair of points.
3. $(-4, 11)$ and $(1, -9)$
4. $(-10, -3)$ and $(-5, 1)$
5. $(-2, -7)$ and $(-8, -4)$
6. Are the three points $(2, 3), (5, 6)$, and $(0, -2)$ on the same line? Explain.
7. Which of the following pairs of points lies on a line with a slope of $-\frac{2}{5}$?
   A. $(4, 0), (-2, 10)$
   B. $(4, 2), (10, 4)$
   C. $(-4, -10), (0, -2)$
   D. $(10, -2), (0, 4)$
8. Juan earns $7 per hour plus $20 per week making picture frames.
   a. Write a function $g(x)$ for Juan’s total earnings if he works $x$ hours in one week.
   b. Without graphing the function, determine the slope.
   c. Describe the meaning of the slope within the context of Juan’s job.
9. Which of the following is not a linear function?
   A. $(4, -6), (7, -12), (8, -14), (10, -18), (2, -2)$
   B. $(-2, -6), (1, 0), (4, -30), (0, 2), (7, -96)$
   C. $(-4, 9), (0, 7), (2, 6), (6, 4), (8, 3)$
   D. $(2, 18), (6, 50), (-3, -22), (0, 2), (3, 26)$
10. The slope of a line is 0. It passes through the point $(-3, 4)$. Identify two other points on the line. Justify your answers.

Lesson 6-2
11. The value of $y$ varies directly as $x$ and $y = 125$ when $x = 25$. What is the value of $y$ when $x = 2$?
12. Which is the graph of a direct variation?
   A. [Graph A]
   B. [Graph B]
   C. [Graph C]
   D. [Graph D]
13. The amount of gas left in the gas tank of a car varies indirectly with the number of miles driven. There are 9 gallons of gas left after 24 miles. How much gas is left after the car is driven 120 miles?
Jeremy collected the following data on stacking chairs. Use the data for Items 14 and 15.

<table>
<thead>
<tr>
<th>Number of Chairs</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
</tr>
</tbody>
</table>

14. Write a linear function that models the data.
15. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack? Justify your answer.

Lesson 6-3

16. Write the equation of a line in slope-intercept form that has a slope of $-8$ and a y-intercept of $(0, 3)$.

17. Find the slope and the y-intercept of the line whose equation is $-5x + 3y - 8 = 0$.

18. Which of the following is the slope-intercept form of the equation of the line in the graph?

19. What is the equation in point-slope form of the line that passes through the points $(2, -3)$ and $(-5, 8)$?

20. David is ordering tea from an online store. Black tea costs $0.80 per ounce and green tea costs $1.20 per ounce. He plans to spend a total of $12 on the two types of tea.
   a. Write an equation that represents the different amounts of black tea, $x$, and green tea, $y$, that David can buy.
   b. Graph the equation.
   c. What is the x-intercept? What does it represent?
   d. Suppose David decides to buy 10 ounces of black tea. How many ounces of green tea will he buy?

21. Which is a true statement about the line $x - 4y = 8$?
   A. The x-intercept of the line is $(2, 0)$.
   B. The y-intercept of the line is $(0, 2)$.
   C. The slope of the line is $\frac{1}{4}$.
   D. The line passes through the origin.

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

22. Aidan stated that for any value of $b$, the line $y = 2x + b$ has the same slope as the line that passes through $(2, 5)$ and $(-1, -1)$. Do you agree with Aidan? Explain why or why not.